Transmission Line Theory

(Part 2)

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Outline

- Summary on lossy transmission lines
- Lossless transmission lines
- Reflection coefficient
- Power along the line
- Return loss
- Standing wave ratio
- Summary on reflection parameters

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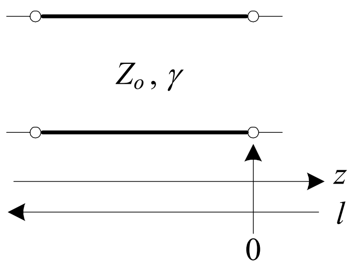
Lossy Transmission Line



$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z}$$

$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{+\gamma z}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \equiv \alpha + j\beta$$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \Gamma_z(z) = \frac{V_o^-}{V_o^+} e^{+2\gamma z}$$

Since $Z_o \equiv \frac{V_o^+}{I_o^+} = \frac{V_o^-}{-I_o^-}$ then $I(z) = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{+\gamma z}$

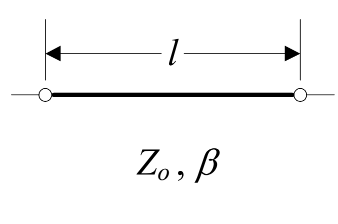
Since $\Gamma = \frac{V_o^-}{V_o^+}$ then $V(z) = V_o^+ [e^{-\gamma z} + \Gamma e^{+\gamma z}]$

$$I(z) = \frac{V_o^+}{Z_o} [e^{-\gamma z} - \Gamma e^{+\gamma z}]$$

Lossless Transmission Line

- Conductive and dielectric losses are zero
- It is a distortion-less transmission line
- Ideally, practical interconnects should behave as lossless transmission lines
- For analytical purposes (first order approximations), many practical interconnects can be treated as lossless

Symbol:



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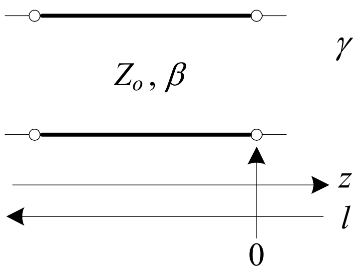
Lossless Transmission Line (cont)



$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = j\omega\sqrt{LC} = j\beta$$

$$\beta = \omega\sqrt{LC}$$

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z}$$

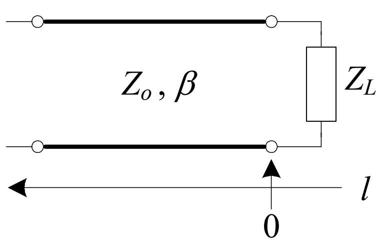
$$I(z) = I_o^+ e^{-j\beta z} + I_o^- e^{+j\beta z}$$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}}$$ (Z_o is frequency independent)

$$\lambda = \frac{v_p}{f} = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}}$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$ (All waves propagate at the same speed)

Reflection Coefficient, Γ



$$Z_o = \sqrt{\frac{L}{C}} \qquad \beta = \omega\sqrt{LC}$$

$$\Gamma_l(l) = \frac{V_o^- e^{-j\beta l}}{V_o^+ e^{j\beta l}} = \frac{V_o^-}{V_o^+} e^{-2j\beta l}$$

$$\Gamma = \Gamma_l(l = 0) = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$V(z) = V_o^+ [e^{-j\beta z} + \Gamma e^{+j\beta z}]$$

$$I(z) = \frac{V_o^+}{Z_o} [e^{-j\beta z} - \Gamma e^{+j\beta z}]$$

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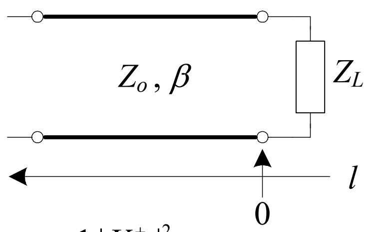
Average Power along the Transmission Line



$$V(z) = V_o^+[e^{-j\beta z} + \Gamma e^{+j\beta z}]$$

$$I(z) = \frac{V_o^+}{Z_o}[e^{-j\beta z} - \Gamma e^{+j\beta z}]$$

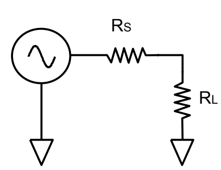
$$P_{av} = \frac{1}{2}\mathrm{Re}\{V(z)I(z)^*\}$$

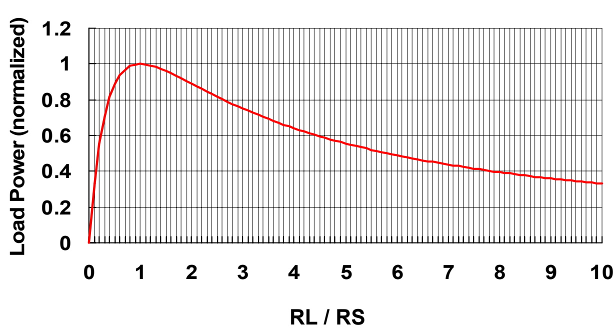
$$P_{av} = \frac{1}{2}\frac{|V_o^+|^2}{Z_o}\mathrm{Re}\{1 - \Gamma^* e^{-2j\beta z} + \Gamma e^{2j\beta z} - |\Gamma|^2\}$$

$$P_{av} = \frac{1}{2}\frac{|V_o^+|^2}{Z_o}(1 - |\Gamma|^2)$$

- The average power flow is constant along the line
- The power delivered is the incident power minus the reflected power
- If $\Gamma = 0$, maximum power is delivered to the load
- If $|\Gamma| = 1$, no power is delivered to the load

Power Transfer Efficiency





Maximum power is transferred when $R_L = R_S$

(Hewlett-Packard's RF Design and Measurement Seminar, 2000)

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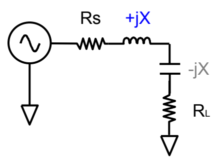
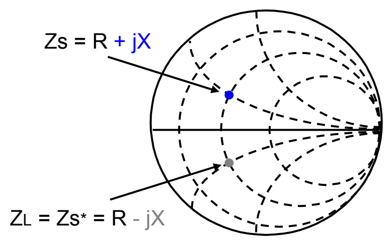
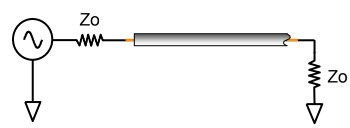
Power Transfer Efficiency (cont)

For complex impedances, maximum power transfer occurs when $Z_L = Z_S^*$ (conjugate match)





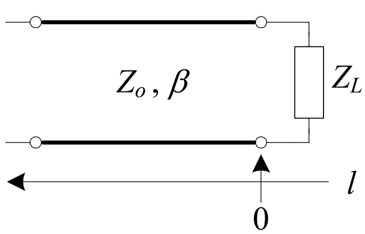
At high frequencies, maximum power transfer occurs when $R_S = R_L = Z_o$



(Hewlett-Packard's RF Design and Measurement Seminar, 2000)

Return Loss, *RL*

- It is a way to measure the ability of the TL to avoid energy reflection
- It is the ratio of the reflected power to the incident power



$$P_{av} = \frac{1}{2}\frac{|V_o^+|^2}{Z_o}(1-|\Gamma|^2)$$

$$RL = -10\log|\Gamma|^2 \quad \text{(dB)}$$

$$RL = -20\log|\Gamma| \quad \text{(dB)}$$

$$RL(\Gamma = 0) = \infty \text{ dB} \qquad RL(|\Gamma| = 1) = 0 \text{ dB}$$

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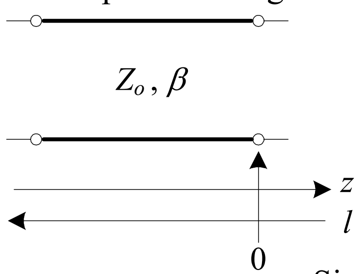
Standing Wave Ratio, *SWR*

- It is a way to measure the mismatch of a line
- A mismatch implies a “standing wave” along the line
- It is the ratio of the maximum to the minimum wave amplitude along the line



$$V(z) = V_o^+[e^{-j\beta z} + \Gamma e^{+j\beta z}]$$

$$e^{+j\beta z}V(z) = V_o^+[1 + \Gamma e^{+j2\beta z}]$$

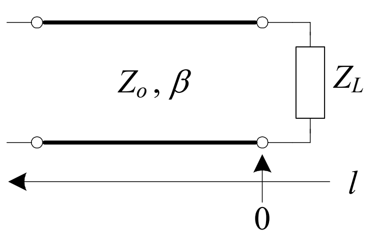
$$|V(z)| = |V_o^+||1 + \Gamma e^{+j2\beta z}|$$

$$|V(l)| = |V_o^+||1 + \Gamma e^{-j2\beta l}|$$

Since $\Gamma = |\Gamma| e^{j\theta}$

$$|V(l)| = |V_o^+| \; |1 + |\Gamma| e^{+j(\theta - 2\beta l)}|$$

Standing Wave Ratio, *SWR* (cont)



$$|V(l)| = |V_o^+| \; |1 + |\Gamma| e^{+j(\theta - 2\beta l)}|$$

$|V(l)|$ is maximum when $e^{+j(\theta - 2\beta l)} = 1$

$|V(l)|$ is minimum when $e^{+j(\theta - 2\beta l)} = -1$

$$|V(l)|_{\max} = V_{\max} = |V_o^+|(1 + |\Gamma|)$$

$$|V(l)|_{\min} = V_{\min} = |V_o^+|(1 - |\Gamma|)$$

$$SWR = \frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \qquad 1 \le SWR \le \infty$$

- The distance between two maxima or two minima is $\lambda/2$
- The distance between a maximum and a minimum is $\lambda/4$

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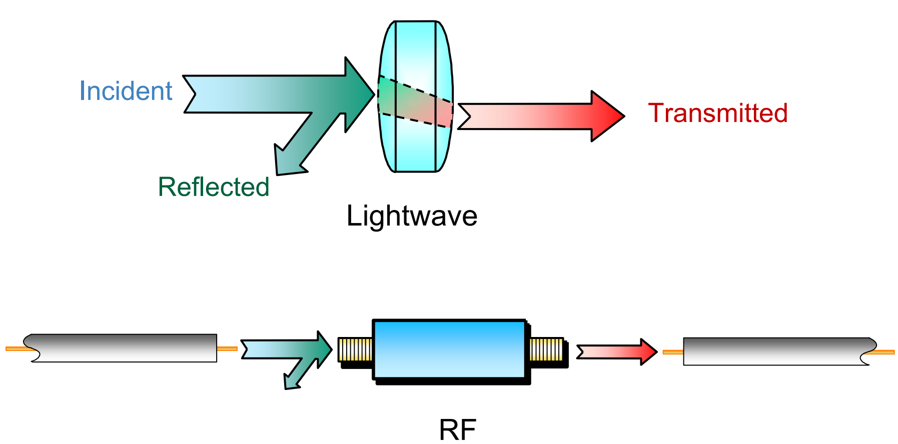
Reflection Parameters - Summary

- Reflection coefficient, Γ_l
- Return Loss, *RL*
- Standing Wave Ratio, *SWR*

Lightwave Analogy to RF Energy



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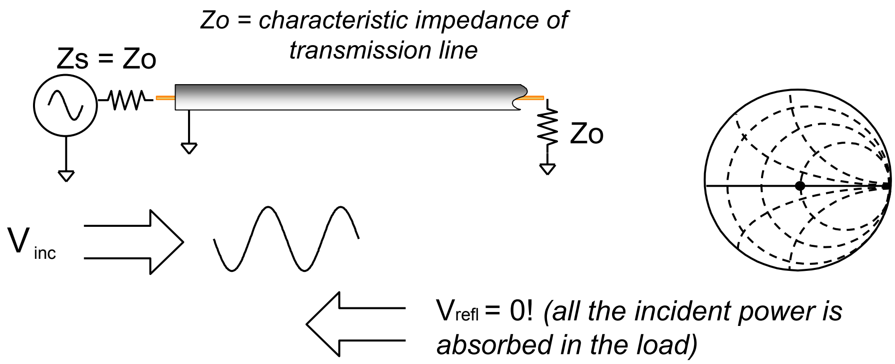
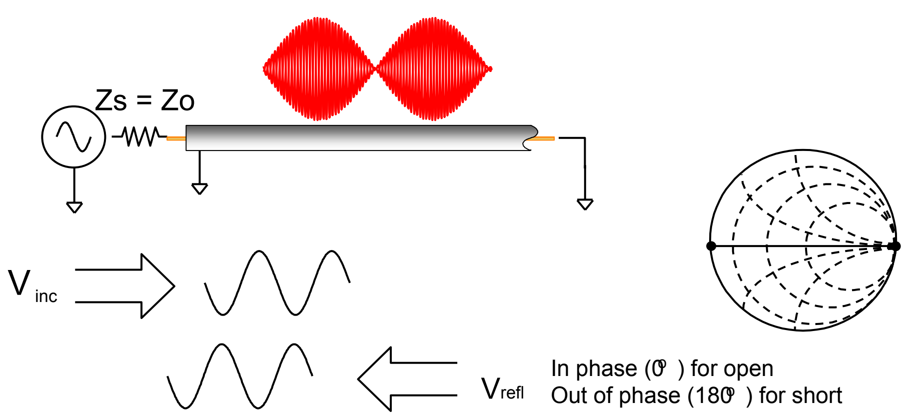
Transmission Line Terminated with Zo



For reflection, a transmission line terminated in Zo behaves like an infinitely long transmission line

(Hewlett-Packard's RF Design and Measurement Seminar, 2000)

Transmission Line Terminated with Short, Open



For reflection, a transmission line terminated in a short or open reflects all power back to source

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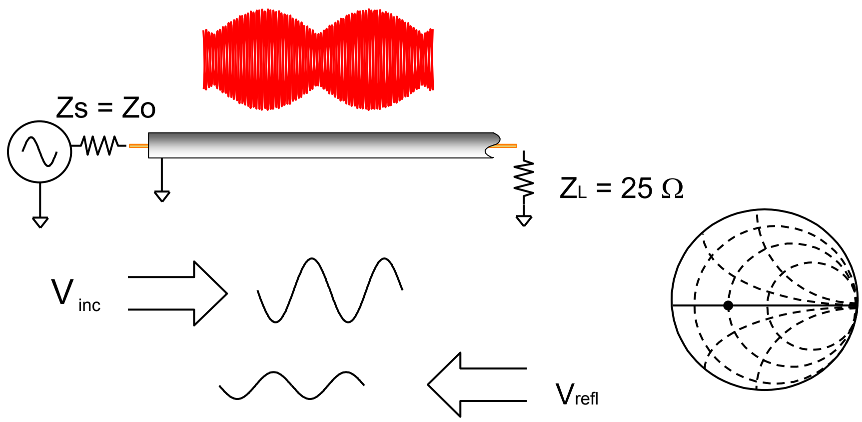
Transmission Line Terminated with 25 Ω



Standing wave pattern does not go to zero as with short or open

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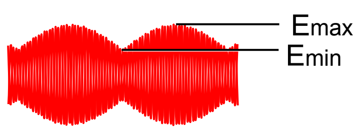
Reflection Parameters

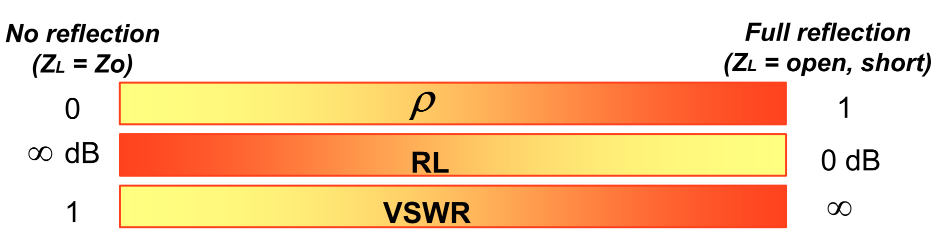
Reflection Coefficient $\Gamma = \frac{V_{reflected}}{V_{incident}} = \rho \angle \Phi = \frac{Z_L - Z_O}{Z_L + Z_O}$

Return loss = -20 log(ρ), $\rho = |\Gamma|$



Voltage Standing Wave Ratio

$$\textbf{VSWR} = \frac{E_{max}}{E_{min}} = \frac{1+\rho}{1-\rho}$$



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