Transmission Line Theory

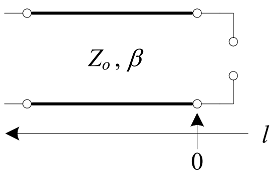
(Part 4)

Dr. José Ernesto Rayas Sánchez

Outline

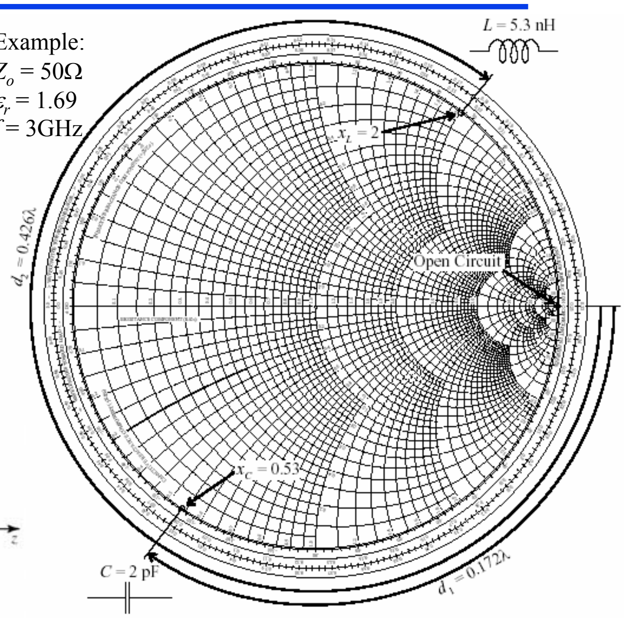
- Open circuit transformations
- Short circuit transformations
- Impedance-admittance transformations
- The quarter-wave transformer
- Lossy vs lossless transmission lines
- The low-loss TL
- The lossy distortionless TL

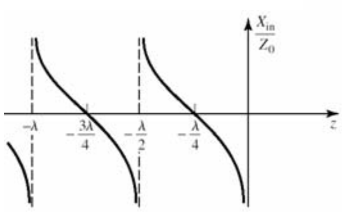
Open Circuit Transformations



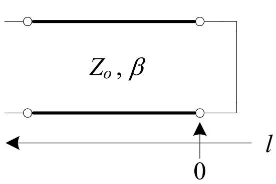
Given Z_o, ε_r and f, we can calculate the lengths for a desired C or L

Example:
$Z_o = 50\Omega$
$\varepsilon_r = 1.69$
$f = 3$GHz





Short Circuit Transformations



Given Z_o, ε_r and f, we can calculate the lengths for a desired L or C

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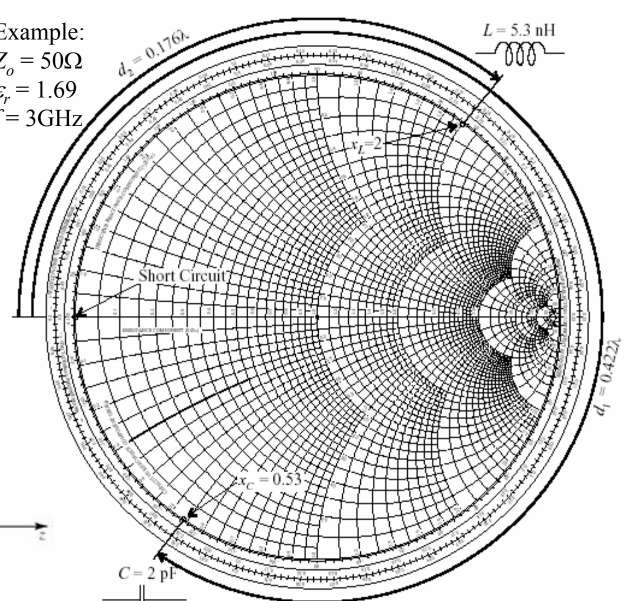

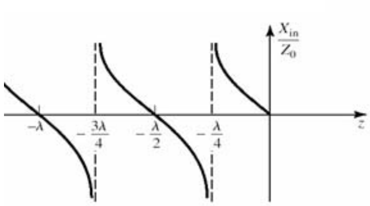

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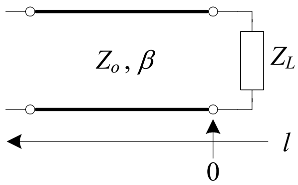
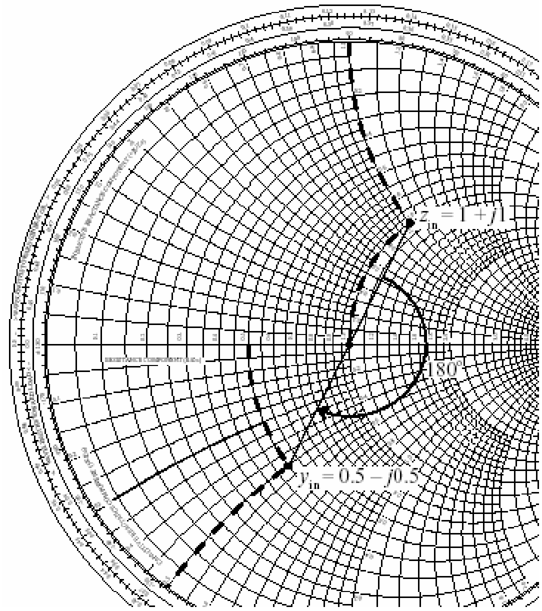
Impedance–Admittance Transformations



$$Z_{in}(l) = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)}$$

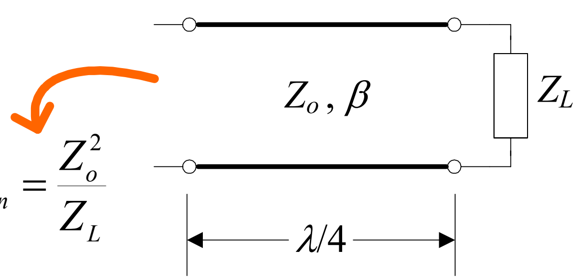
$$Z_{in}(l = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \cdots) = \frac{Z_o^2}{Z_L}$$

$$z_{in}(l = \frac{\lambda}{4}) = \frac{1}{z_L} = y_{in}$$



Quarter-Wave Transformer

- Simple technique for impedance matching
- It can achieve perfect match at a single frequency



$$Z_{in} = \frac{Z_o^2}{Z_L}$$

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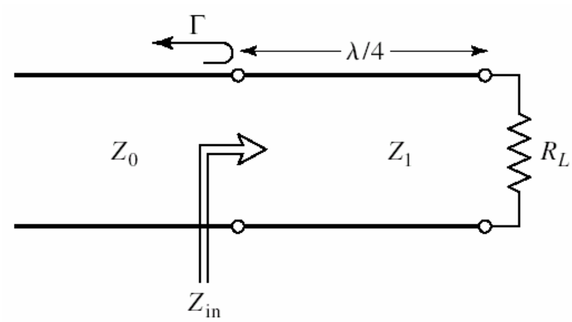
Quarter-Wave Transformer (cont)



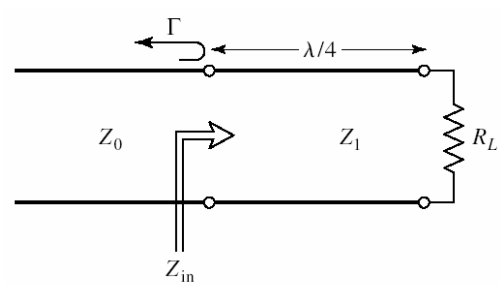
$$Z_{in} = \frac{Z_1^2}{R_L}$$

To make $\Gamma = 0$,

$$Z_1 = \sqrt{R_L Z_o}$$

Z_1 must be the geometric mean of Z_o and R_L

Quarter-Wave Transformer - Example



$Z_o = 50\Omega$
$R_L = 100\Omega$
$Z_1 = ?$
$\Gamma(f) = ?$

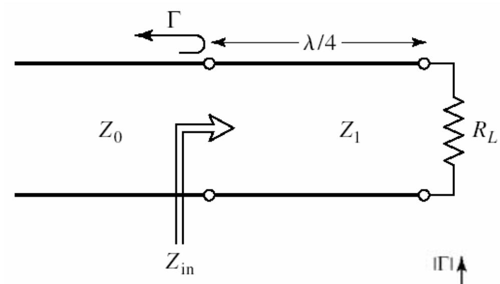
$$Z_1 = \sqrt{(50)(100)} = 70.71\Omega$$

$\Gamma = 0$ only at the frequencies at which the electrical length of the matching section is 90°, 270°, …

Since $\Gamma_l = \dfrac{Z_{in}(l) - Z_o}{Z_{in}(l) + Z_o}$ and $Z_{in}(l) = Z_o \dfrac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)}$

$$\beta l = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda_o}{4}\right) = \left(\frac{2\pi f}{v_p}\right)\left(\frac{v_p}{4 f_o}\right) = \frac{\pi}{2}\frac{f}{f_o}$$

Quarter-Wave Transformer - Example



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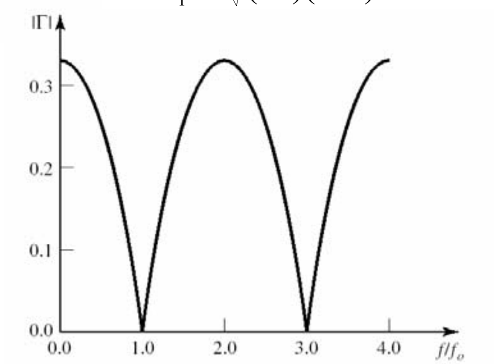

Lossy Transmission Lines

- In practice, all transmission lines have losses due to some finite conductivity and/or lossy dielectric
- These losses are usually small
- For analysis purposes (first-order approximations), these losses may be neglected
- There are two special cases of interest:
  - The low-loss line
  - The lossy distortionless line

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Lossy *vs* Lossless Transmission Lines

$$V(z) = V_o^+[e^{-\gamma z} + \Gamma e^{+\gamma z}]$$
$$I(z) = \frac{V_o^+}{Z_o}[e^{-\gamma z} - \Gamma e^{+\gamma z}]$$
$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$\gamma \equiv \alpha + j\beta$$
$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$
$$\Gamma_z(z) = \Gamma e^{+2\gamma z}$$

$$V(z) = V_o^+[e^{-j\beta z} + \Gamma e^{+j\beta z}]$$
$$I(z) = \frac{V_o^+}{Z_o}[e^{-j\beta z} - \Gamma e^{+j\beta z}]$$
$$\beta = \omega\sqrt{LC} \qquad Z_o = \sqrt{\frac{L}{C}}$$
$$\Gamma_z(z) = \Gamma e^{+2j\beta z}$$

$$Z_o \equiv \frac{V_o^+}{I_o^+} = \frac{V_o^-}{-I_o^-} \qquad \Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o} \qquad \lambda = \frac{v_p}{f} = \frac{2\pi}{\beta} \qquad v_p = \frac{\omega}{\beta}$$

The Low-Loss Transmission Line

- It is a line where $R << \omega L$ and $G << \omega C$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$\gamma = \sqrt{(j\omega L)(j\omega C)\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)}$$
$$\gamma = j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}}$$
$$\gamma \approx j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)}$$

Using $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{2\cdot 4}x^2 + \frac{1\cdot 3}{2\cdot 4\cdot 6}x^3 - \ldots$

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The Low-Loss Transmission Line (cont)

- It is a line where $R << \omega L$ and $G << \omega C$

$$\gamma \approx j\omega\sqrt{LC}\left[1-\frac{j}{2}\left(\frac{R}{\omega L}+\frac{G}{\omega C}\right)\right]$$

$$\gamma \equiv \alpha + j\beta$$

$\beta \approx \omega\sqrt{LC}$ β is almost a linear function of ω (no dispersion)

$$\alpha \approx \frac{1}{2}\left(R\sqrt{\frac{C}{L}}+G\sqrt{\frac{L}{C}}\right) \qquad Z_o = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \approx \sqrt{\frac{L}{C}}$$

$$\alpha \approx \frac{1}{2}\left(\frac{R}{Z_o}+GZ_o\right)$$

The Lossy Distorsionless Transmission Line

- It is a line where $R/L = G/C$

Since $\gamma = j\omega\sqrt{LC}\sqrt{1-j\left(\frac{R}{\omega L}+\frac{G}{\omega C}\right)-\frac{RG}{\omega^2 LC}}$

$$\gamma = j\omega\sqrt{LC}\sqrt{1-2j\frac{R}{\omega L}-\frac{R^2}{\omega^2 L^2}}$$

$$\gamma = j\omega\sqrt{LC}\left(1-j\frac{R}{\omega L}\right)$$

$$\gamma \equiv \alpha + j\beta$$

$$\beta = \omega\sqrt{LC} \qquad \alpha = R\sqrt{\frac{C}{L}}$$

β is a linear function of ω (non dispersive TL)

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