Frequency-Domain Analysis of Transmission Line Circuits

(Part 4)

Dr. José Ernesto Rayas Sánchez

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Outline

- Walker's formulae for 2-coupled microstrip lines
- Variation of the LC-parameters with the separation of the lines
- Variation of Z_o with the separation of the lines
- Variation of v_p with the separation of the lines
- Conclusions

Dr. J.E. Rayas Sánchez

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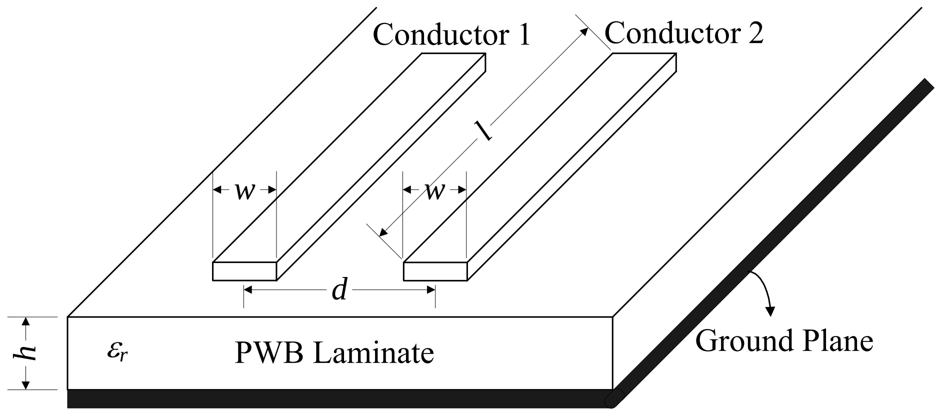
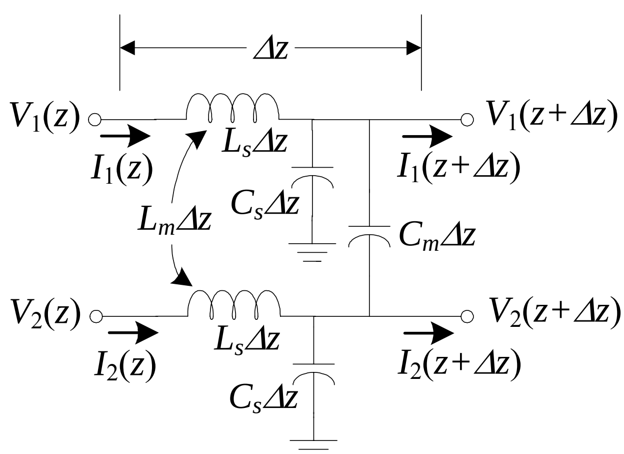
Two-Coupled Microstrip Lines



In the following analysis, we neglect the thickness of each conductor and losses

Lossless Transmission Line Model



Dr. J. E. Rayas Sánchez
http://iteso.mx/~erayas erayas@iteso.mx

Walker's Formulae for Cs

$$C_s = \varepsilon_r \varepsilon_o K_C \left(\frac{w}{h}\right) \quad \text{F/m}$$

$$\varepsilon_o = 8.854 \times 10^{-12} \quad \text{(F/m)}$$
$$\mu_o = 4\pi \times 10^{-7} \quad \text{(H/m)}$$

$$L_s = \frac{\mu_r \mu_o}{K_L}\left(\frac{h}{w}\right) \quad \text{H/m}$$

$$C_m = \frac{\varepsilon_r \varepsilon_o}{4\pi} K_C K_L \left(\frac{w}{h}\right)^2 \ln\left[1 + \left(\frac{2h}{d}\right)^2\right] \quad \text{F/m}$$

$$L_m = \frac{\mu_r \mu_o}{4\pi} \ln\left[1 + \left(\frac{2h}{d}\right)^2\right] \quad \text{H/m}$$

where K_C and K_L are the fringing factors given by …

 C.S. Walker, *Capacitance, Inductance and Crosstalk Analysis*. Norwood, MA: Artech House, 1990.

Walker's Formulae (cont)

$$K_C = \left[\frac{120\pi}{Z_{o(\varepsilon_r=1)}}\left(\frac{h}{w}\right)\sqrt{\frac{\varepsilon_e}{K_L \varepsilon_r}}\right]^2 \qquad K_L = \frac{120\pi}{Z_{o(\varepsilon_r=1)}}\left(\frac{h}{w}\right)$$

where

$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\frac{1}{\sqrt{1 + 10h/w}}$$

$$\text{if } \frac{w}{h} \le 1, \quad Z_{o(\varepsilon_r=1)} = 60\ln\left(\frac{8h}{w} + \frac{w}{4h}\right) \quad \Omega$$

$$\text{if } \frac{w}{h} \ge 1, \quad Z_{o(\varepsilon_r=1)} = \frac{120\pi}{\left(\frac{w}{h}\right) + 2.42 - 0.44\left(\frac{h}{w}\right) + \left(1 - \frac{h}{w}\right)^6} \quad \Omega$$

 C.S. Walker, *Capacitance, Inductance and Crosstalk Analysis*. Norwood, MA: Artech House, 1990.

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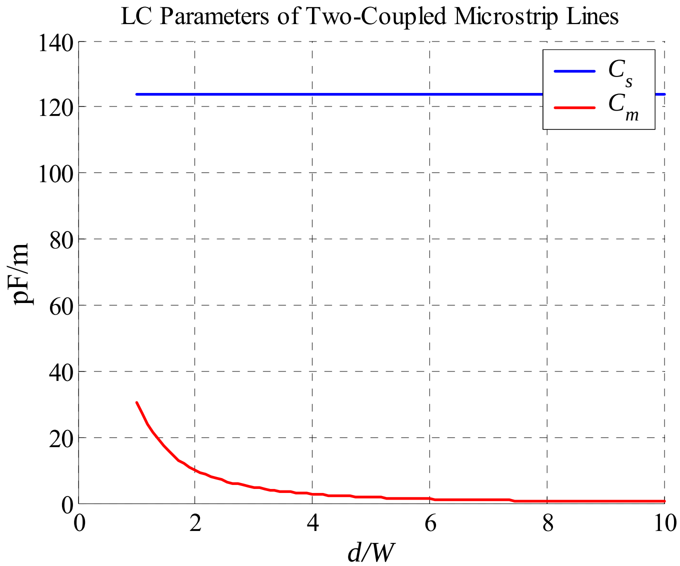
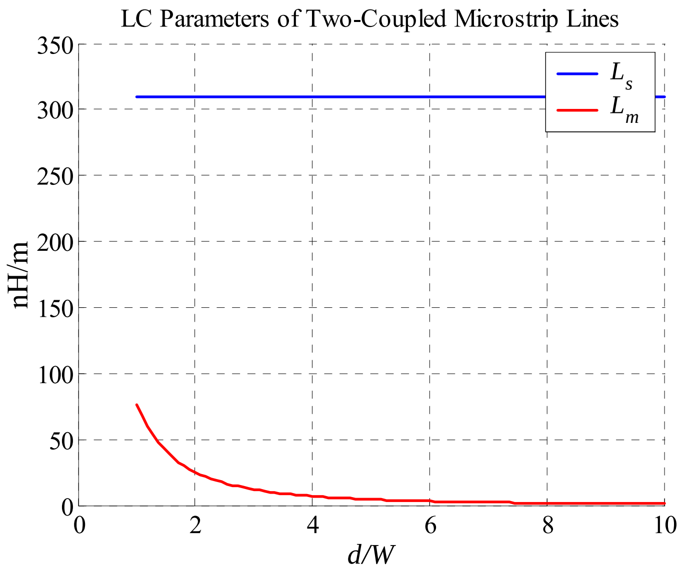
Variation of Capacitances – Example



$W = 74.65\text{mil}; H = 40\text{mil}; \varepsilon_r = 4.5$

Variation of Inductances – Example



$W = 74.65\text{mil}; H = 40\text{mil}; \varepsilon_r = 4.5$

Z_o and v_p for Even and Odd Modes

- If Z_o is the characteristic impedance of each isolated conductor, and v_p is the propagation velocity or wave speed in each isolated conductor

$$Z_o = \sqrt{\frac{L_s}{C_s}} \qquad v_p = \frac{1}{\sqrt{L_s C_s}}$$

- Since

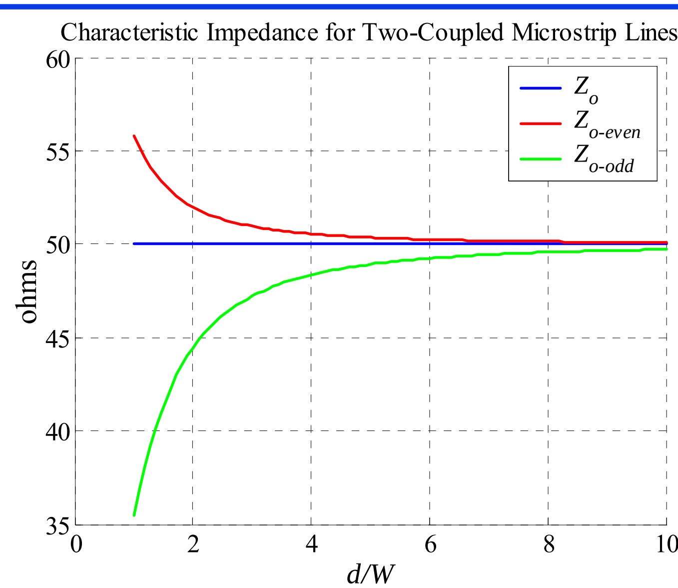
$$Z_{o-even} = \sqrt{\frac{L_s + L_m}{C_s}} \qquad v_{p-even} = \frac{1}{\sqrt{(L_s + L_m)C_s}}$$

$$Z_{o-odd} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \qquad v_{p-odd} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}$$

- then

$$Z_{o-odd} < Z_o < Z_{o-even} \qquad v_{p-even} < v_p$$

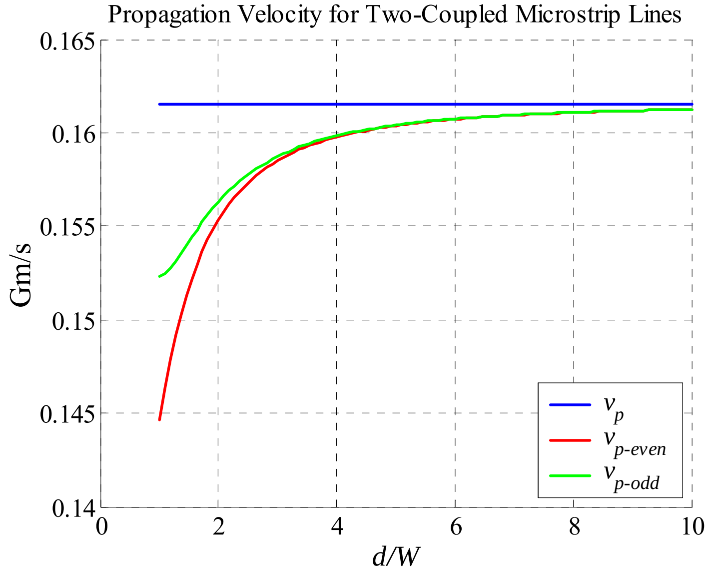
Variation of Z_o for Even and Odd Modes



$W = 74.65\text{mil};\ H = 40\text{mil};\ \varepsilon_r = 4.5$

Dr. J. E. Rayas Sánchez
http://iteso.mx/~erayas erayas@iteso.mx

Variation of v_p for Even and Odd Modes



$W = 74.65\text{mil};\ H = 40\text{mil};\ \varepsilon_r = 4.5$

Conclusions

- It has been verified that

$$Z_{o-odd} < Z_o < Z_{o-even} \qquad v_{p-even} < v_{p-odd} < v_p$$

- If the coupled lines are very separated (large d)

$$C_m, L_m \rightarrow 0 \qquad Z_{o-odd}, Z_{o-even} \rightarrow Z_o \qquad v_{p-odd}, v_{p-even} \rightarrow v_p$$

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