Time-Domain Analysis of Transmission Line Circuits

(Part 2)

Dr. José Ernesto Rayas Sánchez

Outline

- Usual definition of under-driven and over-driven lines
- Conditions for under-driven lines
- Conditions for over-driven lines
- Examples of under-driven and over-driven lines
- Bouncing diagrams for multiple sections of transmission lines
- Example of transient response in two sections of transmission lines

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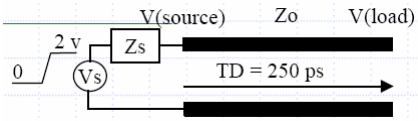
Example – Underdriven Transmission Line



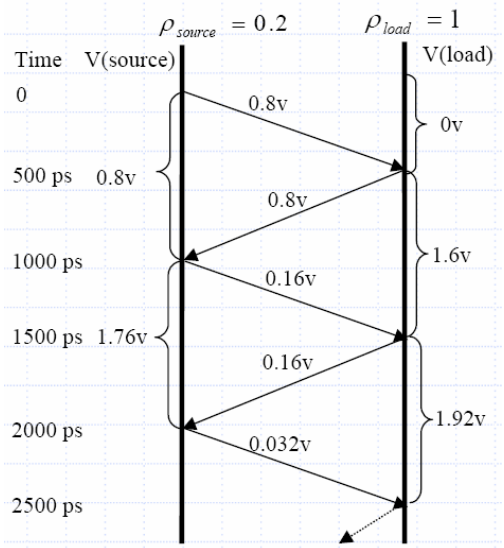
Assume Zs=75 ohms
Zo=50ohms
Vs=0-2 volts

$$V_{initial} = Vs\frac{Zo}{Zs+Zo} = (2)\left(\frac{50}{75+50}\right) = 0.8$$

$$\rho_{source} = \frac{Zs-Zo}{Zs+Zo} = \frac{75-50}{75+50} = 0.2$$

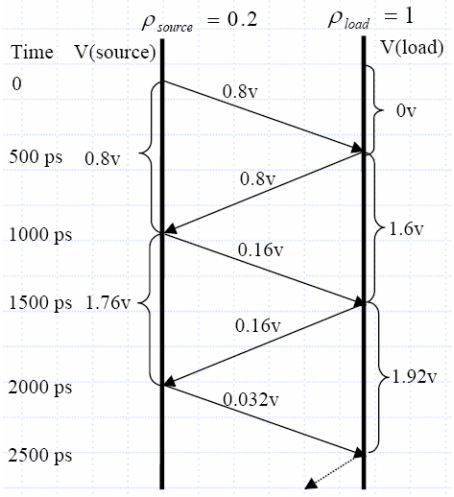
$$\rho_{load} = \frac{Zl-Zo}{Zl+Zo} = \frac{\infty-50}{\infty+50} = 1$$

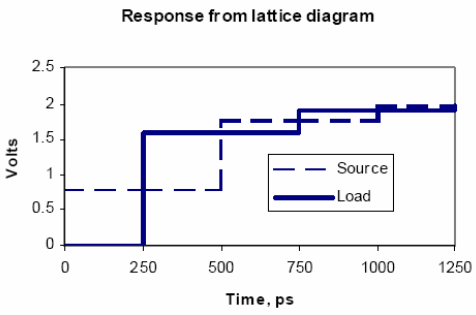
$$(\Gamma_S > 0)$$



 (S.H. Hall, G.W. Hall and J.A. McCall, High-Speed Digital System Design, Wiley, 2000)

Example – Underdriven Transmission Line (cont)

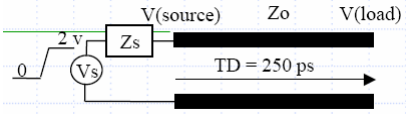




$$(\Gamma_S > 0)$$

 (S.H. Hall, G.W. Hall and J.A. McCall, High-Speed Digital System Design, Wiley, 2000)

Example – Overdriven Transmission Line



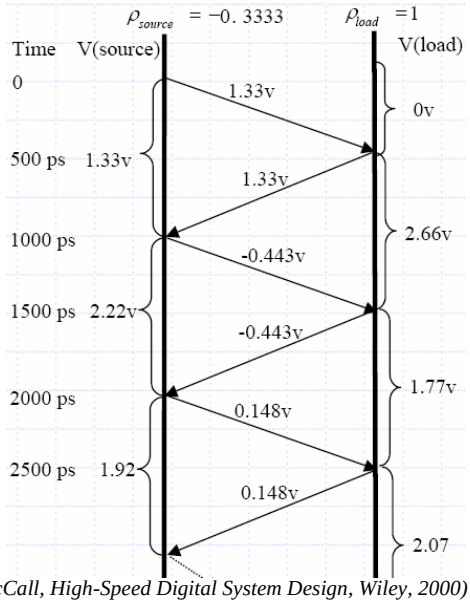
Assume Zs=25 ohms
Zo=50ohms
Vs=0-2 volts

$$V_{initial} = Vs\frac{Zo}{Zs+Zo} = (2)\left(\frac{50}{25+50}\right) = 1.3333$$

$$\rho_{source} = \frac{Zs-Zo}{Zs+Zo} = \frac{25-50}{25+50} = -0.33333$$

$$\rho_{load} = \frac{Zl-Zo}{Zl+Zo} = \frac{\infty-50}{\infty+50} = 1$$

$$(\Gamma_S < 0)$$



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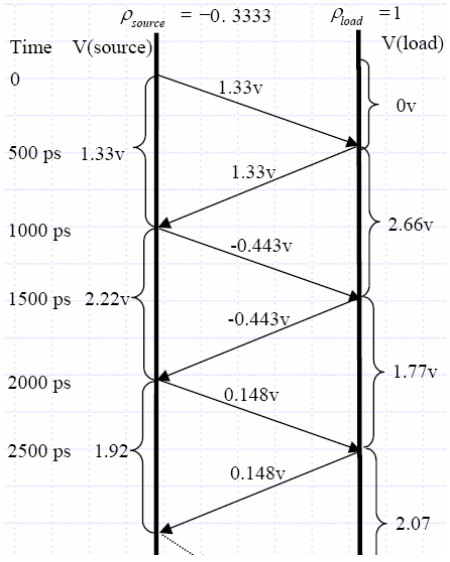
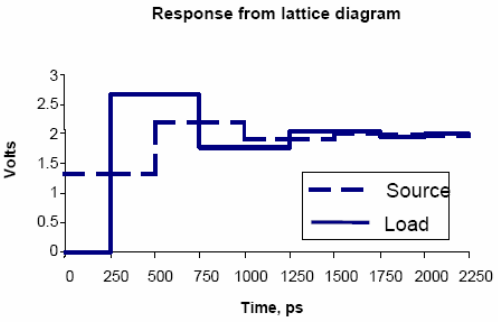
Example – Overdriven Transmission Line

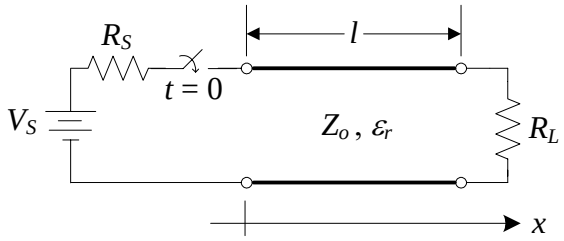




$$(\Gamma_S < 0)$$

 (S.H. Hall, G.W. Hall and J.A. McCall, High-Speed Digital System Design, Wiley, 2000)

Underdriven and Overdriven Lines Revised



$$\Gamma_L = \frac{R_L - Z_o}{R_L + Z_o} \qquad \Gamma_S = \frac{R_S - Z_o}{R_S + Z_o}$$

$$V_o^+ = \frac{V_S Z_o}{R_S + Z_o}$$

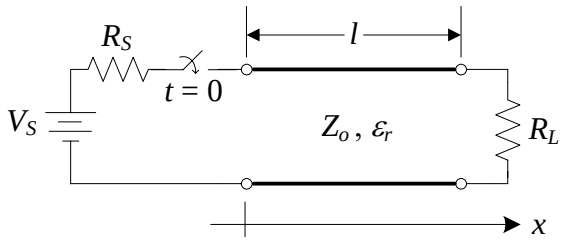
Let $v(x = l, t = t_d) = v_d$
$v(x = l, t \to \infty) = v_\infty$

A line is underdriven if $v_d < v_\infty$
A line is overdriven if $v_d > v_\infty$

$$v_\infty = \frac{V_S R_L}{R_S + R_L} \qquad v_d = V_o^+ (1 + \Gamma_L)$$

$$v_d = \frac{V_S Z_o}{R_S + Z_o}\left(1 + \frac{R_L - Z_o}{R_L + Z_o}\right) = \frac{V_S Z_o}{R_S + Z_o}\left(\frac{2R_L}{R_L + Z_o}\right)$$

Conditions for Underdriven Lines



$$v_\infty = \frac{V_S R_L}{R_S + R_L}$$

$$v_d = \frac{V_S Z_o}{R_S + Z_o}\left(\frac{2R_L}{R_L + Z_o}\right)$$

$v_d < v_\infty$ if

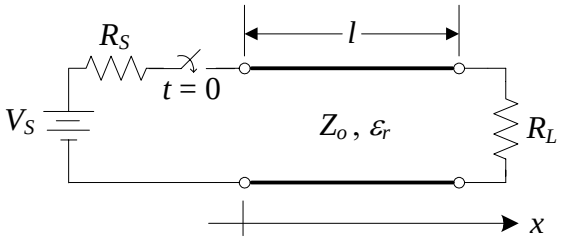
$$\frac{2Z_o}{R_S + Z_o} < \frac{R_L + Z_o}{R_L + R_S}$$

if $R_S > Z_o$, let $R_S = aZ_o$, $a > 1$
$(R_L - Z_o) < a(R_L - Z_o)$
$\rightarrow R_L > Z_o$

if $R_S < Z_o$, let $R_S = aZ_o$, $0 < a < 1$
$(R_L - Z_o) < a(R_L - Z_o)$
$\rightarrow R_L < Z_o$

Hence, $\Gamma_S \Gamma_L > 0 \leftrightarrow$ underdriven line

Conditions for Overdriven Lines



$$v_\infty = \frac{V_S R_L}{R_S + R_L}$$

$$v_d = \frac{V_S Z_o}{R_S + Z_o}\left(\frac{2R_L}{R_L + Z_o}\right)$$

$v_d > v_\infty$ if

$$\frac{2Z_o}{R_S + Z_o} > \frac{R_L + Z_o}{R_L + R_S}$$

if $R_S > Z_o$, let $R_S = aZ_o$, $a > 1$

$(R_L - Z_o) > a(R_L - Z_o)$

$\rightarrow R_L < Z_o$

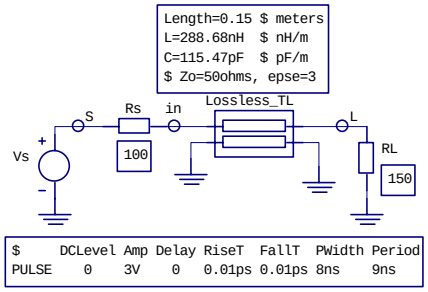
if $R_S < Z_o$, let $R_S = aZ_o$, $0 < a < 1$

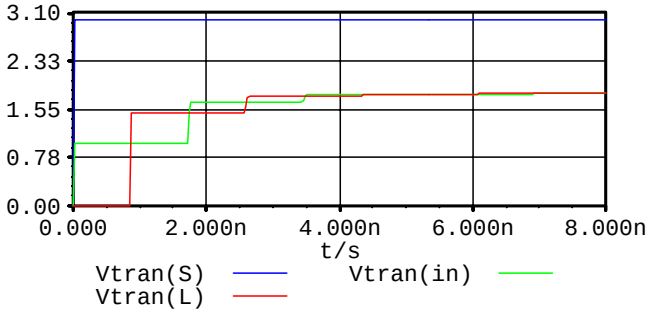
$(R_L - Z_o) > a(R_L - Z_o)$

$\rightarrow R_L > Z_o$

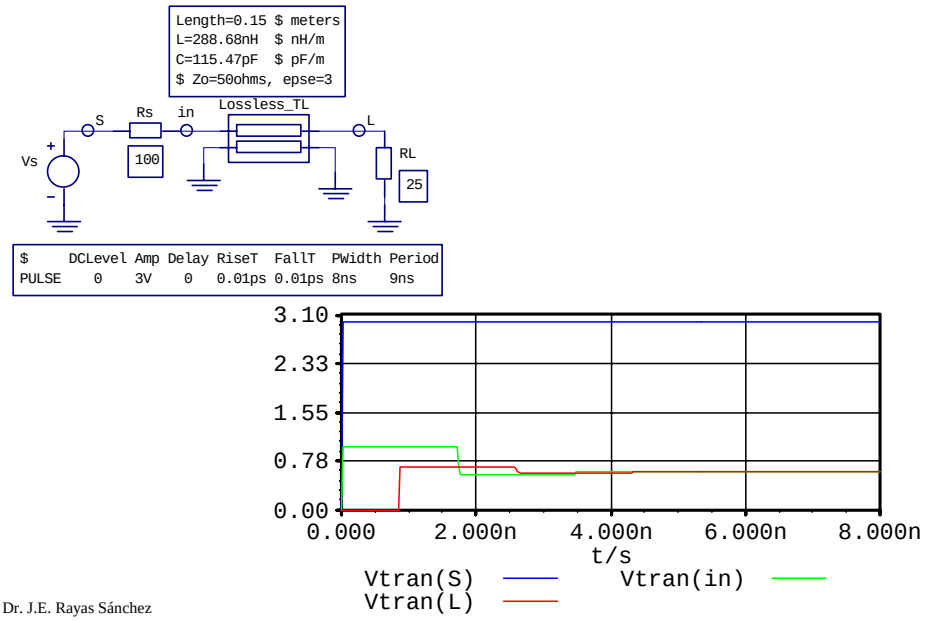
Hence, $\Gamma_S \Gamma_L < 0$ $\leftrightarrow$ overdriven line

Example 1: $\Gamma_S > 0$ and $\Gamma_L > 0$

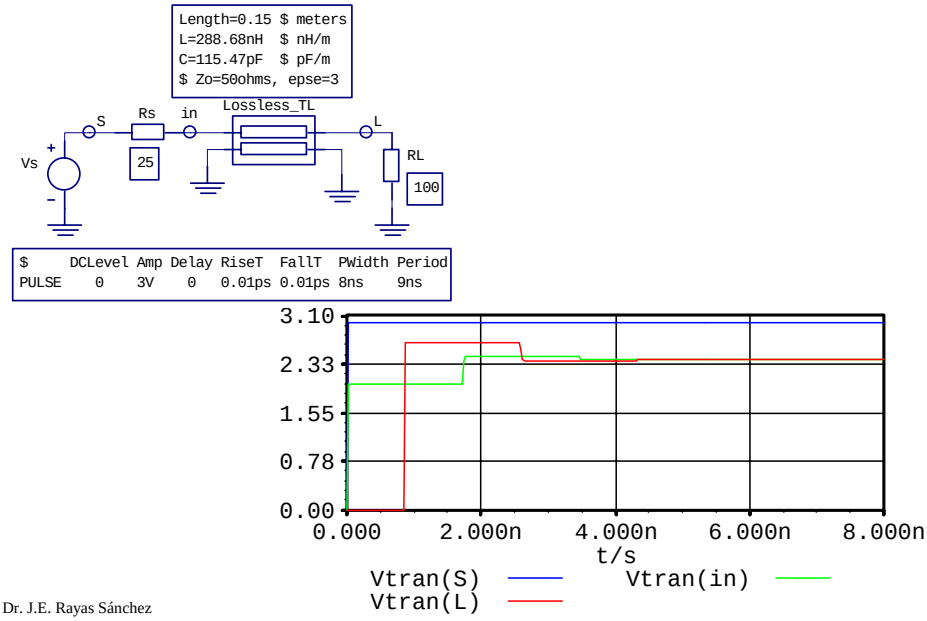




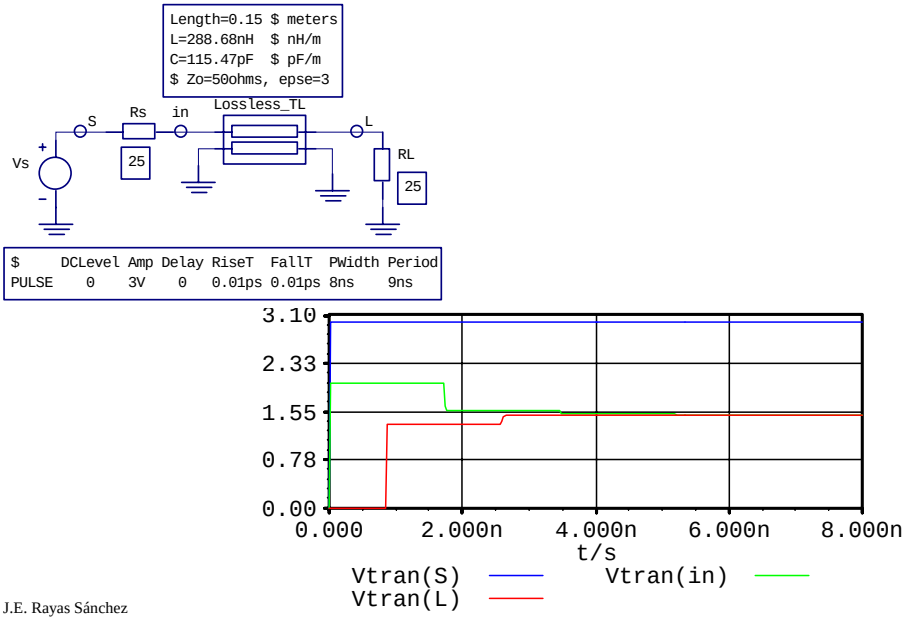
Example 2: $\Gamma_S > 0$ and $\Gamma_L < 0$



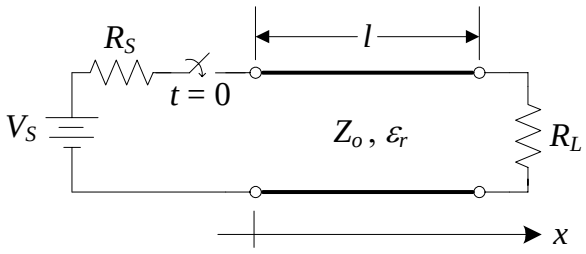
Example 3: $\Gamma_S < 0$ and $\Gamma_L > 0$



Example 4: $\Gamma_S < 0$ and $\Gamma_L < 0$



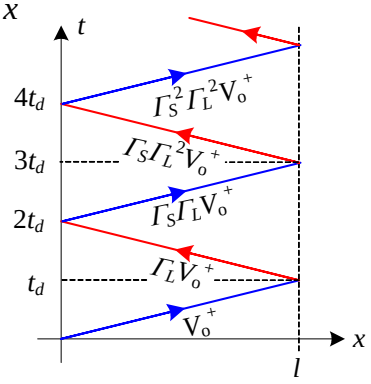
Lattice Diagrams for One TL Section



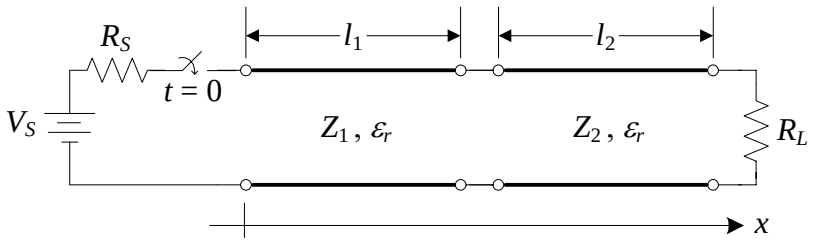
$$\Gamma_S = \frac{R_S - Z_o}{R_S + Z_o} \qquad \Gamma_L = \frac{R_L - Z_o}{R_L + Z_o}$$

$$V_o^+ = \frac{V_S Z_o}{R_S + Z_o} \qquad v_p = \frac{c}{\sqrt{\varepsilon_e}}$$

$$t_d = \frac{l}{v_p}$$



Lattice Diagrams for Multiple TL Sections

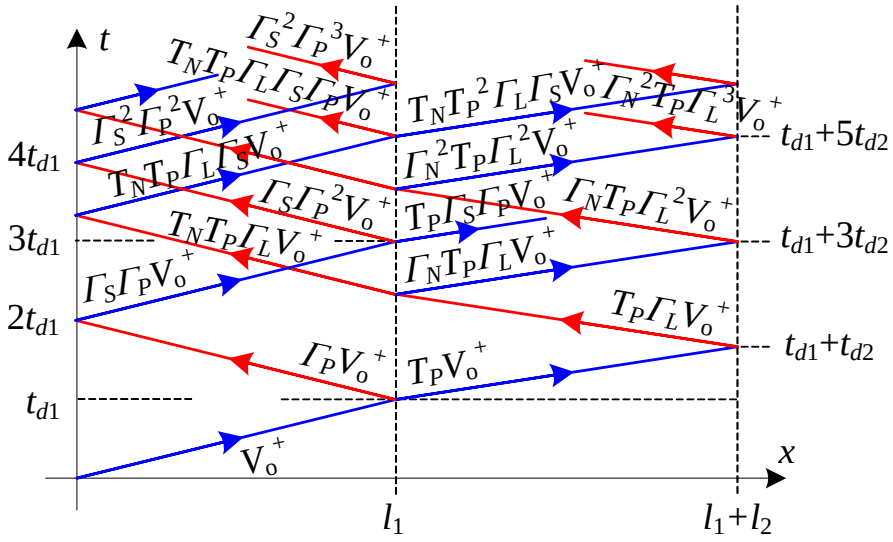


$$\Gamma_S = \frac{R_S - Z_1}{R_S + Z_1} \qquad \Gamma_L = \frac{R_L - Z_2}{R_L + Z_2} \qquad \Gamma_P = \frac{Z_2 - Z_1}{Z_2 + Z_1} \qquad \Gamma_N = \frac{Z_1 - Z_2}{Z_1 + Z_2} = -\Gamma_P$$

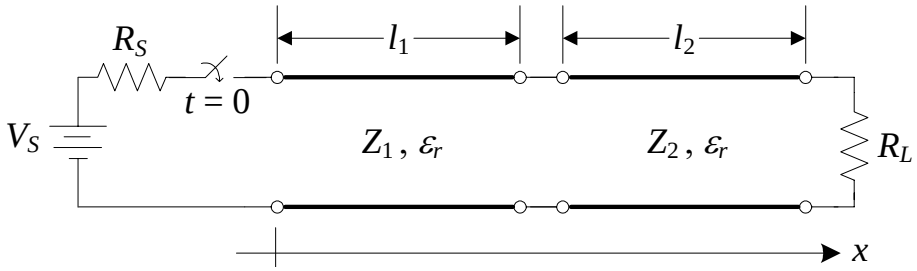
$$V_o^+ = \frac{V_S Z_1}{R_S + Z_1} \qquad v_{p1} = \frac{c}{\sqrt{\varepsilon_{e1}}} \qquad v_{p2} = \frac{c}{\sqrt{\varepsilon_{e2}}} \qquad T_P = 1 + \Gamma_P \qquad T_N = 1 + \Gamma_N$$

$$t_{d1} = \frac{l_1}{v_{p1}} \qquad t_{d2} = \frac{l_2}{v_{p2}}$$

Lattice Diagrams for Multiple TL Sections (cont)

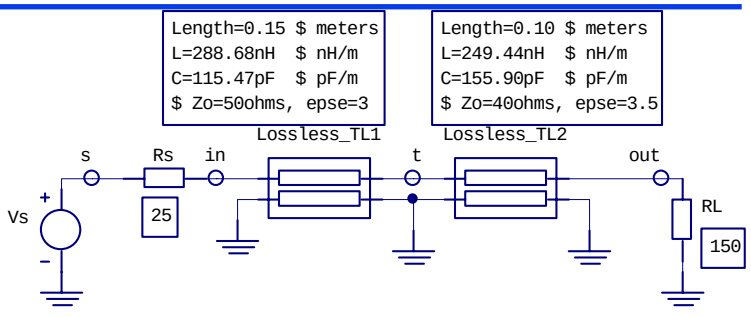


Example of Transient Response in 2-TL Sections



$R_S = 25\Omega$; $R_L = 150\Omega$; $V_S = 3V$;
$Z_1 = 50\Omega$; $Z_2 = 40\Omega$;
$\varepsilon_{e1} = 3$; $\varepsilon_{e2} = 3.5$;
$l_1 = 15cm$; $l_2 = 10cm$

Example Simulated with Aplac




```
$     DCLevel Amp Delay RiseT  FallT  PWidth Period
PULSE    0    3V    0   0.01ps 0.01ps 12ns   13ns
```

```
Sweep
"Lossless Transmission Line"
LOOP 300 TIME LIN 0 12ns
W=0 grid Y "" "" -0 3.2

Show W=0 Y=Vtran(s) Y=Vtran(in) Y=Vtran(t) Y=Vtran(out)
EndSweep
```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Example Simulated with Aplac (cont)