Transmission Line Theory

(Part 1)

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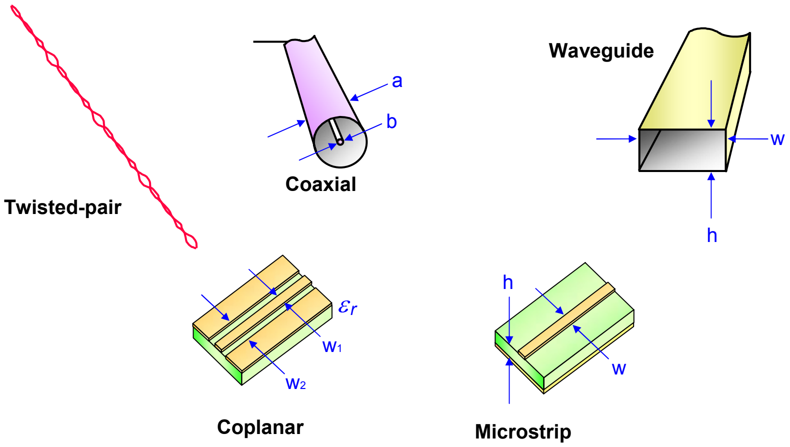
Outline

- Common transmission media
- Modeling uniform interconnects
- Interconnect parasitics and their physical significance
- From lumped circuits to distributed circuits
- Fundamental transmission line equations

Common Transmission Media

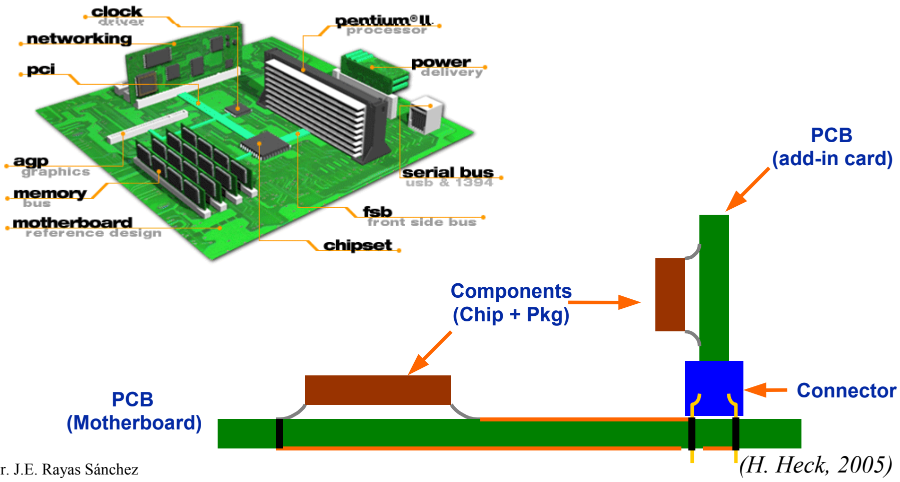
- Uniform Interconnects



(Hewlett-Packard's RF Design and Measurement Seminar, 2000)

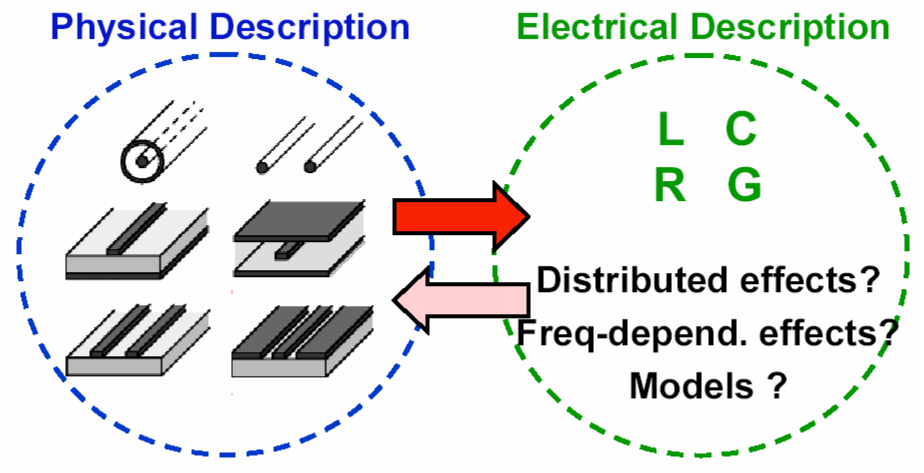
Common Transmission Media (cont)

- Practical interconnects can be decomposed in segments of uniform interconnects (if necessary)



(H. Heck, 2005)

Modeling Uniform Interconnects



Physical Description

Electrical Description

L C
R G

Distributed effects?
Freq-depend. effects?
Models ?

(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

Modeling Uniform Interconnects (cont)

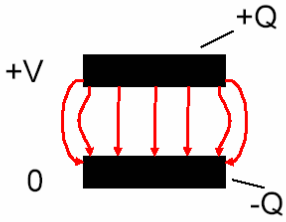
Cross-sectional view of typical uniform interconnects:

- Parasitic effects associated to each transmission media
  - – Capacitance between conductors, C
  - – Resistance of conductors (conductor losses), *R*
  - – Inductance of conductor loops, *L*
  - – Dielectric conductivity (dielectric losses), *G*
- *R*, *C*, *L,* and *G* must be determined per unit length

Interconnect Shunt Capacitance

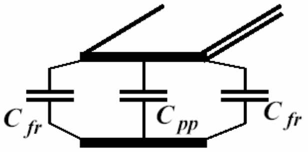
- capacitance

$$\boxed{C = Q/V}$$



- parallel-plate capacitor and field-fringing effect

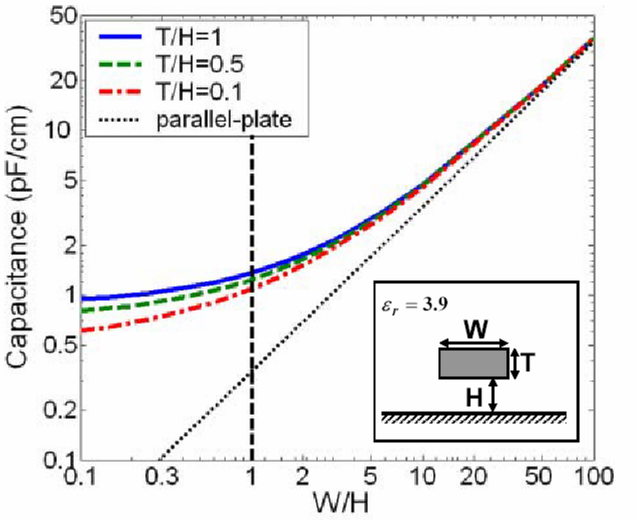
$$\boxed{C_{pp}/l = \varepsilon_0 \varepsilon_r \frac{w}{d}}$$



➢ Actual capacitance is **larger** than parallel-plate capacitance due to fringing fields

(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

Fringing Capacitance Effect

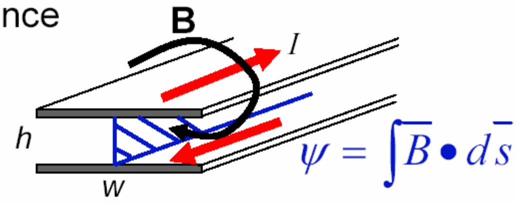


(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

Interconnect Series Inductance

- **External** Inductance

$$\boxed{\frac{L}{l} = \frac{\psi / l}{I}}$$



$$\psi = \int \overline{B} \bullet d\overline{s}$$

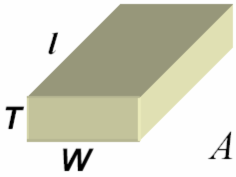
- parallel-plate inductance $\frac{L_{pp}}{l} = \mu_0 \frac{h}{w}$
- closely spaced return path means smaller inductance
- **Internal** Inductance
  - associated with flux inside conductors

(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

Interconnect Series DC Resistance

- DC resistance

$$\boxed{\frac{R}{l} = \frac{\rho}{A}}$$

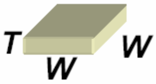


$$A = W * T$$

- Sheet resistance

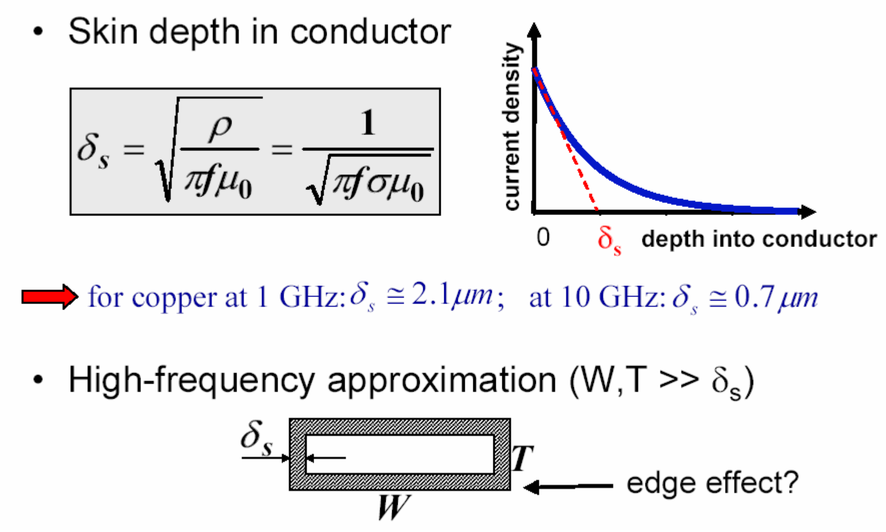
$$R = \rho l / A = \rho \frac{W}{W\,T} = \rho / T$$

$$\boxed{R_s = \rho / T}$$ (ohms per square)



(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

Skin Effect



- Skin depth in conductor

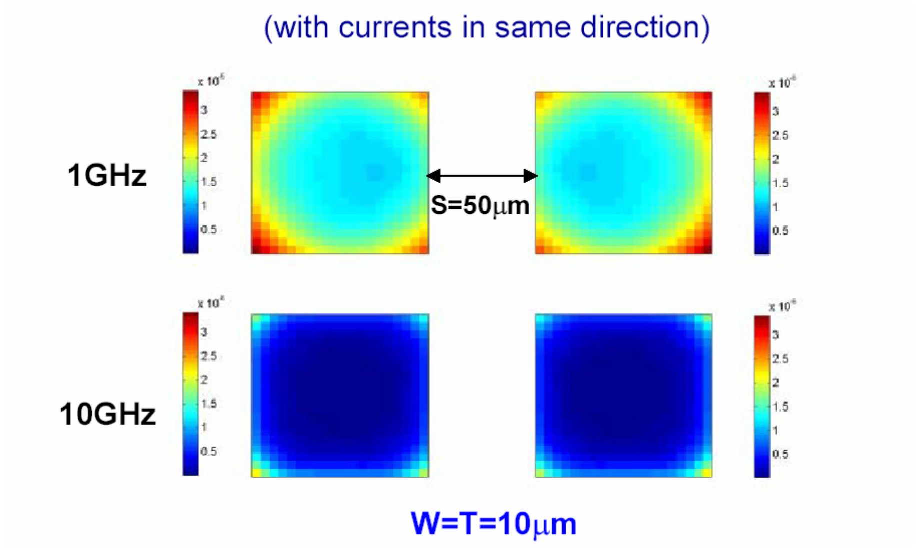
$$\delta_s = \sqrt{\frac{\rho}{\pi f \mu_0}} = \frac{1}{\sqrt{\pi f \sigma \mu_0}}$$

for copper at 1 GHz: $\delta_s \cong 2.1\mu m$; at 10 GHz: $\delta_s \cong 0.7\mu m$

- High-frequency approximation (W,T >> δ_s)

(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

EM-Simulation of Conductor Current Distribution



(with currents in same direction)

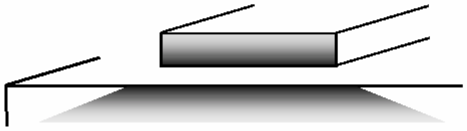
(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

Proximity Effect

- Opposing high frequency (HF) currents in close proximity are drawn to each other

HF current distribution in an **isolated** wire

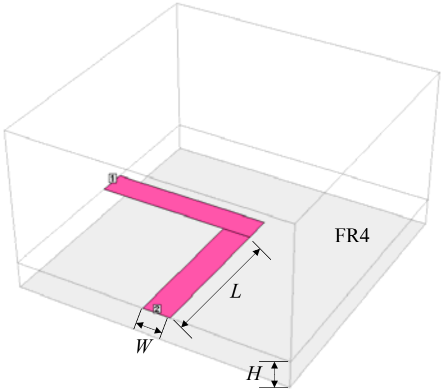
HF current distributions in two wires in close proximity carrying **opposing** currents



Return current in ground plane close to signal trace

(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

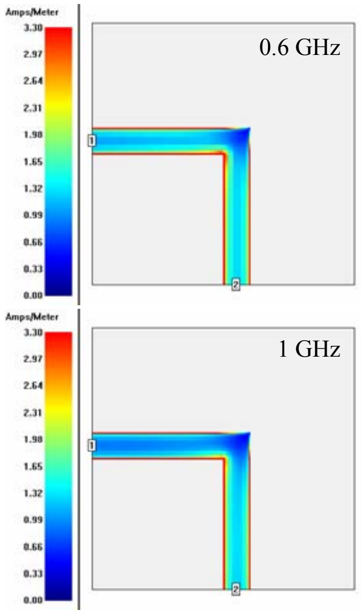
Edge and Indy Effects – Example



$W = 5$ mm $\quad L = 25$ mm

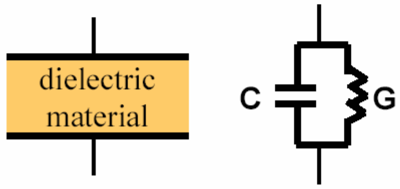
$H = 5$ mm $\quad \varepsilon_r = 4.5$

dielectric loss tan = 0.025



Interconnect Shunt Conductance

- shunt loss due to
  - ohmic losses (free charge carriers)
  - out-of-phase polarization (power loss due to frictional damping forces)
- simple model



loss tangent:

$$\tan\delta_d = G/\omega C$$
$$= \varepsilon''/\varepsilon'$$
where $\varepsilon = \varepsilon' - j\varepsilon''$

(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

Loss Tangent of Typical Materials

Dielectric Material	Loss Tangent	Dielectric Constant
Ceramic (Alumina)	0.001	9.4
Glass-epoxy	0.03	4.0
Glass (Quartz)	0.00006	3.8
Polyimide	0.01	3.5
Silicon (100 Ω-cm)	0.51	11.8
Silicon (10 Ω-cm)	5.1	11.8
Teflon	0.00015	2.1

at 3GHz

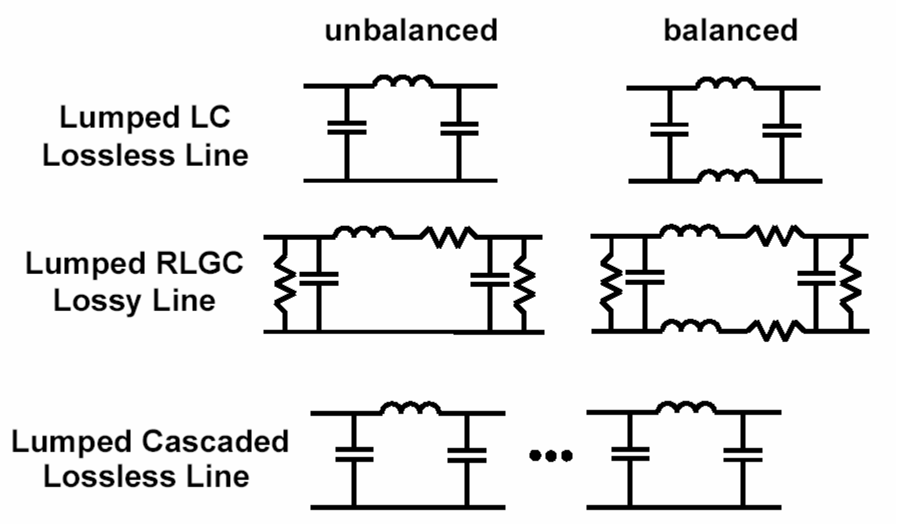
(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

Equivalent **Circuit** Models for Interconnects

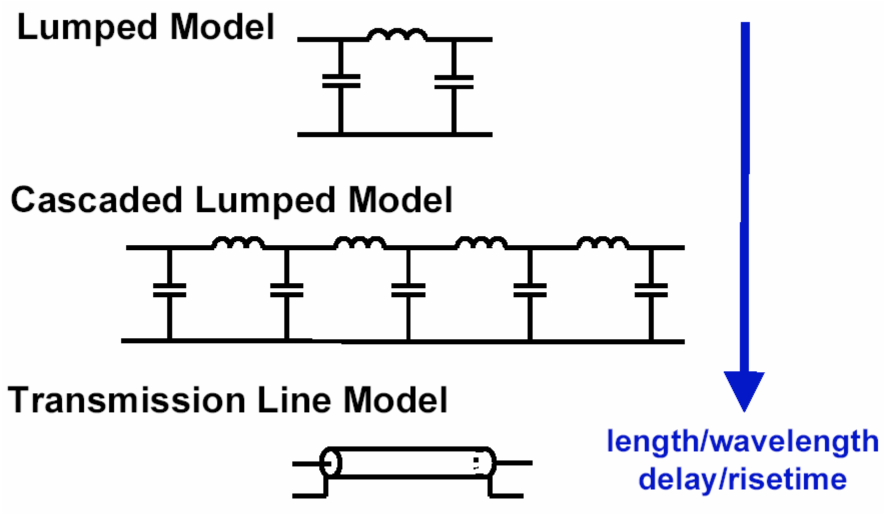
- On-chip Interconnects
  - lumped C if very short
  - lumped RC if very short and R significant
  - cascaded lumped RC if short
  - distributed RC if long
  - RLC distributed line (high performance VLSI circuits and microwave ICs)

- Off-chip Interconnects (→ usually distributed models)
  - LC line if losses can be ignored
  - RLC line on low dielectric loss PCBs
  - RLGC line (if dielectric loss is significant)

Lumped Equivalent Circuit Models

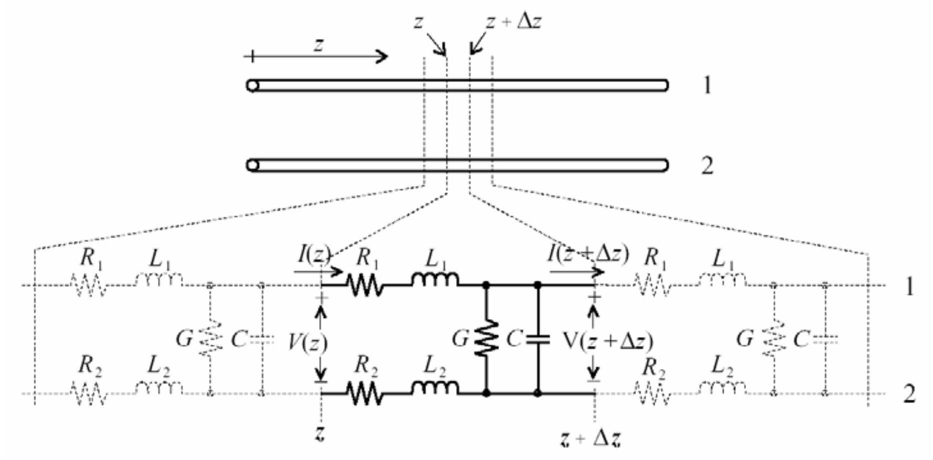


From Lumped Circuits to Distributed Circuits



(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

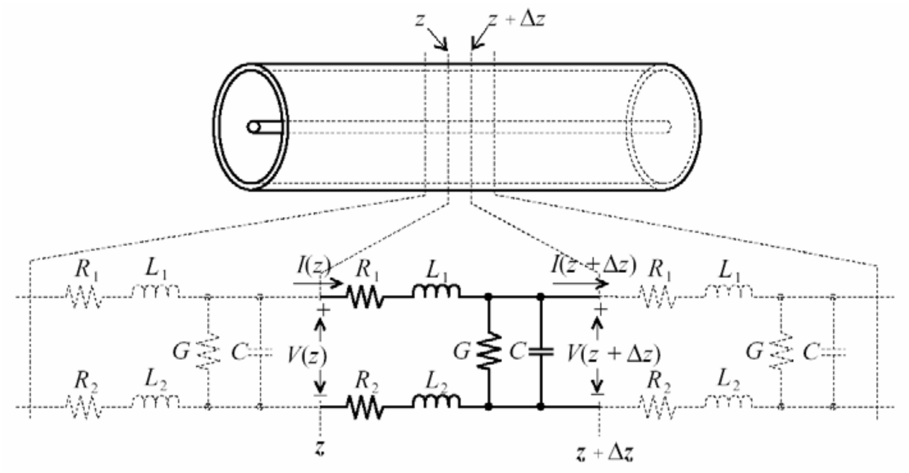
Transmission Line Model



The interconnect is modeled using an infinite number of RLCG sections

(R. Ludwig and P. Bretchko, RF Circuit Design, Prentice Hall, 2000)

Transmission Line Model



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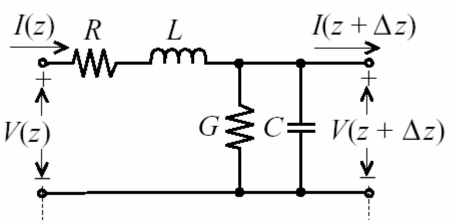
Transmission Line Model



- Generic equivalent circuit for each section (R, L, C and G are per unit length)
- The interconnect is modeled using an infinite number of these sections, making $\Delta z \rightarrow 0$

(R. Ludwig and P. Bretchko, RF Circuit Design, Prentice Hall, 2000)

Transmission Line Equations

- Time-Domain (Telegrapher Equations)

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

- Telegrapher Equations in Frequency-Domain

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z)$$

$$\frac{dI(z)}{dz} = -(G + j\omega C)V(z)$$

Transmission Line Equations (cont)

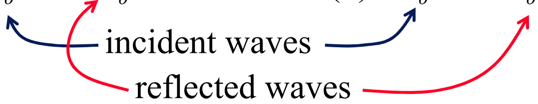
- Wave Equation (frequency domain)

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \qquad \frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0$$

where $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \equiv \alpha + j\beta$
is the complex propagation constant

- Solutions to the wave equation are

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z} \qquad I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{+\gamma z}$$



incident waves

reflected waves

Transmission Line Equations (cont)

- Characteristic Impedance of the TL

$$Z_o \equiv \frac{V_o^+}{I_o^+} = \frac{V_o^-}{-I_o^-}$$

then

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

- Reflection Coefficient along the line, Γ_z

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z}$$

$$\Gamma_z(z) = \frac{V_o^- e^{+\gamma z}}{V_o^+ e^{-\gamma z}} = \frac{V_o^-}{V_o^+} e^{+2\gamma z}$$

Transmission Line Equations (cont)

- Solutions in the frequency domain

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{+\gamma z} \qquad I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{+\gamma z}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \equiv \alpha + j\beta$$

- Solutions in the time domain

$$v(z,t) = |V_o^+| \cos(\omega t - \beta z + \phi^+) e^{-\alpha z} + |V_o^-| \cos(\omega t + \beta z + \phi^-) e^{+\alpha z}$$

- Wavelength

$$\lambda = \frac{v_p}{f} = \frac{2\pi}{\beta}$$

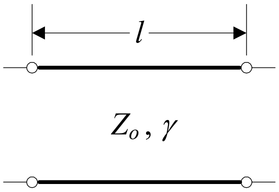
- Phase velocity, wave velocity or propagation speed

$$v_p = \frac{dz}{dt} = \frac{\omega}{\beta}$$

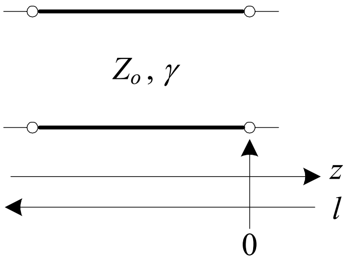
(speed at which a constant phase point travels down the line)

Transmission Line Symbol

- (Lossy) Transmission Line

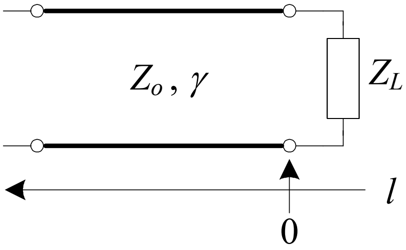


- Length along the line



Reflection Coefficient

- Reflection coefficient along the line



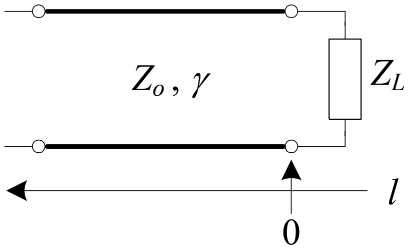
$$\Gamma_l(l) = \frac{V_o^- e^{-\gamma l}}{V_o^+ e^{+\gamma l}} = \frac{V_o^-}{V_o^+} e^{-2\gamma l}$$

- Reflection coefficient at the load

$$\Gamma = \Gamma_l(l = 0) = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

Input Impedance

- Input Impedance along the line



$$Z_{in}(l) = \frac{V(l)}{I(l)} = Z_o \frac{Z_L + Z_o \tanh(\gamma l)}{Z_o + Z_L \tanh(\gamma l)}$$