Time-Domain Analysis of Transmission Line Circuits

(Part 1)

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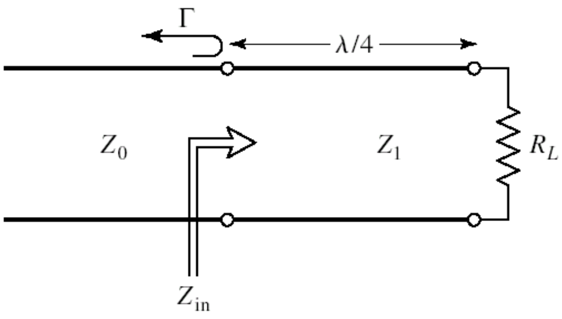
Outline

- Quarter-wave transformer – steady state response
- Quarter-wave transformer – transient response
- Reflection coefficient revised
- Concept of "transient impedance"
- Applying DC to transmission lines
- Lattice (or bouncing or reflection) diagrams
- Building transient signals from bouncing diagrams

Quarter-Wave Transformer



$$Z_{in} = \frac{Z_1^2}{R_L}$$

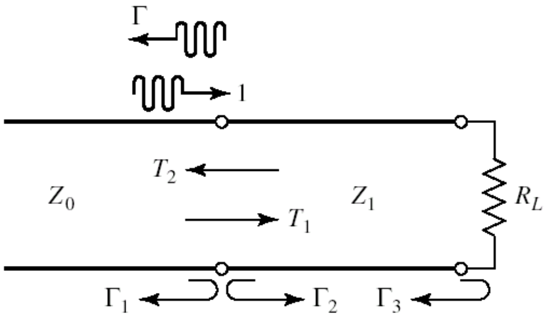
To make $\Gamma = 0$,

$$Z_1 = \sqrt{R_L Z_o}$$

Z_1 must be the geometric mean of Z_o and R_L

Γ is the steady-state reflection coefficient

Quarter-Wave Transformer – Transient Response



$$\Gamma_1 = \frac{Z_1 - Z_o}{Z_1 + Z_o}$$

$$\Gamma_2 = \frac{Z_o - Z_1}{Z_o + Z_1} = -\Gamma_1$$

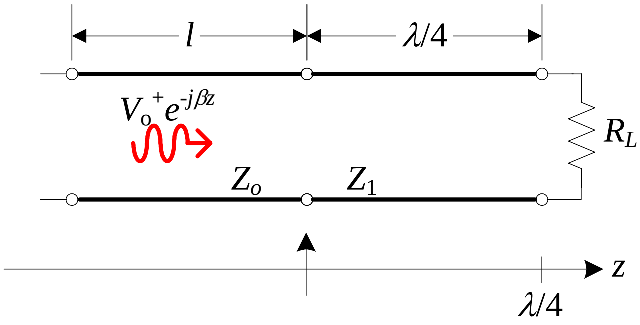
$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1}$$

$$T_1 = 1 + \Gamma_1 = \frac{2Z_1}{Z_1 + Z_o}$$

$$T_2 = 1 + \Gamma_2 = \frac{2Z_o}{Z_o + Z_1}$$

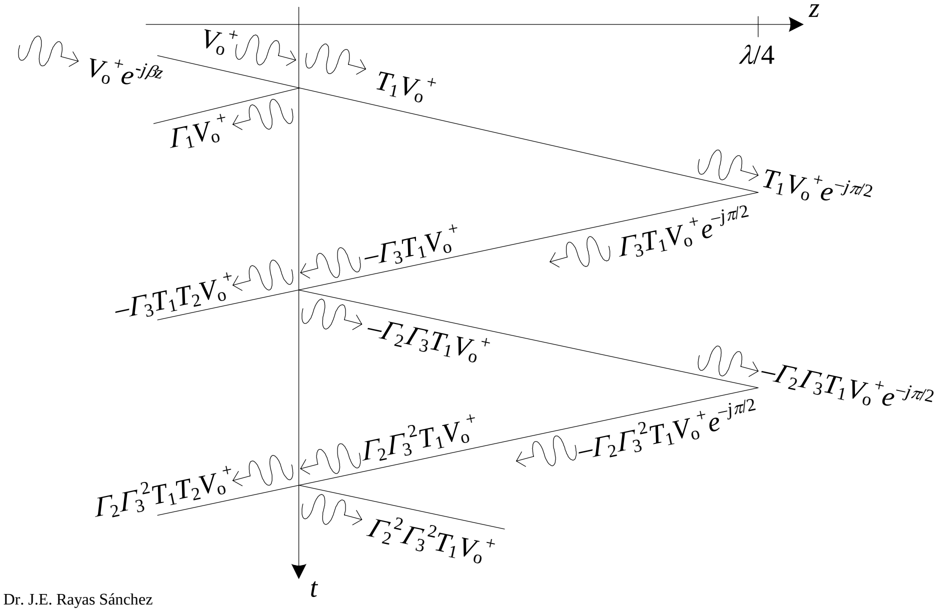
(D. M. Pozar, Microwave Engineering, Wiley, 2005)

Quarter-Wave Transformer – Transient Response

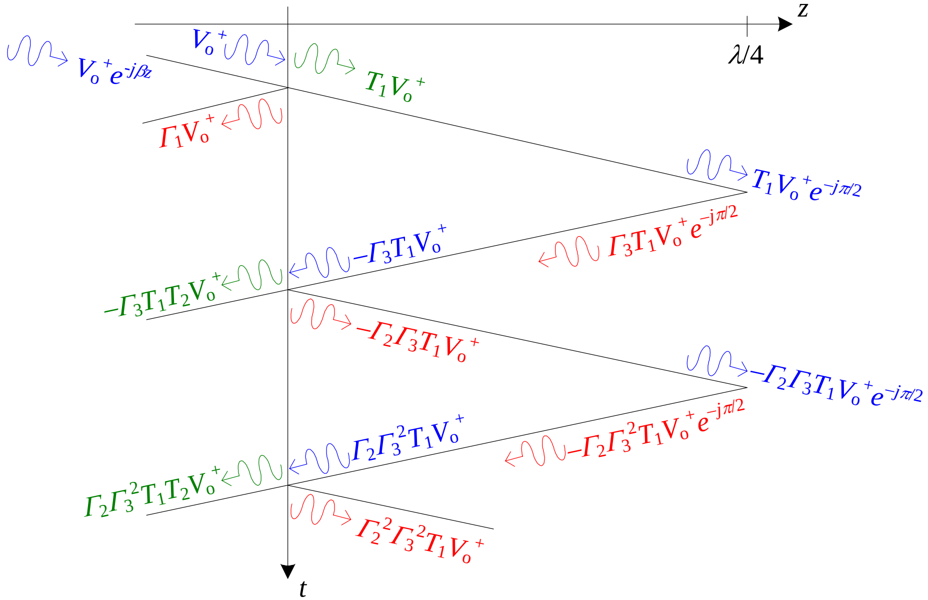


Initially, the incoming wave has only the incident component: $V(z) = V_o^+ e^{-j\beta z}$

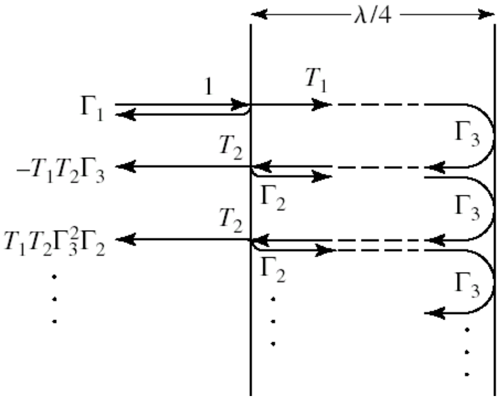
Quarter-Wave Transformer – Transient Response



Quarter-Wave Transformer – Transient Response



Quarter-Wave Transformer – Transient Response



$$\Gamma = \Gamma_1 - T_1T_2\Gamma_3 + T_1T_2\Gamma_2\Gamma_3^2 - T_1T_2\Gamma_2^2\Gamma_3^3 + \dots$$

$$\Gamma = \Gamma_1 - T_1T_2\Gamma_3\sum_{n=0}^{\infty}(-\Gamma_2\Gamma_3)^n$$

Quarter-Wave Transformer – Transient Response

$$\Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2 \Gamma_3)^n$$

Since $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, for $|x| < 1$

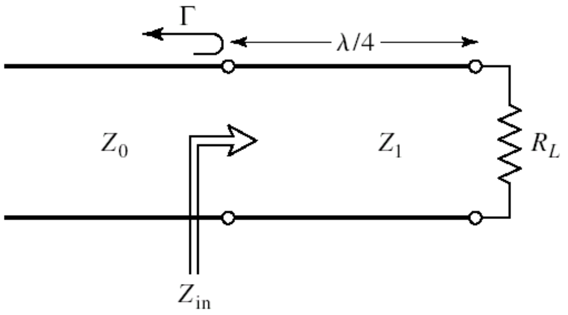
$$\Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} = \frac{\Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3}$$

using

$$\Gamma_1 = \frac{Z_1 - Z_o}{Z_1 + Z_o} \quad \Gamma_2 = -\Gamma_1 \quad \Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1} \quad T_1 = \frac{2Z_1}{Z_1 + Z_o} \quad T_2 = \frac{2Z_o}{Z_o + Z_1}$$

$$\Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3 = \frac{2(Z_1^2 - Z_o R_L)}{(Z_1 + Z_o)(R_L + Z_1)} = 0 \text{ if } Z_1 = \sqrt{Z_o R_L}$$

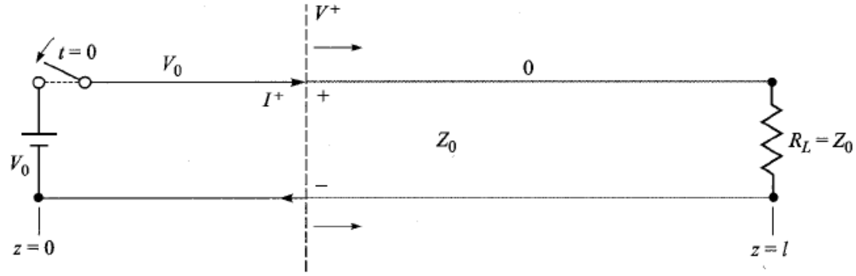
Quarter-Wave Transformer – Transient Response

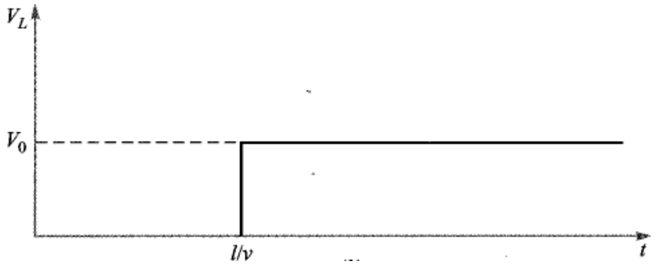


$$\Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2 \Gamma_3)^n = 0 \qquad \text{if } Z_1 = \sqrt{Z_o R_L}$$

If Z_1 is the geometric mean of Z_o and R_L, the sum of the infinite number of partial reflections is zero

Applying DC to Transmission Lines

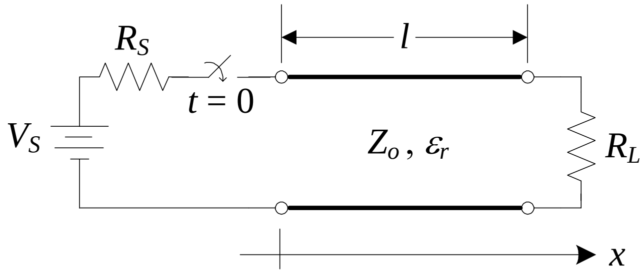




$$I^+ = \frac{V^+}{Z_o}$$

$$v = \frac{c}{\sqrt{\varepsilon_e}}$$

Applying DC to Transmission Lines (cont)

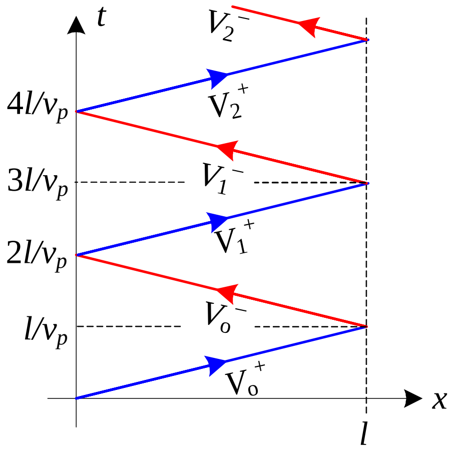


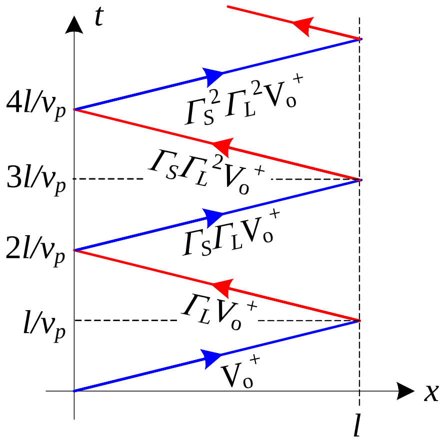
$$\Gamma_S = \frac{R_S - Z_o}{R_S + Z_o} \qquad \Gamma_L = \frac{R_L - Z_o}{R_L + Z_o} \qquad v_p = \frac{c}{\sqrt{\varepsilon_e}}$$

$$V_o^+ = \frac{V_S Z_o}{R_S + Z_o} \qquad I_o^+ = \frac{V_o^+}{Z_o}$$

Lattice (or Bouncing or Reflection) Diagrams

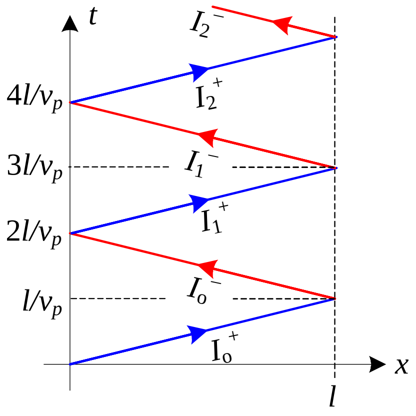
- Voltage Diagrams

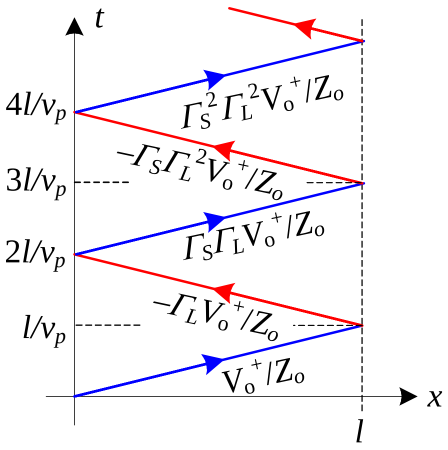




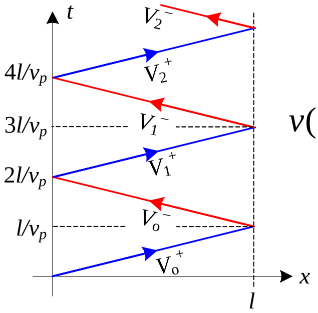
Bouncing Diagrams for Currents

Since $I_o^+ = \dfrac{V_o^+}{Z_o}$ and $I_o^- = \dfrac{-V_o^-}{Z_o}$





Transient Signals from Bouncing Diagrams

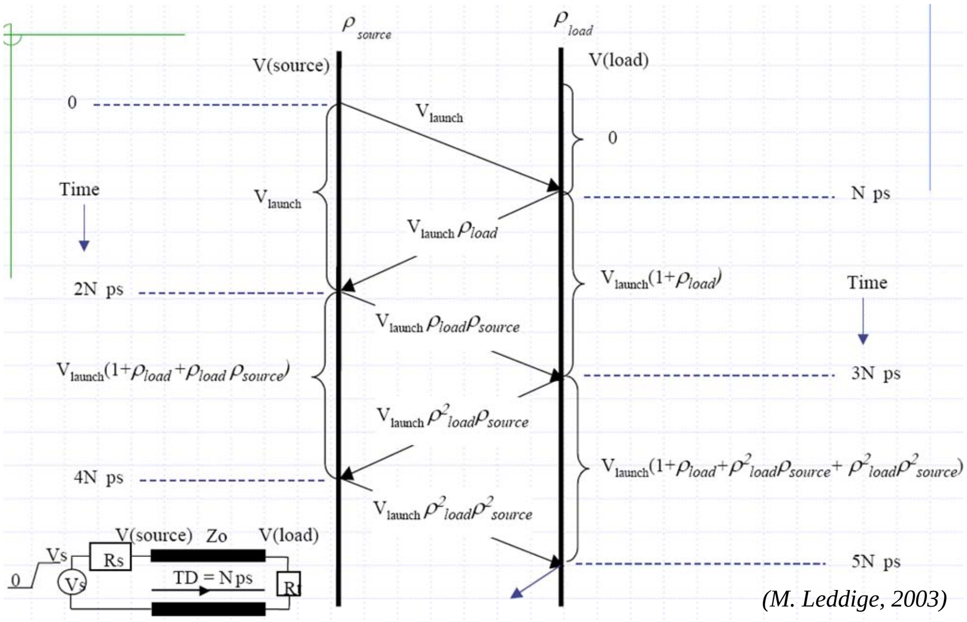


$$t_d = \frac{l}{v_p}$$

$$v(0,t) = \begin{cases} V_o^+ & 0 \le t < 2t_d \\ V_o^+ + V_o^- + V_1^+ & 2t_d \le t < 4t_d \\ V_o^+ + V_o^- + V_1^+ + V_1^- + V_2^+ & 4t_d \le t < 6t_d \\ \vdots & \vdots \end{cases}$$

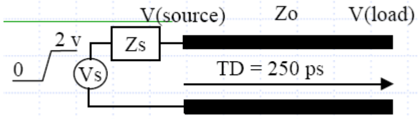
$$v(l,t) = \begin{cases} 0 & 0 \le t < t_d \\ V_o^+ + V_o^- & t_d \le t < 3t_d \\ V_o^+ + V_o^- + V_1^+ + V_1^- & 3t_d \le t < 5t_d \\ \vdots & \vdots \end{cases}$$

Bouncing Diagrams – Other Representation



(M. Leddige, 2003)

Example – Underdriven Transmission Line

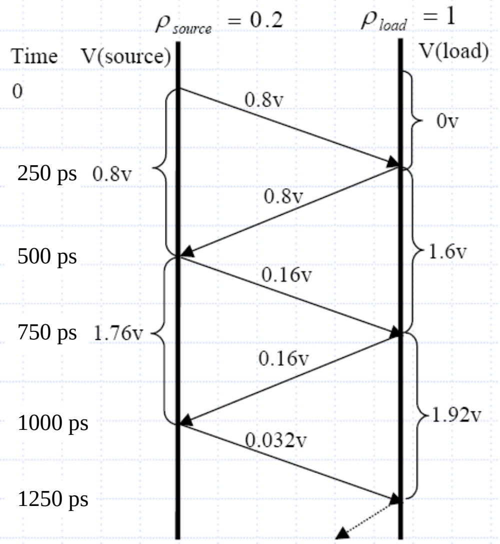


Assume Zs=75 ohms
Zo=50ohms
Vs=0-2 volts

$$V_{initial} = Vs\frac{Zo}{Zs+Zo} = (2)\left(\frac{50}{75+50}\right) = 0.8$$

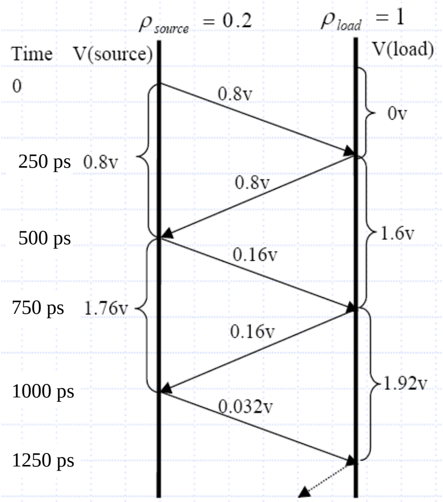
$$\rho_{source} = \frac{Zs-Zo}{Zs+Zo} = \frac{75-50}{75+50} = 0.2$$

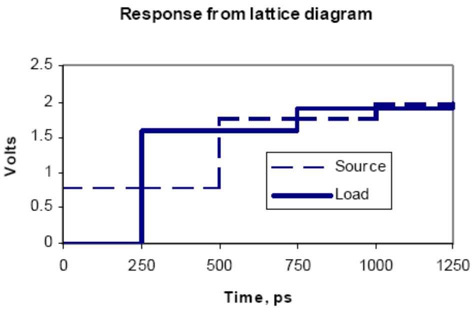
$$\rho_{load} = \frac{Zl-Zo}{Zl+Zo} = \frac{\infty-50}{\infty+50} = 1$$



(S.H. Hall, G.W. Hall and J.A. McCall, High-Speed Digital System Design, Wiley, 2000)

Example – Underdriven Transmission Line (cont)





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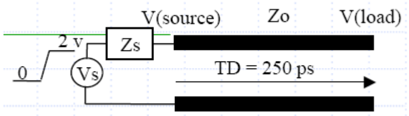
Example – Overdriven Transmission Line



Assume Zs=25 ohms

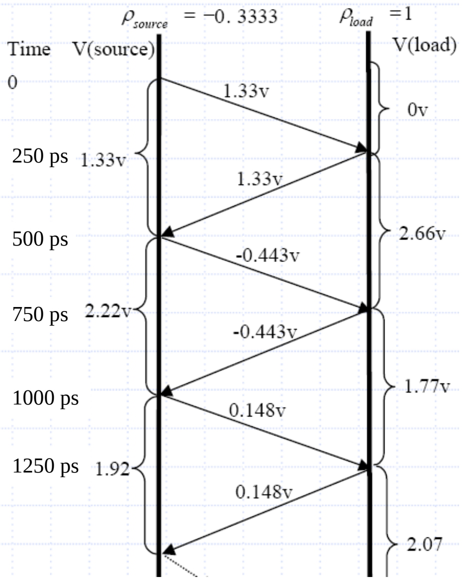
Zo=50ohms

Vs=0-2 volts

$$V_{initial} = Vs\frac{Zo}{Zs+Zo} = (2)\left(\frac{50}{25+50}\right) = 1.3333$$

$$\rho_{source} = \frac{Zs-Zo}{Zs+Zo} = \frac{25-50}{25+50} = -0.33333$$

$$\rho_{load} = \frac{Zl-Zo}{Zl+Zo} = \frac{\infty-50}{\infty+50} = 1$$



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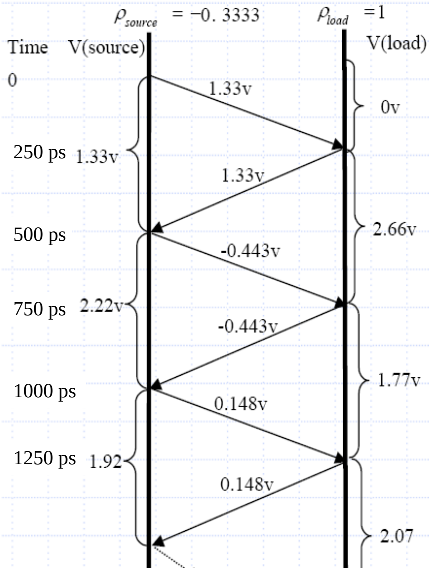
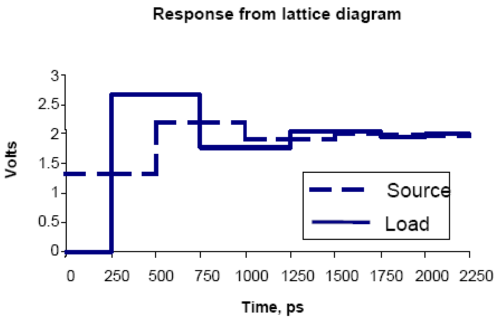
Example – Overdriven Transmission Line





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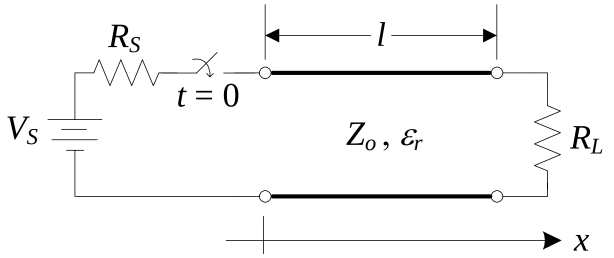
Using the Bouncing Diagrams – Example



$R_S = 25\Omega$; $R_L = 150\Omega$;
$V_S = 3\text{V}$; $Z_o = 50\Omega$;
$\varepsilon_e = 3$; $l = 15\text{cm}$

$$v(x=0,t)=?\,;\; v(x=l,t)=?\,;\; v(x=\frac{3}{4}l,t)=?$$

$$i(x=0,t)=?\,;\; i(x=l,t)=?\,;\; i(x=\frac{3}{4}l,t)=?$$

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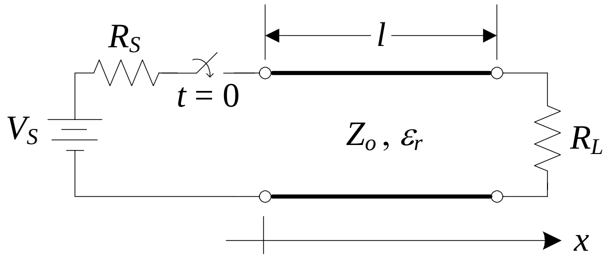
Using the Bouncing Diagrams – Example (cont)



$R_S = 25\Omega$; $R_L = 150\Omega$;
$V_S = 3\text{V}$; $Z_o = 50\Omega$;
$\varepsilon_e = 3$; $l = 15\text{cm}$

$$\Gamma_L = \frac{R_L - Z_o}{R_L + Z_o} = \frac{1}{2} \qquad \Gamma_S = \frac{R_S - Z_o}{R_S + Z_o} = -\frac{1}{3} \qquad V_o^+ = \frac{V_S Z_o}{R_S + Z_o} = 2\text{V}$$

$$v_p = \frac{c}{\sqrt{\varepsilon_e}} = 173.2\text{Mm/s} \qquad t_d = \frac{l}{v_p} = 866.02\text{ps}$$

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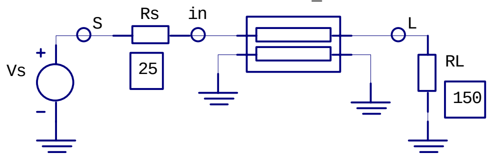
Example Simulated with APLAC

```
Length=0.15 $ meters
L=288.68nH  $ nH/m
C=115.47pF  $ pF/m
$ Zo=50ohms, epse=3
```

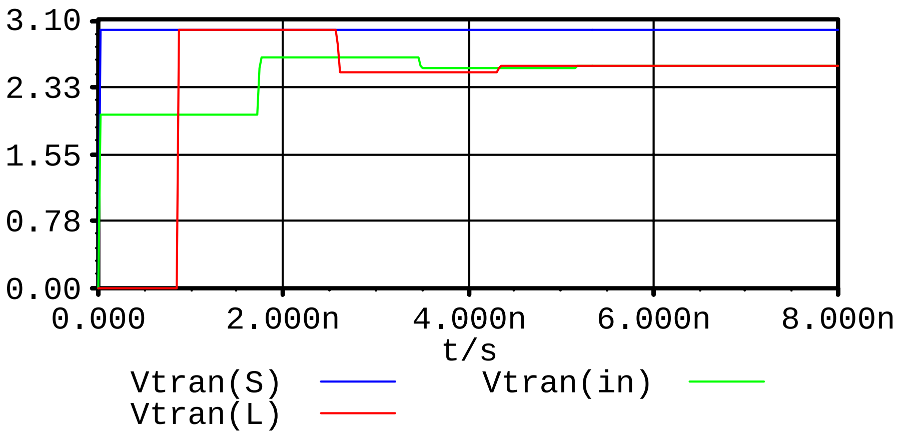


```
$     DCLevel Amp Delay RiseT  FallT  PWidth Period
PULSE    0    3V    0   0.01ps 0.01ps 8ns    9ns
```

```
Sweep
"Lossless Transmission Line"
LOOP 300 TIME LIN 0 8ns
W=0 grid Y "" "" 0 3.1
W=1 grid Y "" "" 0 45m

Show W=0 Y=Vtran(S) Y=Vtran(in) Y=Vtran(L)
Show W=1 Y=(Vtran(S)-Vtran(in))/25 Y=Vtran(L)/150
EndSweep
```

Example Simulated with APLAC (cont)



Example Simulated with APLAC (cont)