Transmission Line Theory

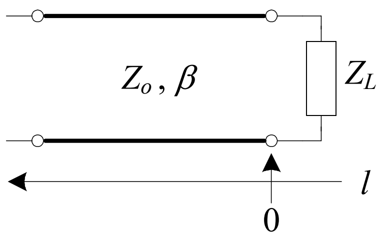
(Part 3)

Dr. José Ernesto Rayas Sánchez

Outline

- Input impedance in lossless transmission lines (TL)
- Special cases of lossless terminated TL
- Insertion loss
- Transmission coefficient
- Smith Chart interpretation
- Basic Smith Chart applications

Input Impedance in Lossless TL



$$V(z) = V_o^+ (e^{-j\beta z} + \Gamma e^{+j\beta z})$$

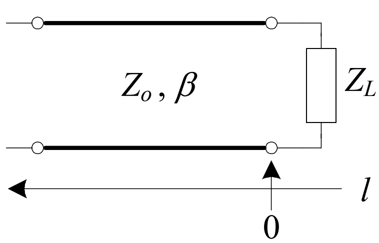
$$I(z) = \frac{V_o^+}{Z_o} (e^{-j\beta z} - \Gamma e^{+j\beta z})$$

$$Z_{in}(l) = \frac{V(l)}{I(l)} = Z_o \frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} = Z_o \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}}$$

$$Z_{in}(l) = Z_o \frac{1 + \Gamma_l}{1 - \Gamma_l}$$

$$\Gamma_l = \frac{Z_{in}(l) - Z_o}{Z_{in}(l) + Z_o}$$

Input Impedance in Lossless TL



$$V(z) = V_o^+ (e^{-j\beta z} + \Gamma e^{+j\beta z})$$

$$I(z) = \frac{V_o^+}{Z_o} (e^{-j\beta z} - \Gamma e^{+j\beta z})$$

$$Z_{in}(l) = \frac{V(l)}{I(l)} = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)}$$

Since $\beta = \frac{2\pi}{\lambda}$

βl is the electrical length

$$Z_{in}(l = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \cdots) = Z_L \qquad Z_{in}(l = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \cdots) = \frac{Z_o^2}{Z_L}$$

(Period = $\lambda/2$)

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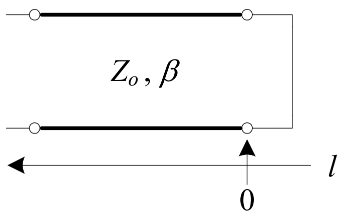
Short-Circuited Lossless TL

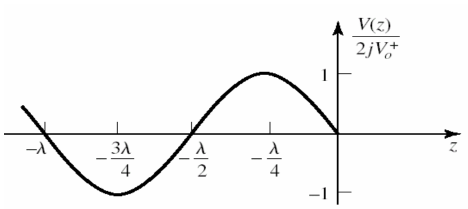


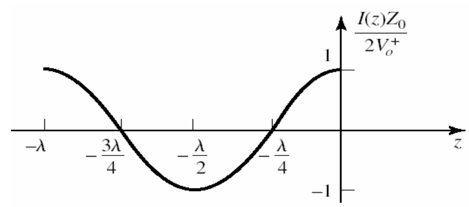
$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = -1$$

$$V(z) = V_o^+(e^{-j\beta z} + \Gamma e^{+j\beta z}) = V_o^+(e^{-j\beta z} - e^{+j\beta z})$$

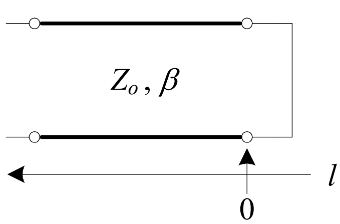
$$I(z) = \frac{V_o^+}{Z_o}(e^{-j\beta z} - \Gamma e^{+j\beta z}) = \frac{V_o^+}{Z_o}(e^{-j\beta z} + e^{+j\beta z})$$

$$V(z) = -2jV_o^+ \sin \beta z \qquad I(z) = \frac{2V_o^+}{Z_o}\cos \beta z$$





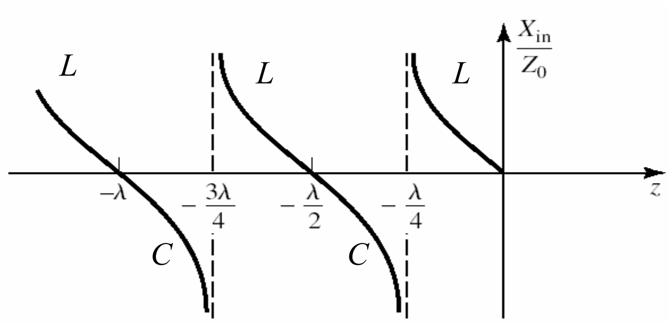
Short-Circuited Lossless TL (cont)



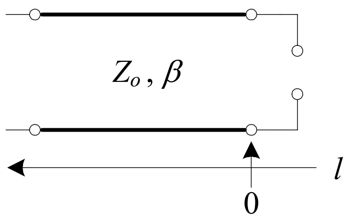
$$Z_{in}(l) = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)}$$

$$Z_{in}(l) = jZ_o \tan(\beta l)$$

Z_{in} is purely reactive



Open-Circuited Lossless TL

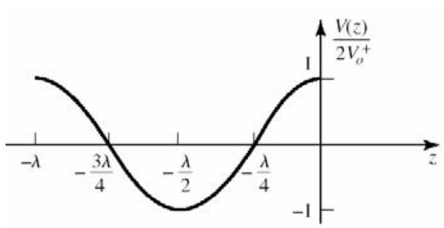


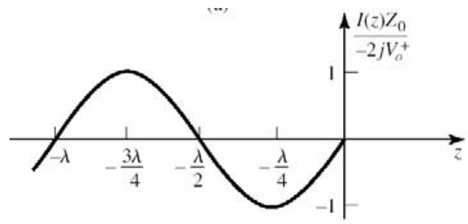
$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = +1$$

$$V(z) = V_o^+ (e^{-j\beta z} + e^{+j\beta z})$$

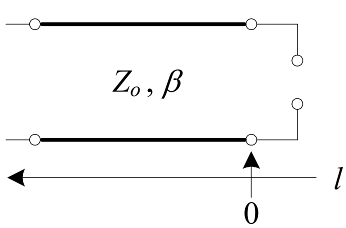
$$I(z) = \frac{V_o^+}{Z_o} (e^{-j\beta z} - e^{+j\beta z})$$

$$V(z) = 2V_o^+ \cos \beta z \qquad I(z) = \frac{j2V_o^+}{Z_o} \sin \beta z$$





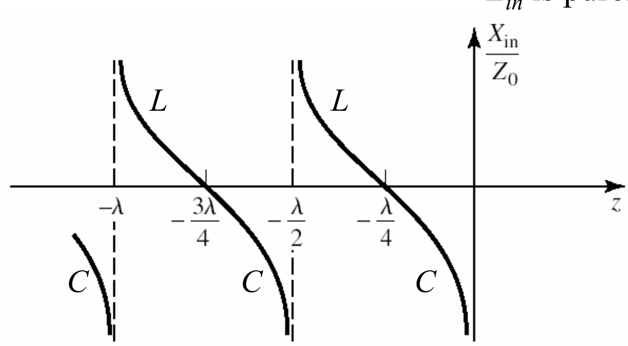
Open-Circuited Lossless TL (cont)



$$Z_{in}(l) = Z_o \frac{Z_L + jZ_o \tan(\beta l)}{Z_o + jZ_L \tan(\beta l)}$$

$$Z_{in}(l) = \frac{Z_o}{j \tan(\beta l)} = -jZ_o \cot(\beta l)$$

Z_{in} is purely reactive



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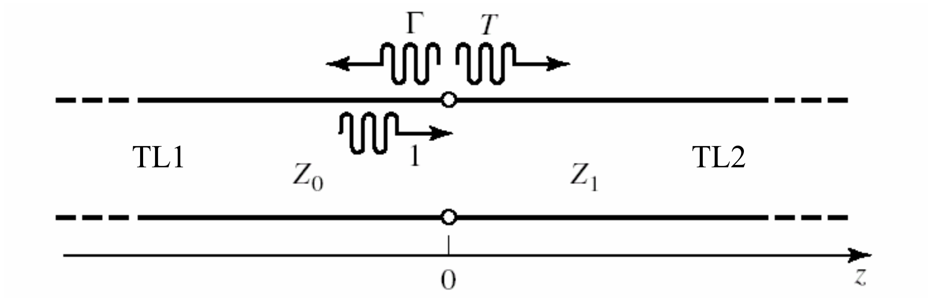
Transmission Coefficient, T

- It is a measure of the capacity of a TL to transmit an incoming wave



$$\Gamma = \frac{Z_1 - Z_o}{Z_1 + Z_o}$$

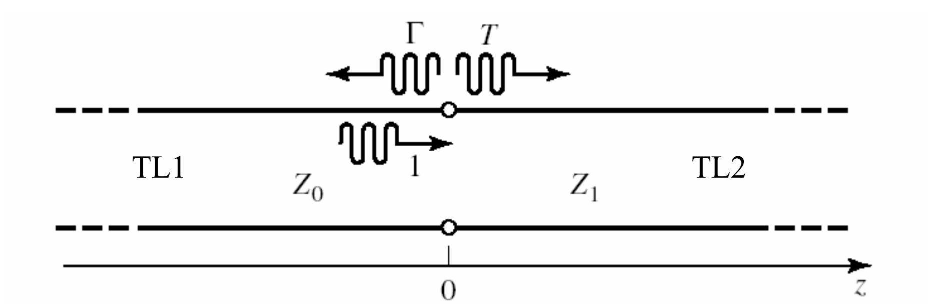
$$V(z) = V_o^+ (e^{-j\beta z} + \Gamma e^{+j\beta z}) \quad \text{for } z \le 0$$

$$V(z) = V_o^+ T e^{-j\beta z} \quad \text{for } z \ge 0 \text{ (assuming TL2 infinite)}$$

$$V(0) = V_o^+ [1 + \Gamma] = T V_o^+ \qquad T = 1 + \Gamma$$

Insertion Loss, IL

- It is a measure of the reflections caused by an inserted TL



$$IL = -20 \log |T| \quad \text{(dB)}$$

$$IL(|T| = 0) = \infty \text{ dB} \qquad IL(|T| = 1) = 0 \text{ dB}$$

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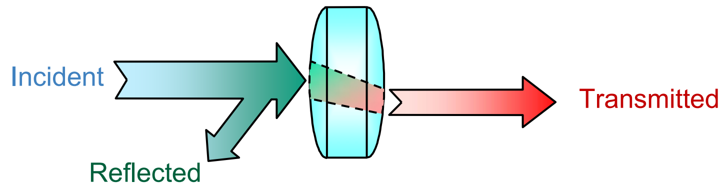
Lightwave Analogy to RF Energy



Lightwave

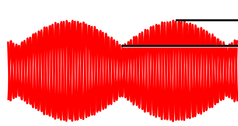

RF

(Hewlett-Packard's RF Design and Measurement Seminar, 2000)

Reflection Parameters

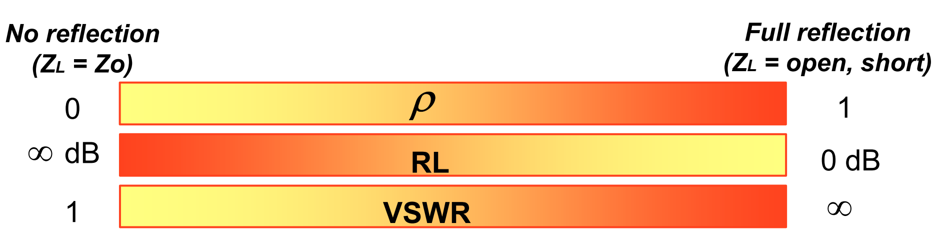
Reflection Coefficient $\Gamma = \dfrac{V_{reflected}}{V_{incident}} = \rho \angle \Phi = \dfrac{Z_L - Z_O}{Z_L + Z_O}$

Return loss = -20 log(ρ), $\rho = |\Gamma|$



Voltage Standing Wave Ratio

VSWR $= \dfrac{E_{max}}{E_{min}} = \dfrac{1+\rho}{1-\rho}$



(Hewlett-Packard's RF Design and Measurement Seminar, 2000)

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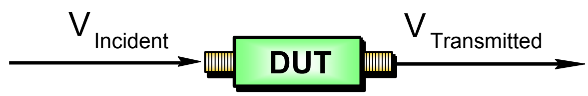
Transmission Parameters



Transmission Coefficient = $T = \frac{V_{Transmitted}}{V_{Incident}} = \tau\angle\phi$

Insertion Loss (dB) = $-20 \text{ Log} \left| \frac{V_{Trans}}{V_{Inc}} \right| = -20 \log \tau$

Gain (dB) = $20 \text{ Log} \left| \frac{V_{Trans}}{V_{Inc}} \right| = 20 \log \tau$

Insertion Phase (deg) = $\angle \frac{V_{Trans}}{V_{Inc}} = \phi$

(Hewlett-Packard's RF Design and Measurement Seminar, 2000)

The Smith Chart

- Developed in 1939 by P. H. Smith at Bell Labs
- Very useful for visualizing transmission line phenomena and impedance matching problems
- It is part of most current CAD tools and modern measurement equipments
- It is essentially a plot of Γ in polar coordinates
- Any impedance can be mapped in the Γ plane

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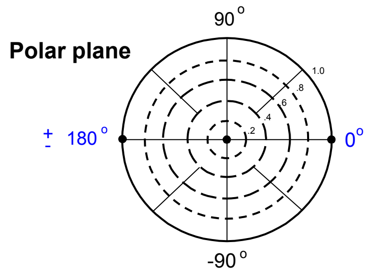
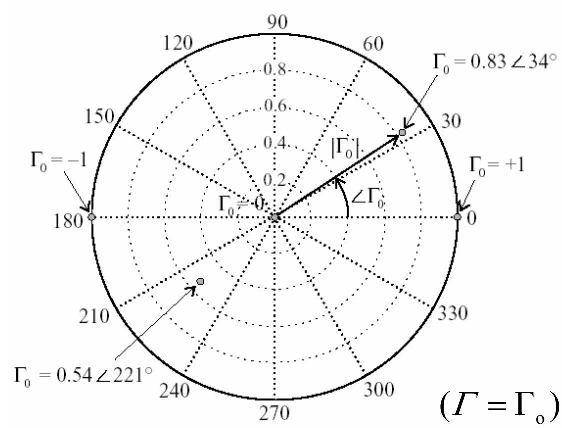
The Smith Chart (cont)



$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = |\Gamma| e^{j\theta}$$



$(\Gamma = \Gamma_o)$

(R. Ludwig and P. Bretchko, RF Circuit Design, Prentice Hall, 2000)

The Smith Chart (cont)

$$\Gamma = \frac{Z - Z_o}{Z + Z_o} = |\Gamma| e^{j\theta}$$

Normalizing Z w.r.t Z_o

$$\frac{z-1}{z+1} = |\Gamma| e^{j\theta} \quad \text{where} \quad z = \frac{Z}{Z_o}$$

$$z = \frac{1 + |\Gamma| e^{j\theta}}{1 - |\Gamma| e^{j\theta}}$$

$$r + jx = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i}$$

Solving for r and x

$$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$x = \frac{2\Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

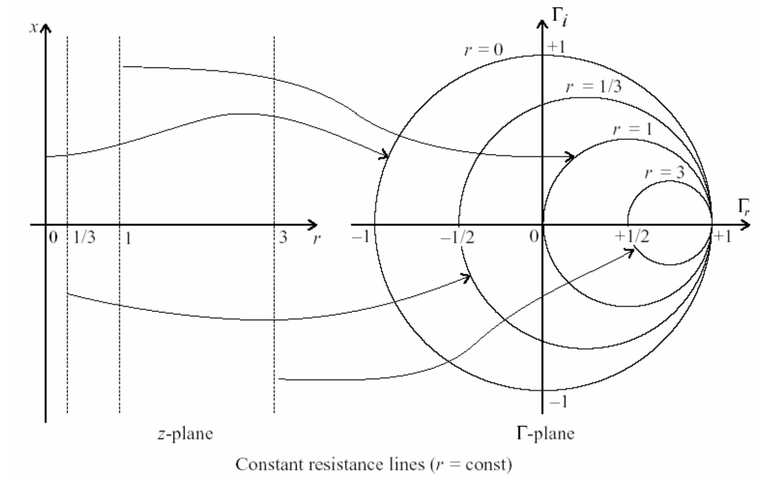
Rearranging

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

The Smith Chart – Circles of Constant r

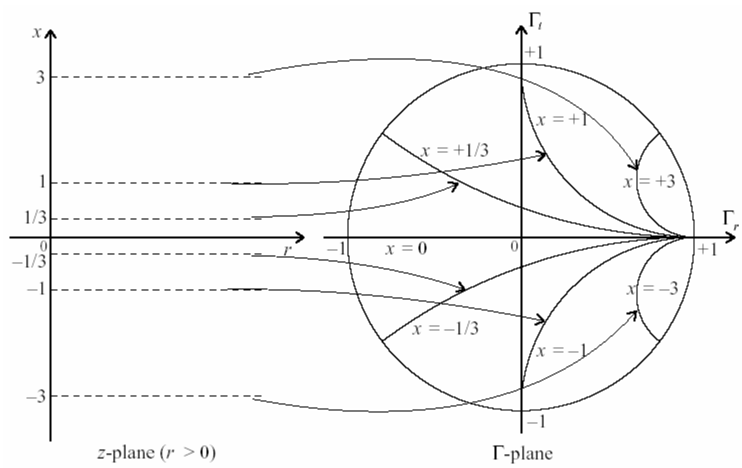
$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2$$



 (R. Ludwig and P. Bretchko, RF Circuit Design, Prentice Hall, 2000)

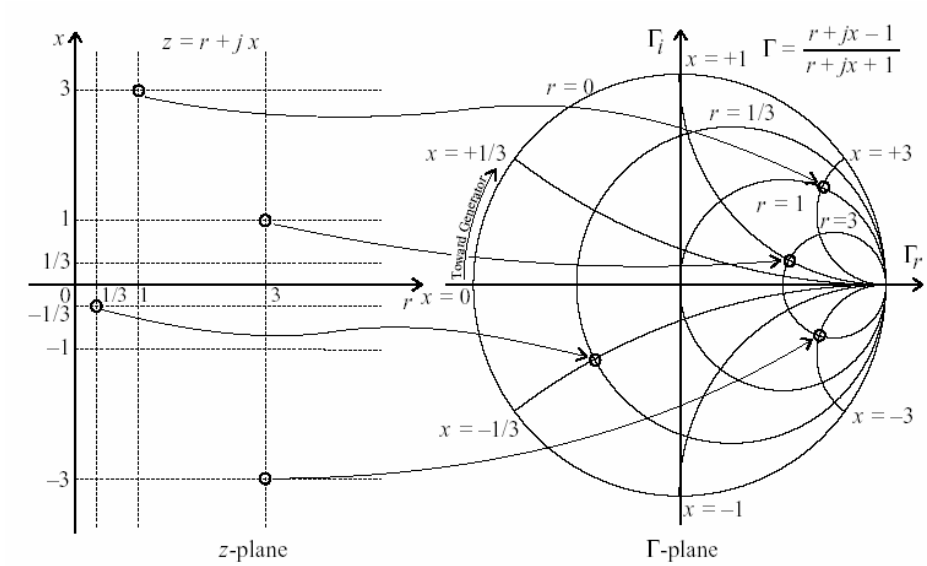
The Smith Chart – Circles of Constant x

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$



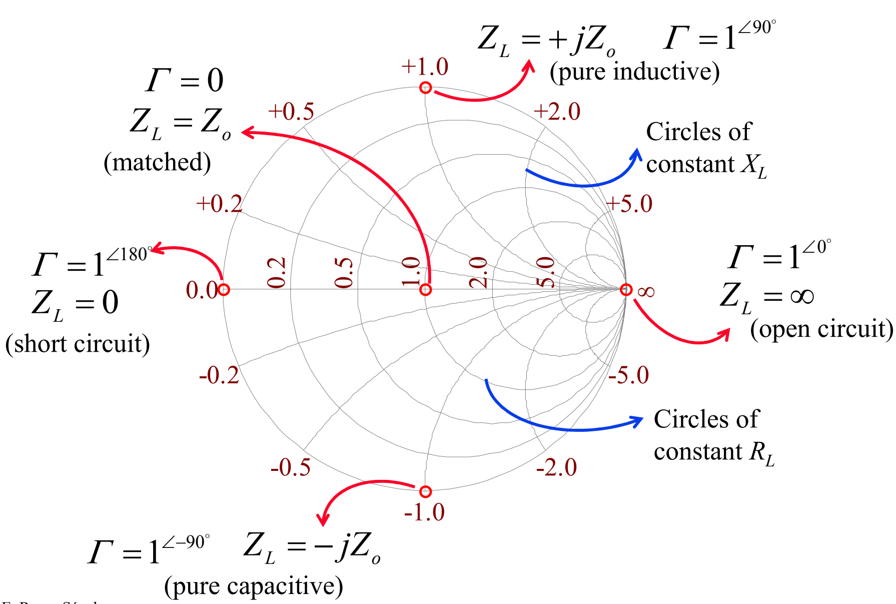
 (R. Ludwig and P. Bretchko, RF Circuit Design, Prentice Hall, 2000)

The Smith Chart – Combining both Circles

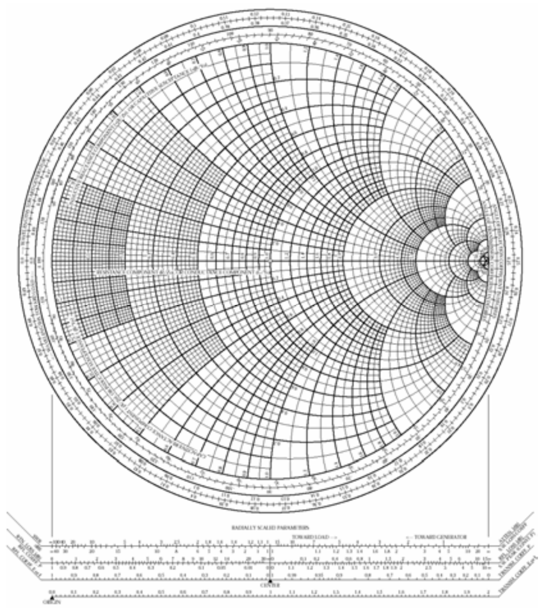


(R. Ludwig and P. Bretchko, RF Circuit Design, Prentice Hall, 2000)

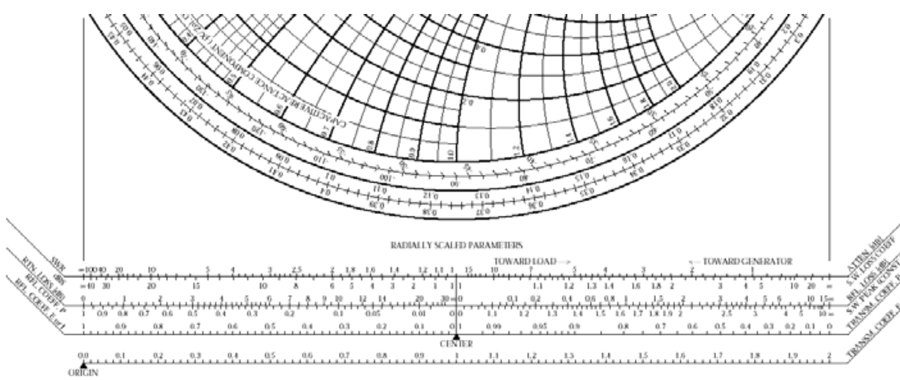
The Smith Chart – Interpretation



Practical Smith Chart



Practical Smith Chart (cont)



Smith Chart – Calculating *Γ, RL, SWR*

Example:

$Z_o = 50\Omega$

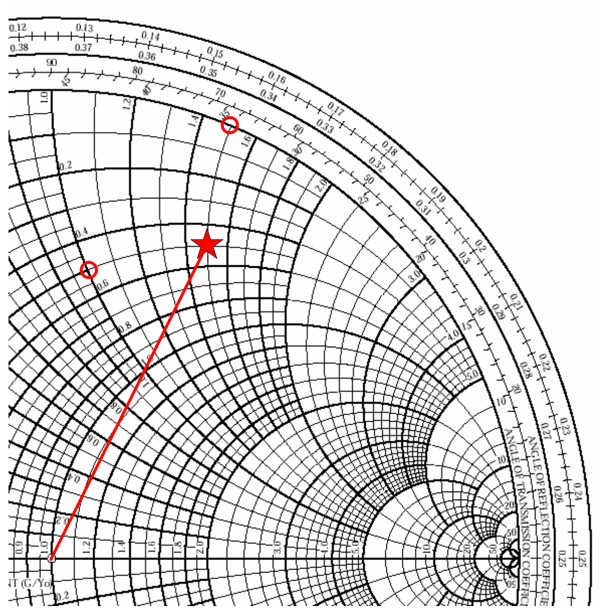
$Z_L = 25 + j75\ \Omega$

$\Gamma = ?$

$RL = ?$

$SWR = ?$

$z_L = 0.5 + j1.5$



Smith Chart – Calculating *Γ, RL, SWR* (cont)

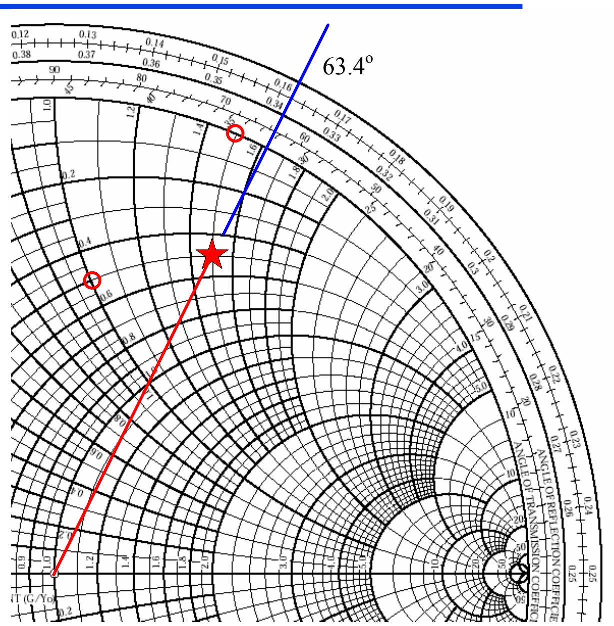
Example:

$Z_o = 50\Omega$

$Z_L = 25 + j75\ \Omega$

$z_L = 0.5 + j1.5$

$\Gamma = |\Gamma|^{\angle 63.4^\circ}$



Smith Chart – Calculating *Γ, RL, SWR* (cont)

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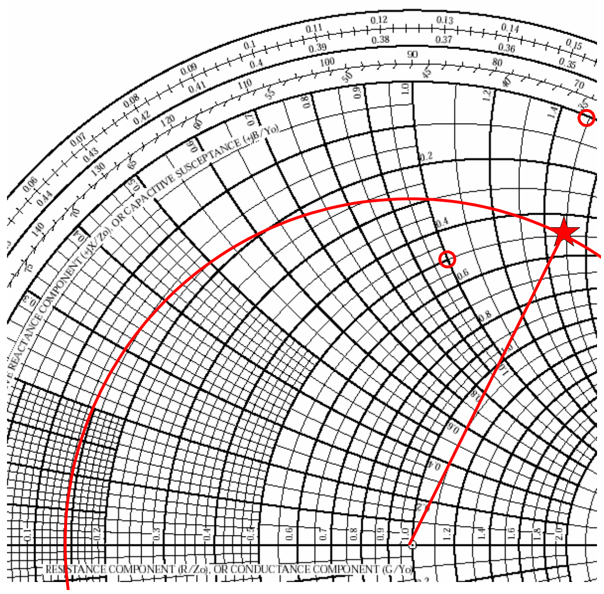


Smith Chart – Calculating *Γ, RL, SWR* (cont)

Example:

$Z_o = 50\Omega$

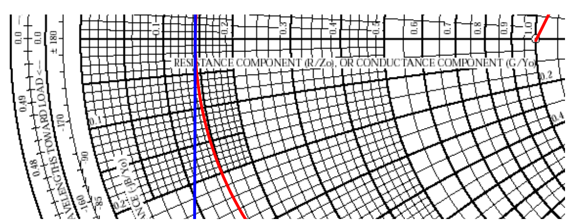
$Z_L = 25 + j75\ \Omega$

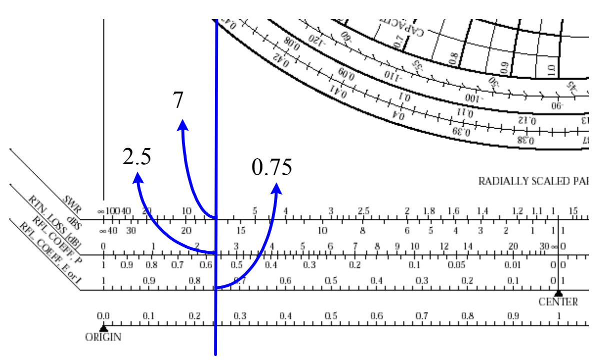
$z_L = 0.5 + j1.5$

$\Gamma \approx 0.75^{\angle 63.4^\circ}$

$SWR \approx 7$

$RL \approx 2.5$ dB





Smith Chart – Circles of *SWR*

Example:

$Z_o = 50\Omega$

$Z_L = 25 + j75\ \Omega$

$z_L = 0.5 + j1.5$

$\Gamma \approx 0.75^{\angle 63.4^\circ}$

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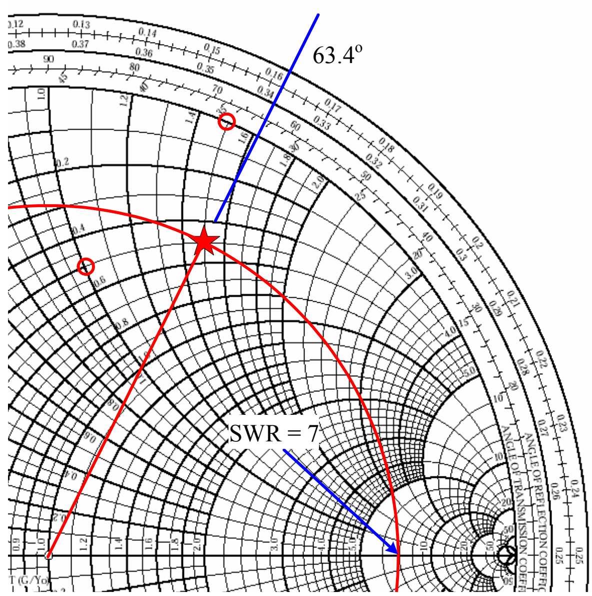


Smith Chart – Circles of *SWR* (cont)

Examples:

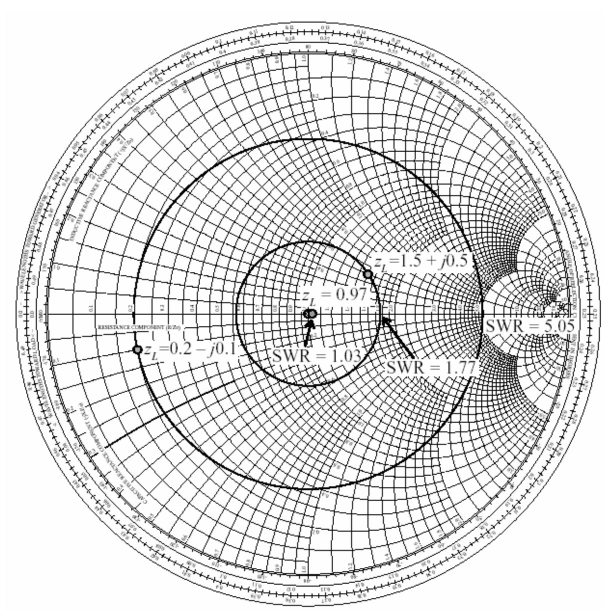
$Z_o = 50\Omega$

a) $Z_L = 50\ \Omega$

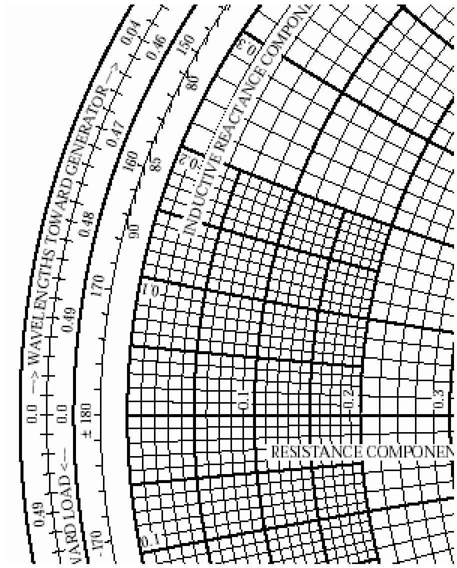
b) $Z_L = 48.5\ \Omega$

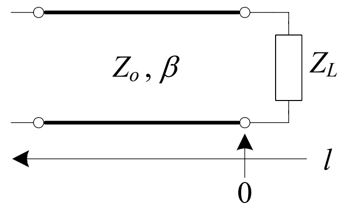
c) $Z_L = 75 + j25\ \Omega$

d) $Z_L = 10 - j5\ \Omega$



Smith Chart – Calculating Γ_l





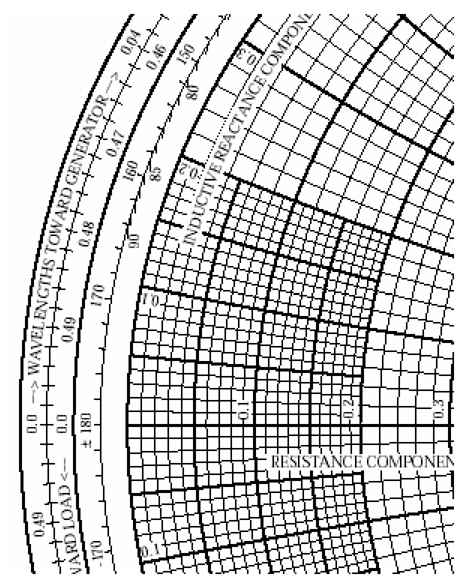
$$\Gamma_l(l) = \frac{V_o^- e^{-j\beta l}}{V_o^+ e^{j\beta l}} = \frac{V_o^-}{V_o^+} e^{-2j\beta l} = \Gamma e^{-2j\beta l}$$

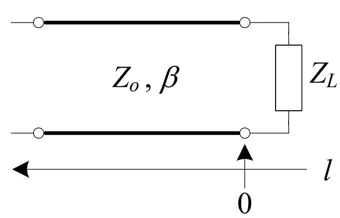
Since $\beta = \dfrac{2\pi}{\lambda}$

$$\Gamma_l(l) = \Gamma e^{-2j\frac{2\pi}{\lambda}l} \qquad \text{(Period} = \lambda/2)$$

$\dfrac{l}{\lambda}$ is the length in wavelngths

Smith Chart – Calculating Z_{in}





$$\Gamma_l(l) = \Gamma e^{-2j\frac{2\pi}{\lambda}l}$$

$\dfrac{l}{\lambda}$ is the length in wavelngths

$$\Gamma_l = \frac{Z_{in}(l) - Z_o}{Z_{in}(l) + Z_o}$$

Smith Chart – Calculating Γ_l and Z_{in}

Example:

$Z_o = 50\Omega$

$Z_L = 25 + j75\ \Omega$

$l = 1\text{cm}$

$f = 1\text{GHz}$

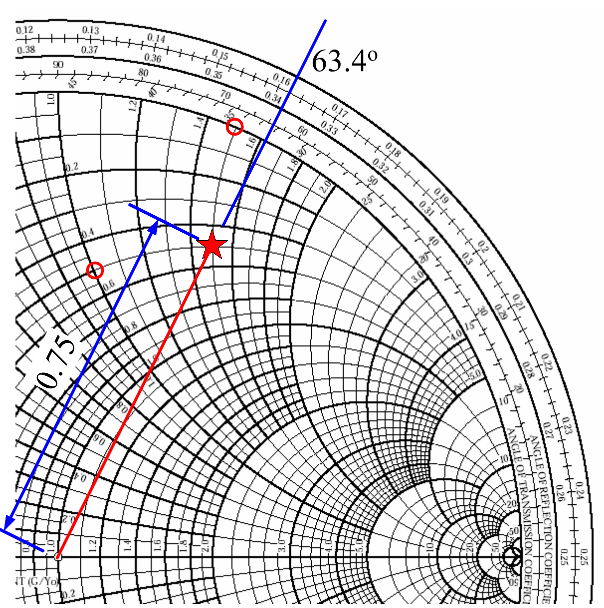
$\varepsilon_r = 4$

$\Gamma_l = ?$

$Z_{in} = ?$

$z_L = 0.5 + j1.5$

$\Gamma \approx 0.75\ ^{\angle 63.4^\circ}$



Smith Chart – Calculating Γ_l and Z_{in} (cont)

Example:

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$Z_L = 25 + j75\ \Omega$

$l = 1\text{cm}$

$f = 1\text{GHz}$

$\varepsilon_r = 4$

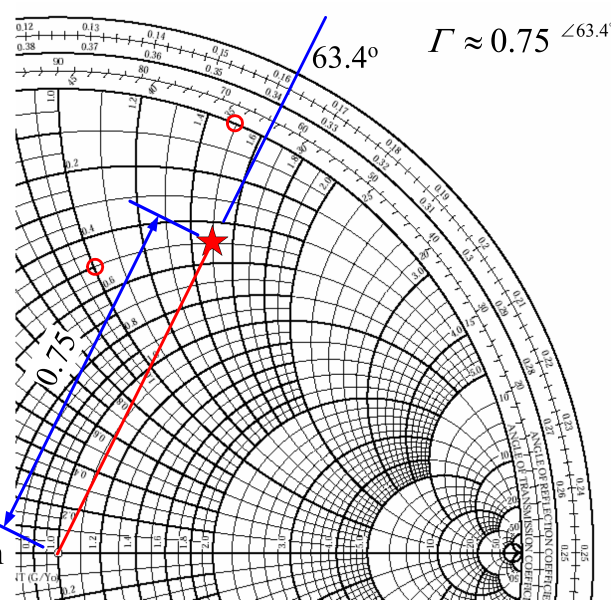
$\Gamma_l = ?$

$Z_{in} = ?$

$$\lambda = \frac{v_p}{f} = \frac{c}{f\sqrt{\varepsilon_r}}$$

$$\lambda = \frac{0.3\text{Gm/s}}{1\text{GHz}\sqrt{4}} = 15\text{cm}$$

$\Gamma \approx 0.75\ ^{\angle 63.4^\circ}$



Smith Chart – Calculating Γ_l and Z_{in} (cont)

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$Z_o = 50\Omega$

$Z_L = 25 + j75\ \ \Omega$

$l = 1\text{cm}$

$f = 1\text{GHz}$

$\varepsilon_r = 4$

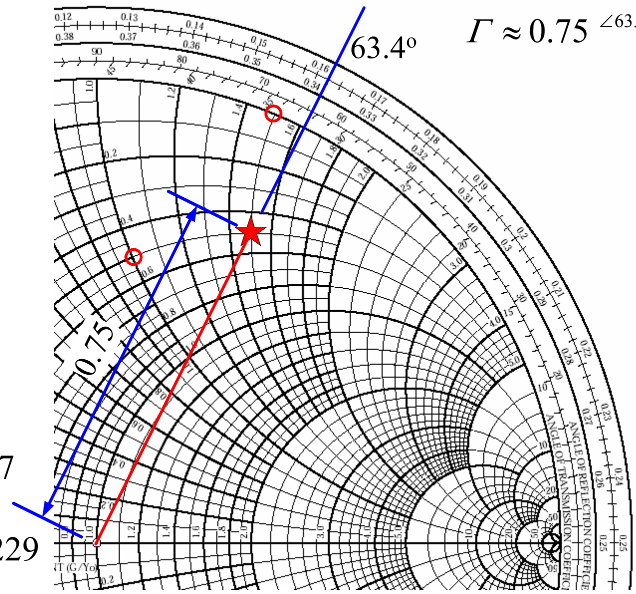
$\Gamma_l = ?$

$Z_{in} = ?$

$$\frac{l}{\lambda} = \frac{1\text{cm}}{15\text{cm}} = 0.067$$

$$0.162 + 0.067 = 0.229$$

$\Gamma \approx 0.75\ ^{\angle 63.4^\circ}$



Smith Chart – Calculating Γ_l and Z_{in} (cont)

Example:

$Z_o = 50\Omega$

$Z_L = 25 + j75\ \ \Omega$

$l = 1\text{cm}$

$f = 1\text{GHz}$

$\varepsilon_r = 4$

$\Gamma_l = ?$

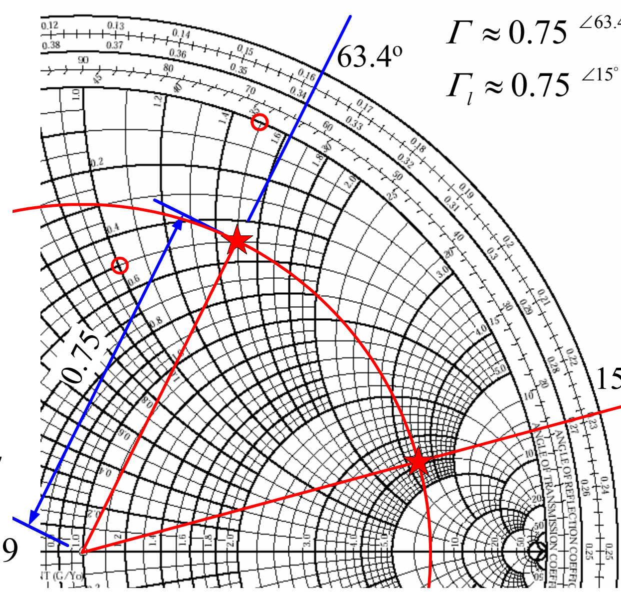
$Z_{in} = ?$

$$\frac{l}{\lambda} = \frac{1\text{cm}}{15\text{cm}} = 0.0667$$

$$0.162 + 0.067 = 0.229$$

$\Gamma \approx 0.75\ ^{\angle 63.4^\circ}$

$\Gamma_l \approx 0.75\ ^{\angle 15^\circ}$



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Smith Chart – Calculating Γ_l and Z_{in} (cont)

Example:

$Z_o = 50\Omega$

$Z_L = 25 + j75\ \Omega$

$l = 1\text{cm}$

$f = 1\text{GHz}$

$\varepsilon_r = 4$

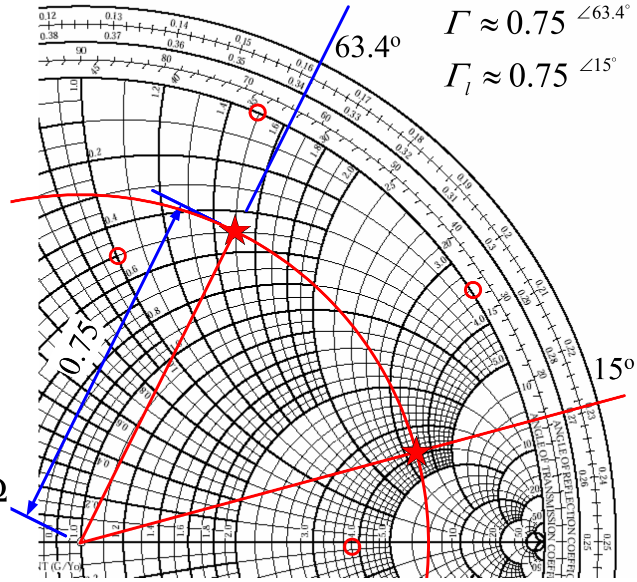
$\Gamma_l = ?$

$Z_{in} = ?$

$z_{in} = 3.8 + j3.4$

$Z_{in} = 50(3.8 + j3.4)\ \Omega$

$Z_{in} \approx 190 + j170\ \Omega$



$\Gamma \approx 0.75\ \angle 63.4^\circ$

$\Gamma_l \approx 0.75\ \angle 15^\circ$

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Smith Chart – Circles of SWR, V_{max} and V_{min}

Since $|V(l)| = |V_o^+|\ |1 + |\Gamma| e^{+j(\theta - 2\beta l)}|$

where $\Gamma = |\Gamma| e^{j\theta}$

We can locate the positions of V_{max} and V_{min} along the line

Example:

$Z_o = 50\Omega$

$Z_L = 60 + j40\ \Omega$

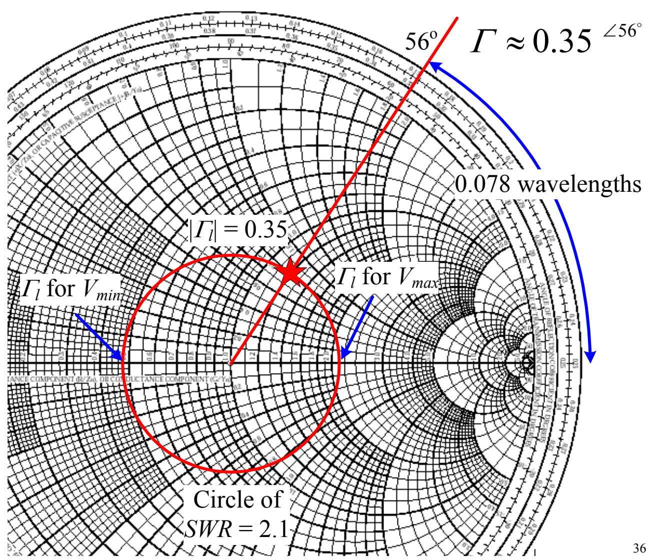
$z_L = 1.2 + j0.8\ \Omega$

$SWR \approx 2.1$

$WL_{\max} = 0.25 - 0.172$

$WL_{\max} = 0.078$

$\Gamma \approx 0.35\ \angle 56^\circ$



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