Time-Domain Analysis of Transmission Line Circuits

(Part 3)

Dr. José Ernesto Rayas Sánchez

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Outline

- Bouncing diagrams for TLs with ideal excitation pulses and resistive terminations
- Example of a TL with resistive terminations and ideal pulse excitation
- Transients in TLs with capacitive loads
- Examples of TLs with capacitive load (over-driven and under-driven cases)
- Conclusions on TLs with capacitive loads

Dr. J.E. Rayas Sánchez

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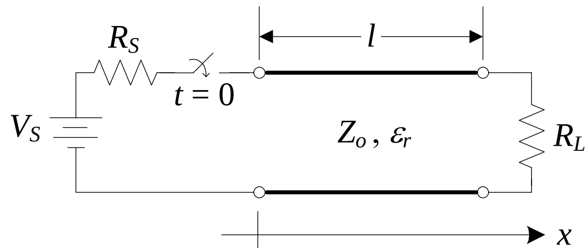
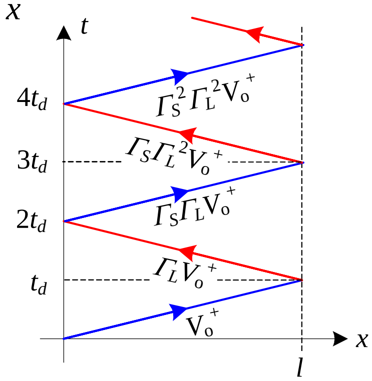
Lattice Diagrams for a DC Excitation



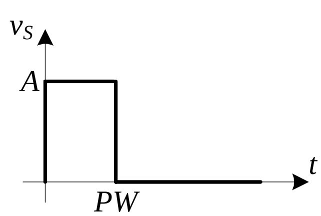
$$\Gamma_S = \frac{R_S - Z_o}{R_S + Z_o} \qquad \Gamma_L = \frac{R_L - Z_o}{R_L + Z_o}$$

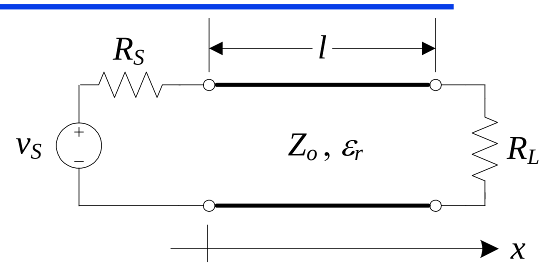
$$V_o^+ = \frac{V_S Z_o}{R_S + Z_o} \qquad v_p = \frac{c}{\sqrt{\varepsilon_e}}$$

$$t_d = \frac{l}{v_p}$$



Lattice Diagrams for Pulses



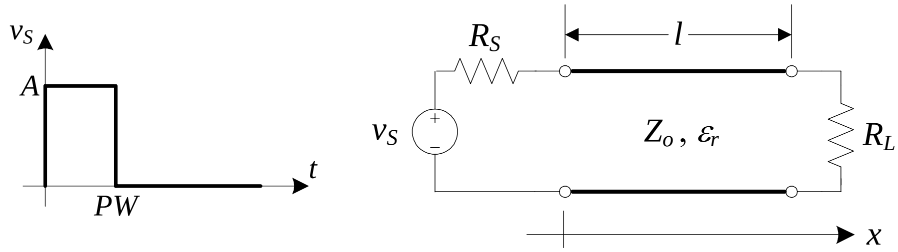


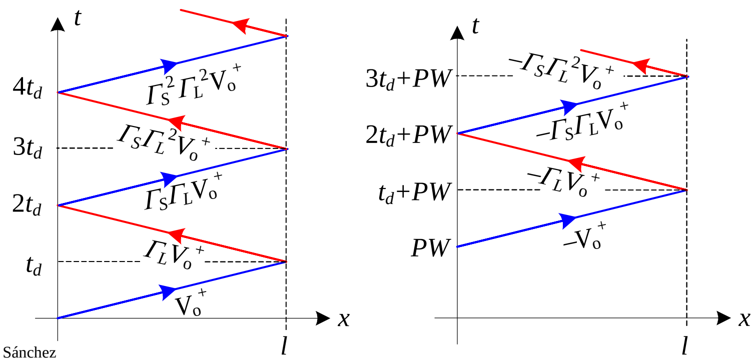
$$v_S = v_{ON} + v_{OFF}$$

where $v_{ON} = Au(t)$ $\quad v_{OFF} = -Au(t - PW)$

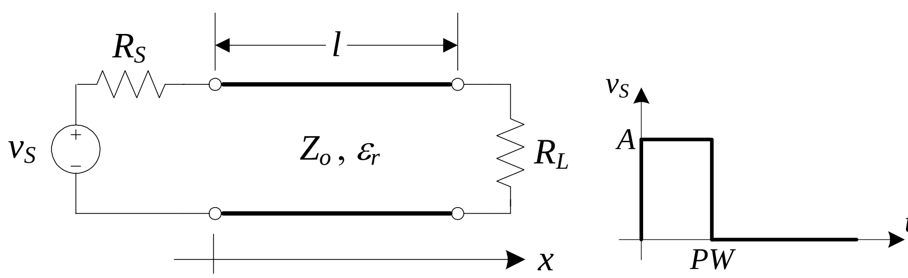
- We built two lattice diagrams, one for v_{ON} and one for v_{OFF}
- The lattice diagram for v_{OFF} is a negative and delayed version of that one for v_{ON}
- The resultant signals are obtained by adding both lattice diagrams

Lattice Diagrams for Pulses (cont)





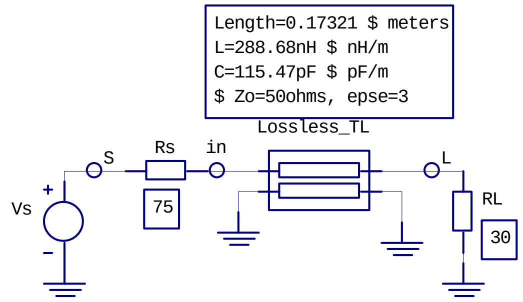
Example of TL with a Pulse Excitation



$R_S = 75\Omega$; $R_L = 30\Omega$; $A = 3\text{V}$; $PW = 2.5\text{ns}$

$Z_o = 50\Omega$; $\varepsilon_e = 3$; $l = 17.321\text{cm}$

Example Simulated with Aplac

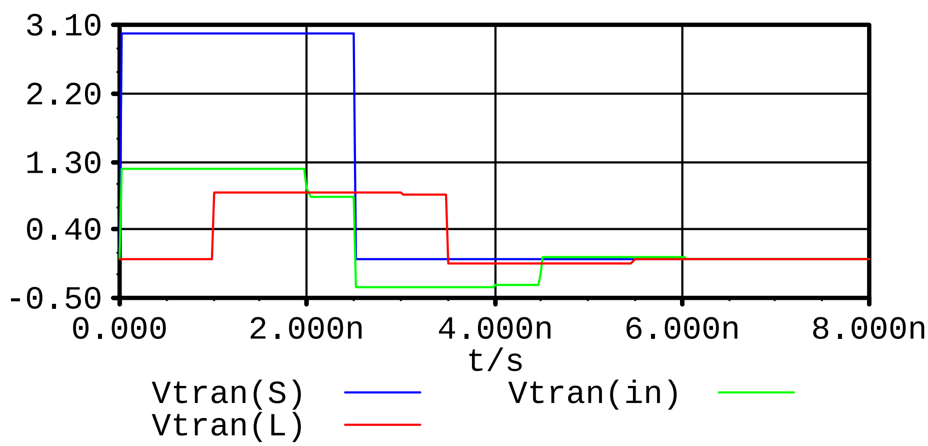



```
$      DCLevel Amp Delay RiseT  FallT  PWidth Period
PULSE    0     3V    0   0.01ps 0.01ps 2.5ns  9ns
```

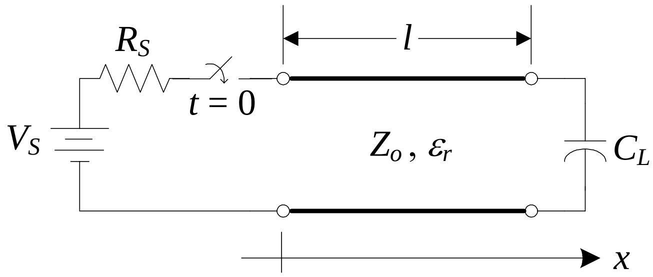
```
Sweep
"Lossless Transmission Line"
LOOP 300 TIME LIN 0 8ns
W=0 grid Y "" "" -0.5 3.1

Show W=0 Y=Vtran(S) Y=Vtran(in) Y=Vtran(L)
EndSweep
```


Example Simulated with Aplac

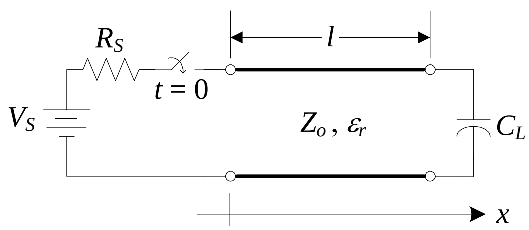


Transients in TLs with Capacitive Loads



- C_L can be treated as a time-dependent impedance
- Initially, C_L is discharged, hence $\Gamma_L = -1$
- At the steady state, C_L is fully charged, hence $\Gamma_L = +1$
- At any other time, Γ_L can not be calculated

Transients in TLs with Capacitive Load ($R_S = Z_o$)



$$R_S = Z_o \qquad t_d = \frac{l}{v_p}$$

$$\tau = Z_o C_L$$

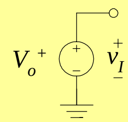
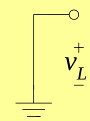
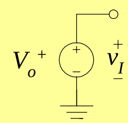
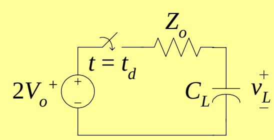
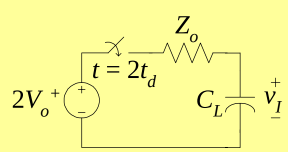
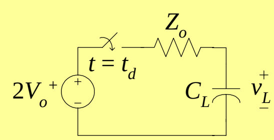
$$v(x=0,t) = v_I(t)$$

$$v(x=l,t) = v_L(t)$$

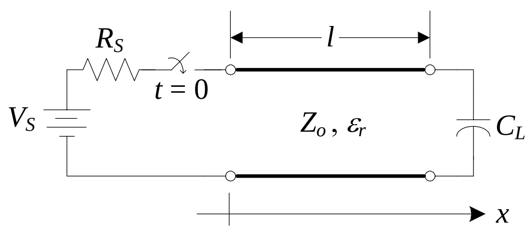
time	$v_I(t)$	$v_L(t)$
$0 \le t \le t_d$	$V_o^+ = \dfrac{V_S Z_o}{R_S + Z_o} = \dfrac{V_S}{2}$	0
$t_d \le t < 2t_d$	$V_S / 2$	$V_S[1 - e^{-(t-t_d)/\tau}]$
$t = 2t_d$	0	$V_S[1 - e^{-t_d/\tau}]$
$t \ge 2t_d$	$V_S[1 - e^{-(t-2t_d)/\tau}]$	$V_S[1 - e^{-(t-t_d)/\tau}]$

Dr. J. E. Rayas Sánchez
http://iteso.mx/~erayas erayas@iteso.mx

Transients in TLs with Capacitive Load ($R_S = Z_o$)

time	$v_I(t)$	$v_L(t)$
$0 \le t \le t_d$		
$t_d \le t < 2t_d$		
$t = 2t_d$	0	$V_S[1-e^{-t_d/\tau}]$
$t \ge 2t_d$		

TL with Capacitive Load – Example 1



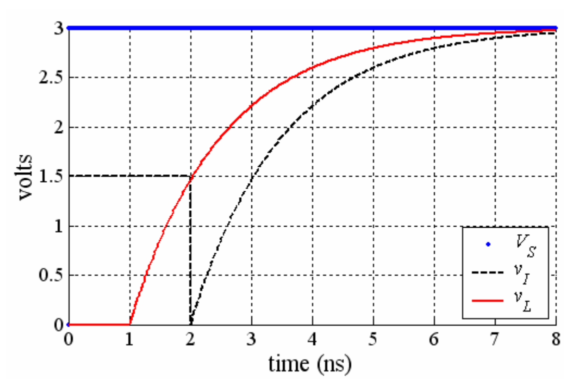
V_S = 3V; R_S = 50Ω
Z_o = 50Ω; ε_e = 3
l = 17.321cm
C_L = 30pF

$$V_o^+ = \frac{V_S}{2} = 1.5\text{V}$$

$$v_I(2t_d) = 0$$

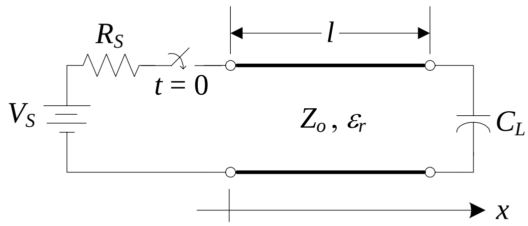
$$2V_o^+[1-e^{-t_d/\tau}] = 1.46\text{V}$$

$$2V_o^+[1-e^{-2t_d/\tau}] = 2.21\text{V}$$



Dr. J. E. Rayas Sánchez
http://iteso.mx/~erayas erayas@iteso.mx

TL with Capacitive Load – Example 2

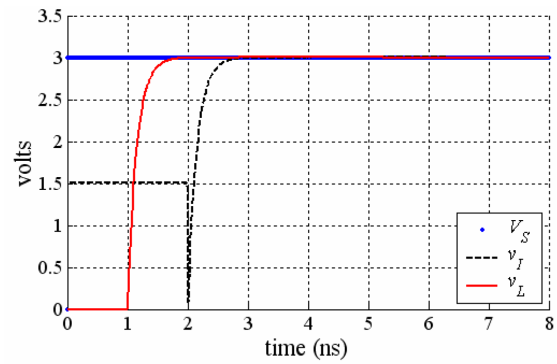


$V_S = 3\text{V};\ R_S = 50\Omega$

$Z_o = 50\Omega;\ \varepsilon_e = 3$

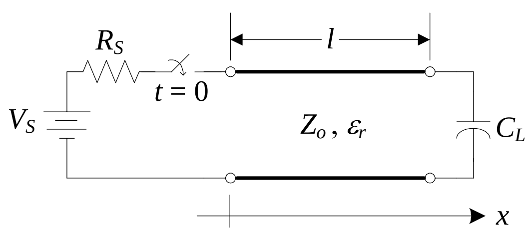
$l = 17.321\text{cm}$

$C_L = 3\text{pF}$



Dr. J.E. Rayas Sánchez

TLs with Capacitive Load and Unmatched Source



$R_S \neq Z_o \qquad t_d = \dfrac{l}{v_p}$

$\tau = Z_o C_L$

$v(x = 0, t) = v_I(t)$

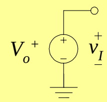
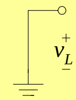
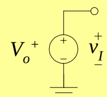
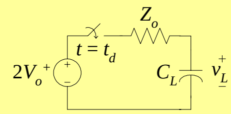
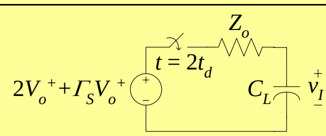
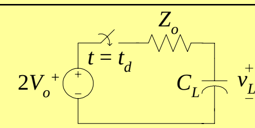
$v(x = l, t) = v_L(t)$

time	$v_I(t)$	$v_L(t)$
$0 \le t \le t_d$	$V_o^+ = \dfrac{V_S Z_o}{R_S + Z_o}$	0
$t_d \le t < 2t_d$	V_o^+	$2V_o^+[1 - e^{-(t-t_d)/\tau}]$
$t = 2t_d$	$V_o^+ - V_o^+ - \Gamma_S V_o^+ = -\Gamma_S V_o^+$	$2V_o^+[1 - e^{-t_d/\tau}]$
$2t_d \le t < 3t_d$	$[2V_o^+ + 2\Gamma_S V_o^+][1 - e^{-(t-2t_d)/\tau}] - \Gamma_S V_o^+$	$2V_o^+[1 - e^{-(t-t_d)/\tau}]$
$t = 3t_d$	$[2V_o^+ + 2\Gamma_S V_o^+][1 - e^{-t_d/\tau}] - \Gamma_S V_o^+$	$2V_o^+[1 - e^{-2t_d/\tau}]$
$t > 3t_d$	?	?

Dr. J.E. Rayas S

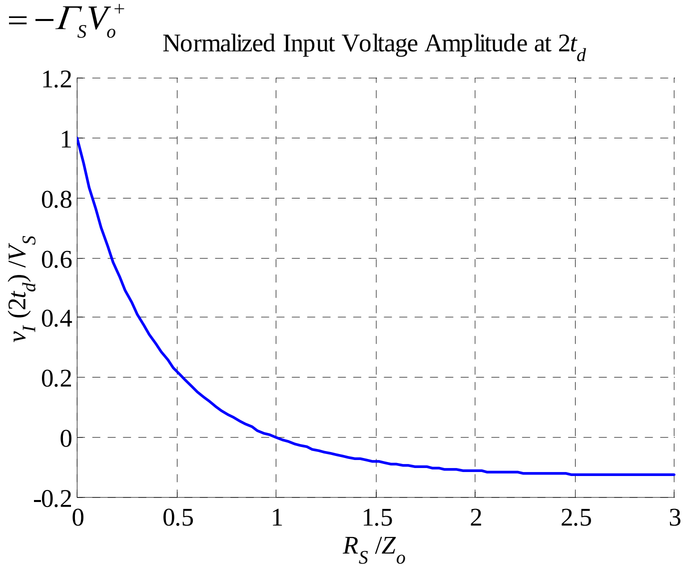
Dr. J. E. Rayas Sánchez
http://iteso.mx/~erayas erayas@iteso.mx

TLs with Capacitive Load and Unmatched Source

time	$v_I(t)$	$v_L(t)$
$0 \le t \le t_d$		
$t_d \le t < 2t_d$		
$t = 2t_d$	$-\Gamma_S V_o^+$	$2V_o^+[1 - e^{-t_d/\tau}]$
$2t_d \le t < 3t_d$		
$t = 3t_d$	$[2V_o^+ + 2\Gamma_S V_o^+][1 - e^{-t_d/\tau}] - \Gamma_S V_o^+$	$2V_o^+[1 - e^{-2t_d/\tau}]$
$t > 3t_d$	?	?

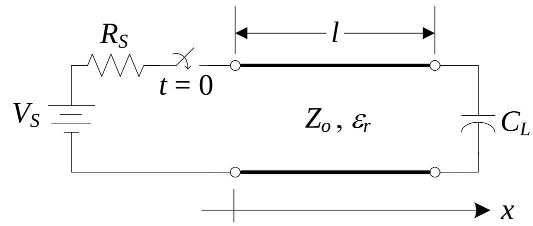
TLs with Capacitive Load and Unmatched Source

$$v_I(2t_d) = -\Gamma_S V_o^+$$



Dr. J. E. Rayas Sánchez
http://iteso.mx/~erayas erayas@iteso.mx

TL with Capacitive Load – Example 3



V_S = 3V; R_S = 25Ω
Z_o = 50Ω; ε_e = 3
l = 17.321cm
C_L = 30pF

$$V_o^+ = V_S Z_o/(R_S + Z_o) = 2\text{V}$$

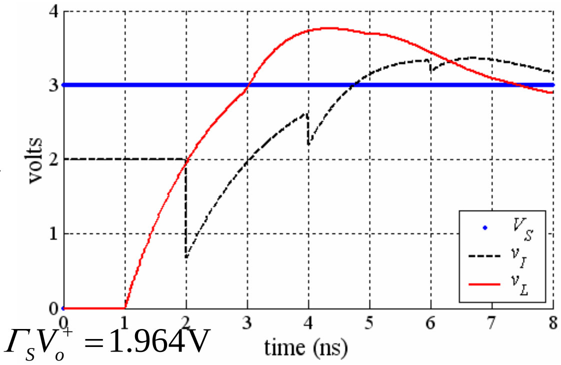
$$\Gamma_S = \frac{R_S - Z_o}{R_S + Z_o} = -0.333$$

$$v_I(2t_d) = -\Gamma_S V_o^+ = 0.667\text{V}$$

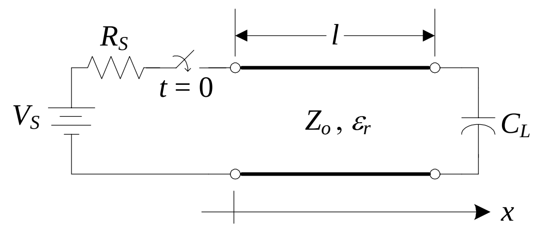
$$2V_o^+[1 - e^{-t_d/\tau}] = 1.946\text{V}$$

$$2V_o^+[1 - e^{-2t_d/\tau}] = 2.946\text{V}$$

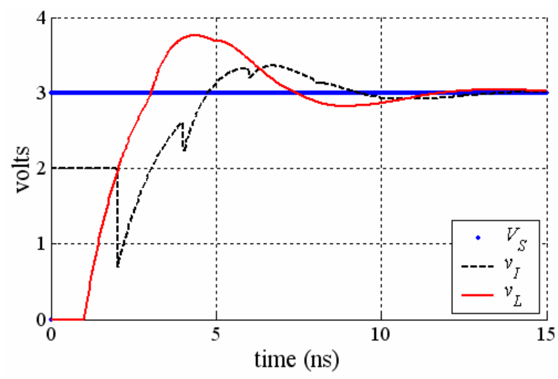
$$[2V_o^+ + 2\Gamma_S V_o^+][1 - e^{-t_d/\tau}] - \Gamma_S V_o^+ = 1.964\text{V}$$



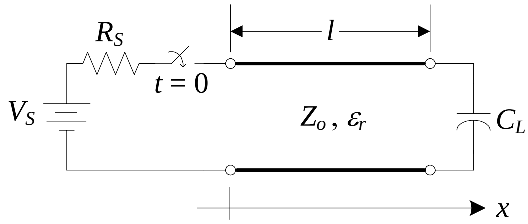
TL with Capacitive Load – Example 3b



V_S = 3V; R_S = 25Ω
Z_o = 50Ω; ε_e = 3
l = 17.321cm
C_L = 30pF



TL with Capacitive Load – Example 4

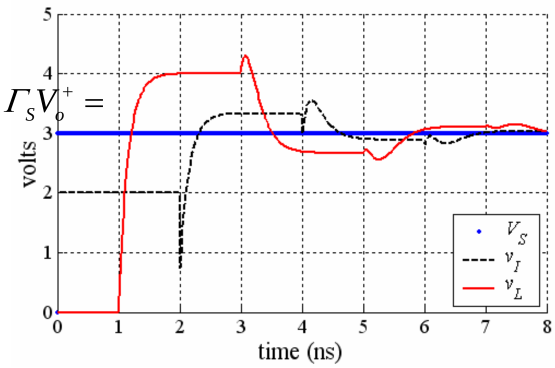


$V_S = 3\text{V};\ R_S = 25\Omega$
$Z_o = 50\Omega;\ \varepsilon_e = 3$
$l = 17.321\text{cm}$
$C_L = 3\text{pF}$

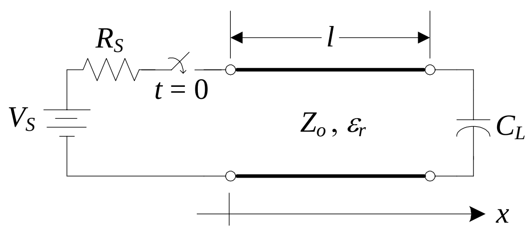
$$2V_o^+[1-e^{-\infty/\tau}] = 2V_o^+ = 4\text{V}$$

$$[2V_o^+ + 2\Gamma_S V_o^+][1-e^{-\infty/\tau}] - \Gamma_S V_o^+ =$$

$$2V_o^+ + \Gamma_S V_o^+ = 3.333\text{V}$$



TL with Capacitive Load – Example 5



$V_S = 3\text{V};\ R_S = 10\Omega$
$Z_o = 50\Omega;\ \varepsilon_e = 3$
$l = 17.321\text{cm}$
$C_L = 3\text{pF}$

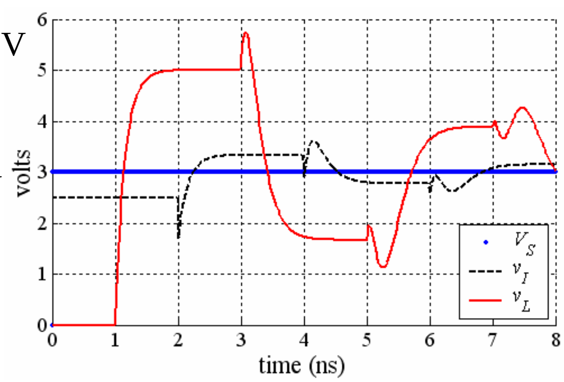
$$V_o^+ = V_S Z_o/(R_S + Z_o) = 2.5\text{V}$$

$$\Gamma_S = \frac{R_S - Z_o}{R_S + Z_o} = -0.6667$$

$$v_I(2t_d) = -\Gamma_S V_o^+ = 1.667\text{V}$$

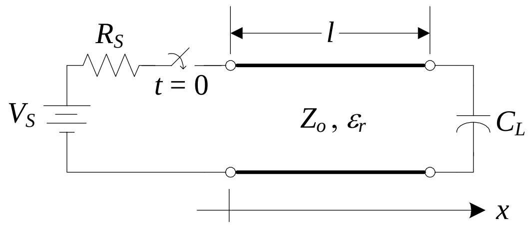
$$2V_o^+[1-e^{-\infty/\tau}] = 2V_o^+ = 5\text{V}$$

$$2V_o^+ + \Gamma_S V_o^+ = 3.331\text{V}$$

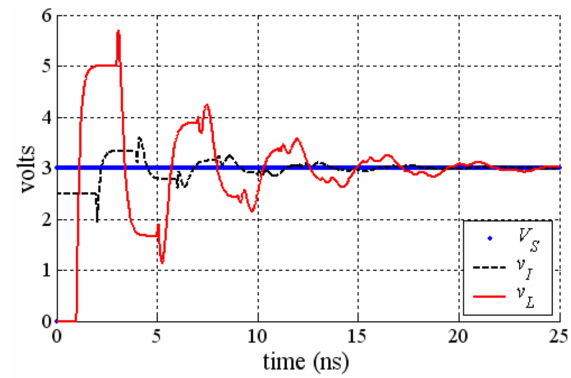


Dr. J. E. Rayas Sánchez
http://iteso.mx/~erayas erayas@iteso.mx

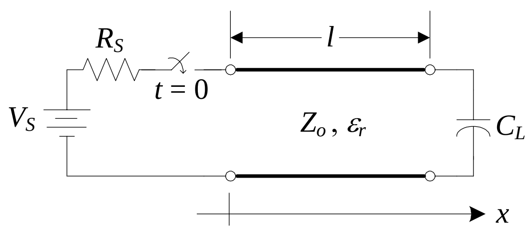
TL with Capacitive Load – Example 5b



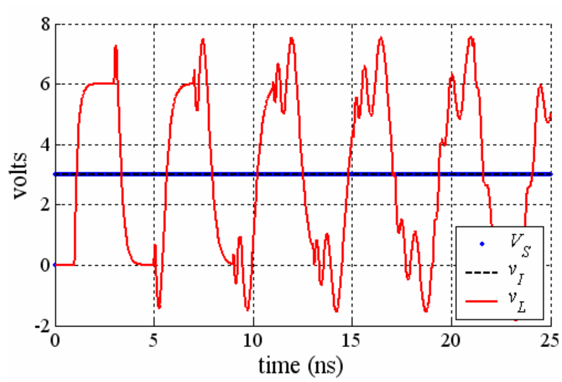
V_S = 3V; R_S = 10Ω
Z_o = 50Ω; ε_e = 3
l = 17.321cm
C_L = 3pF



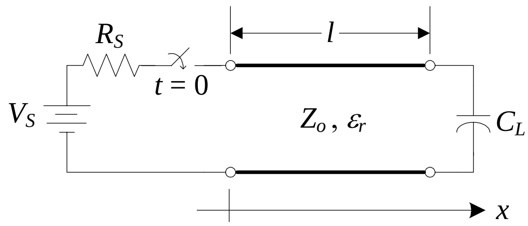
TL with Capacitive Load – Example 6



V_S = 3V; R_S = 0.01Ω
Z_o = 50Ω; ε_e = 3
l = 17.321cm
C_L = 3pF



TL with Capacitive Load – Example 6b

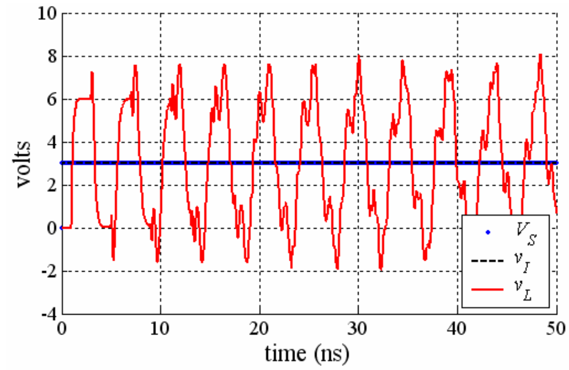


V_S = 3V; R_S = 0.01Ω

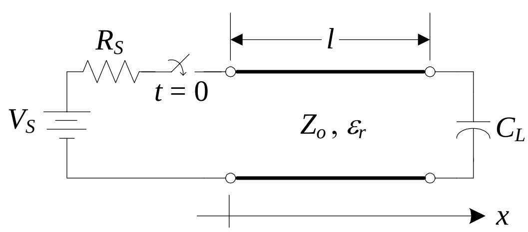
Z_o = 50Ω; ε_e = 3

l = 17.321cm

C_L = 3pF



TL with Capacitive Load – Example 6c

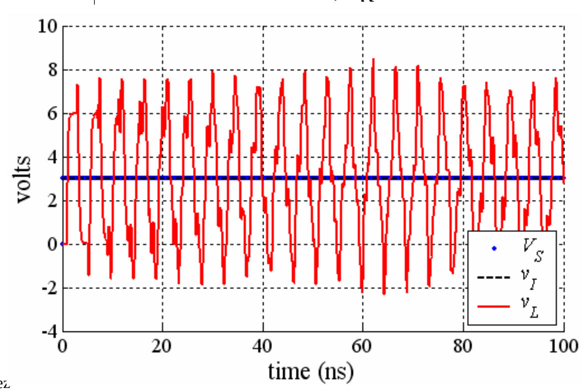


V_S = 3V; R_S = 0.01Ω

Z_o = 50Ω; ε_e = 3

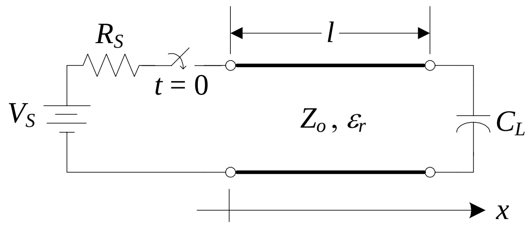
l = 17.321cm

C_L = 3pF



Dr. J. E. Rayas Sánchez
http://iteso.mx/~erayas erayas@iteso.mx

TL with Capacitive Load – Example 6d

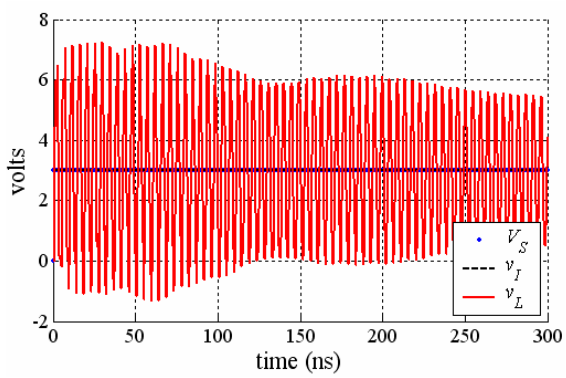


V_S = 3V; R_S = 0.01Ω

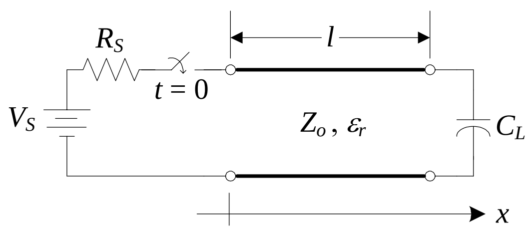
Z_o = 50Ω; ε_e = 3

l = 17.321cm

C_L = 3pF



TL with Capacitive Load – Example 6e

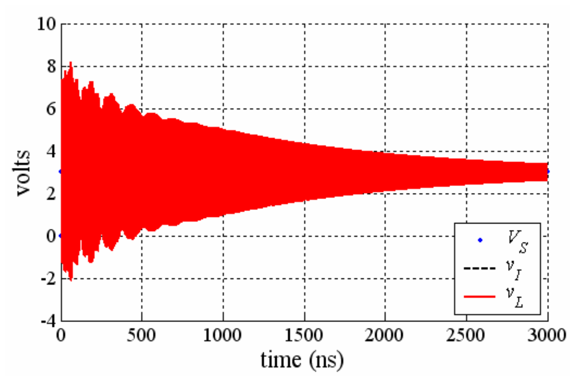


V_S = 3V; R_S = 0.01Ω

Z_o = 50Ω; ε_e = 3

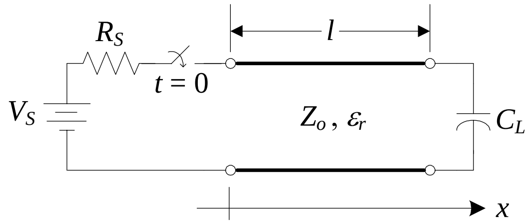
l = 17.321cm

C_L = 3pF

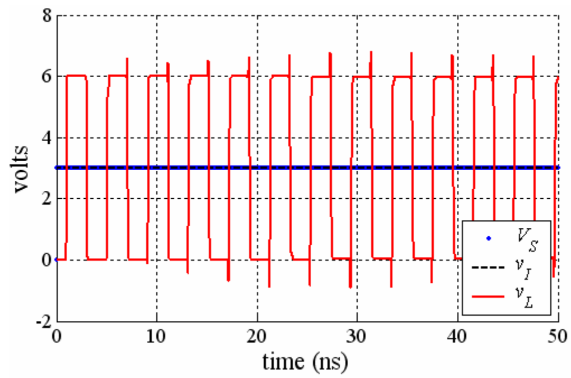


Dr. J. E. Rayas Sánchez
http://iteso.mx/~erayas erayas@iteso.mx

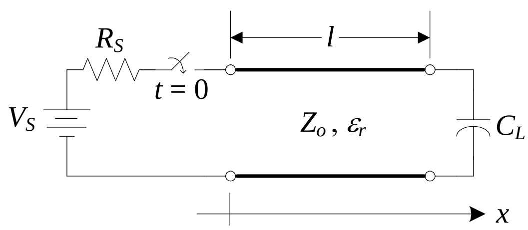
TL with Capacitive Load – Example 7



$V_S = 3\text{V};\ R_S = 0.01\Omega$
$Z_o = 50\Omega;\ \varepsilon_e = 3$
$l = 17.321\text{cm}$
$C_L = 0.3\text{pF}$

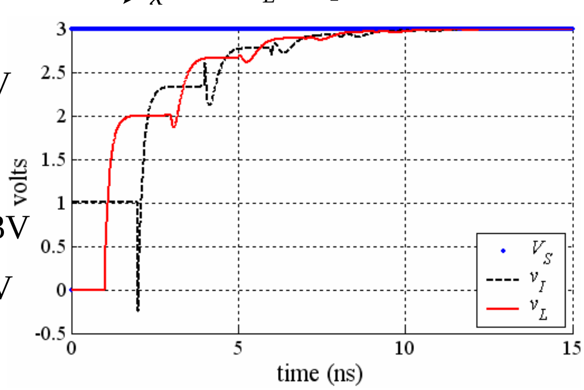


TL with Capacitive Load – Example 8

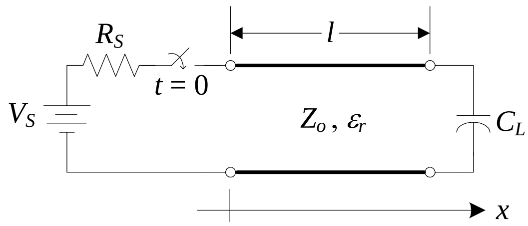


$V_S = 3\text{V};\ R_S = 100\Omega$
$Z_o = 50\Omega;\ \varepsilon_e = 3$
$l = 17.321\text{cm}$
$C_L = 3\text{pF}$

$$V_o^+ = V_S Z_o / (R_S + Z_o) = 1\text{V}$$
$$\Gamma_S = \frac{R_S - Z_o}{R_S + Z_o} = 0.333$$
$$v_I(2t_d) = -\Gamma_S V_o^+ = -0.333\text{V}$$
$$2V_o^+[1 - e^{-\infty/\tau}] = 2V_o^+ = 2\text{V}$$
$$2V_o^+ + \Gamma_S V_o^+ = 2.333\text{V}$$



TL with Capacitive Load – Example 9

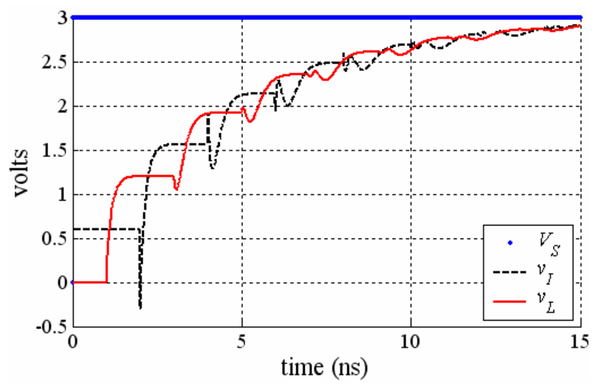


V_S = 3V; R_S = 200Ω

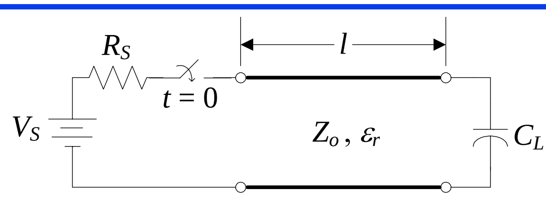
Z_o = 50Ω; ε_e = 3

l = 17.321cm

C_L = 3pF



Transients in TLs with Capacitive Loads



SOME CONCLUSIONS:

- Lattice diagrams can not be directly applied
- If $\Gamma_S = 0$ analytical solutions are easy, otherwise the problem becomes too complex for $t > 3t_d$ (simulation required)
- If $\Gamma_S > 0$, the line behaves as an under-driven line
- If $\Gamma_S < 0$, there will be overshooting voltages at the load (over-driven line)
- If $\Gamma_S << 0$, there will be significant oscillations at the load

Dr. J. E. Rayas Sánchez
http://iteso.mx/~erayas   erayas@iteso.mx