# Frequency-Domain Analysis of Transmission Line Circuits

(Part 3)

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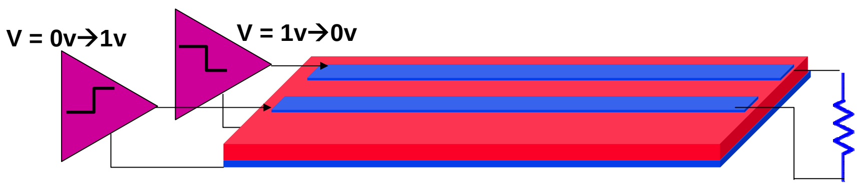
## Outline

- Differential transmission lines
- Common mode signaling
- Differential mode signaling
- Mode conversion
- Even and odd modes
- 2-coupled lossless transmission line theory
- Termination techniques
- Differential or Mixed-Mode S-parameters

## Differential Transmission Lines

For high data rates, differential signaling is more used due to:

- Radiation is reduced (cancellation of fields)
- Receiver rejects signals that are common to both lines (high CMRR at the receiver)
- Signal voltage amplitudes can be smaller

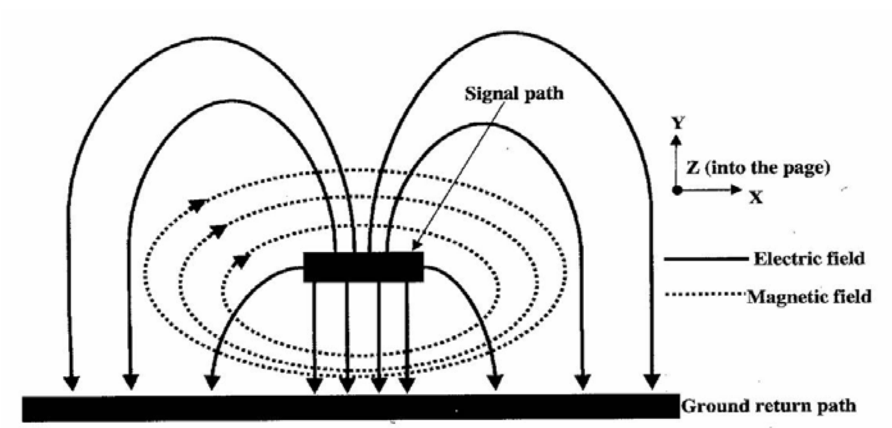





*(M. Resso, 2005)*

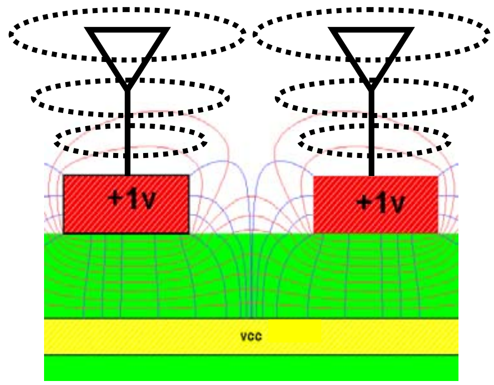


## Electromagnetic Fields in a Microstrip Line



## Common Mode Signaling

As data rates go up, frequencies increase, lines become antennas (both send and receive) and corrupt the communication (BER, crosstalk, etc)




Common Stimulus

E and H fields addition outside

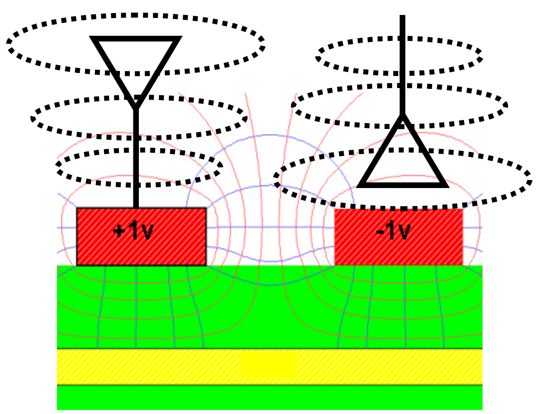


*(M. Resso, 2005)*



## Differential Mode Signaling

Using differential excitations (differential transmission lines), most of the outside electromagnetic field cancels




Differential Stimulus

E and H fields cancellation outside



*(M. Resso, 2005)*



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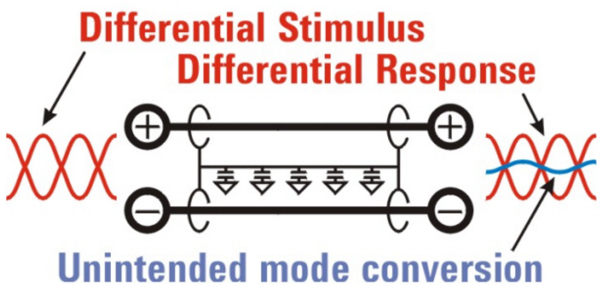
## Mode Conversion

- Is produced by asymmetries in the differential pairs
- Can cause a differential signal to be converted to a common mode signal (radiation, crosstalk, etc.)



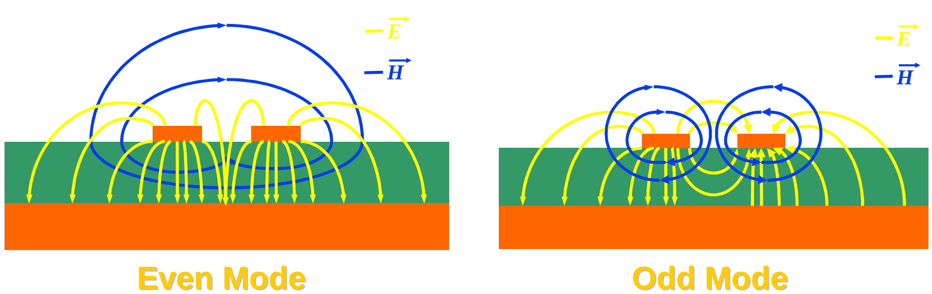



*(M. Resso, 2005)*



## Even Mode and Odd Mode

- Practical differential pairs operate at even and odd modes simultaneously
- Even mode – excited in phase with equal amplitudes
- Odd mode – driven 180º out of phase with equal amplitudes

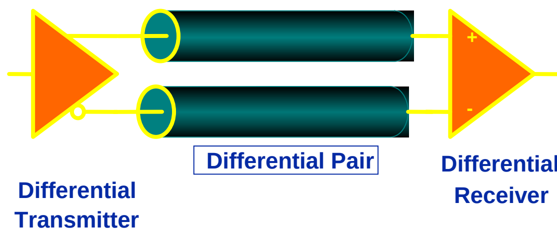





*(H. Heck 2002)*

## Differential Signaling for High-Speed Links

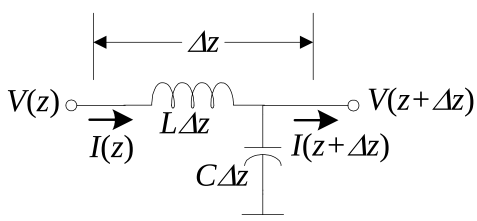
- Differential signaling can operate at much higher data rates
- High speed links operating in excess of ~1 Gb/s use differential signaling (e.g. Infiniband, PCI-Express).
- In fact, differential signals are already used for high speed clocks in desktop




(H. Heck 2002)



## Lossless Transmission Lines




$$\frac{dV(z)}{dz} = -(j\omega L)I(z)$$

$$\frac{dI(z)}{dz} = -(j\omega C)V(z)$$

$$\frac{dV}{dz} = -j\omega LI$$

$$\frac{dI}{dz} = -j\omega CV$$

$$-\frac{d}{dz}\begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} 0 & Z_L \\ Y_C & 0 \end{bmatrix}\begin{bmatrix} V \\ I \end{bmatrix}$$

$$Z_L = j\omega L \qquad Y_C = j\omega C$$

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{+j\beta z}$$

$$I(z) = I_o^+ e^{-j\beta z} + I_o^- e^{+j\beta z}$$

$$\beta = \omega\sqrt{LC}$$

$$Z_o = \sqrt{\frac{L}{C}} \qquad v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$



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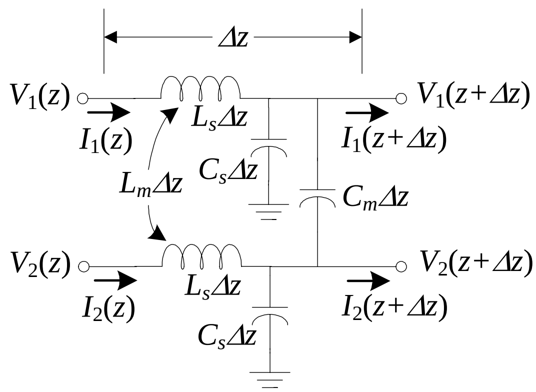
# 2-Coupled Lossless Symmetrical TLs




$$\frac{dV_1}{dz} = -j\omega L_s I_1 - j\omega L_m I_2 \qquad \frac{dI_1}{dz} = -j\omega(C_s + C_m)V_1 + j\omega C_m V_2$$

$$\frac{dV_2}{dz} = -j\omega L_m I_1 - j\omega L_s I_2 \qquad \frac{dI_2}{dz} = +j\omega C_m V_1 - j\omega(C_s + C_m)V_2$$



# 2-Coupled Lossless Symmetrical TLs (cont)

$$\frac{dV_1}{dz} = -j\omega L_s I_1 - j\omega L_m I_2$$

$$\frac{dV_2}{dz} = -j\omega L_m I_1 - j\omega L_s I_2$$

$$\frac{dI_1}{dz} = -j\omega(C_s + C_m)V_1 + j\omega C_m V_2$$

$$\frac{dI_2}{dz} = +j\omega C_m V_1 - j\omega(C_s + C_m)V_2$$

$$\boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \boldsymbol{I} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\boldsymbol{Y}_C = j\omega \begin{bmatrix} (C_s + C_m) & -C_m \\ -C_m & (C_s + C_m) \end{bmatrix}$$

$$\boldsymbol{Z}_L = j\omega \begin{bmatrix} L_s & L_m \\ L_m & L_s \end{bmatrix}$$

$$-\frac{d}{dz}\begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{I} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{Z}_L \\ \boldsymbol{Y}_C & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{I} \end{bmatrix}$$

## LC Matrices of 2-Coupled TLs

$$\boldsymbol{C} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} (C_s + C_m) & -C_m \\ -C_m & (C_s + C_m) \end{bmatrix}$$

$$\boldsymbol{L} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} = \begin{bmatrix} L_s & L_m \\ L_m & L_s \end{bmatrix}$$

$$Z_o = ?$$

$$v_p = ?$$



## Even Mode in 2-Coupled Symmetrical TLs

$$\frac{dV_1}{dz} = -j\omega L_s I_1 - j\omega L_m I_2 \qquad \frac{dV_2}{dz} = -j\omega L_m I_1 - j\omega L_s I_2$$

$$\frac{dI_1}{dz} = -j\omega(C_s + C_m)V_1 + j\omega C_m V_2 \qquad \frac{dI_2}{dz} = j\omega C_m V_1 - j\omega(C_s + C_m)V_2$$

Since $V_1 = V_2$ and $I_1 = I_2$

$$\frac{dV_1}{dz} = -j\omega(L_s + L_m)I_1 \qquad \frac{dI_1}{dz} = -j\omega C_s V_1$$

The effective $L$ and $C$ are

$$L_{eff} = L_s + L_m \qquad C_{eff} = C_s$$

Hence $$Z_{o-even} = \sqrt{\frac{L_s + L_m}{C_s}} \qquad v_{p-even} = \frac{1}{\sqrt{(L_s + L_m)C_s}}$$



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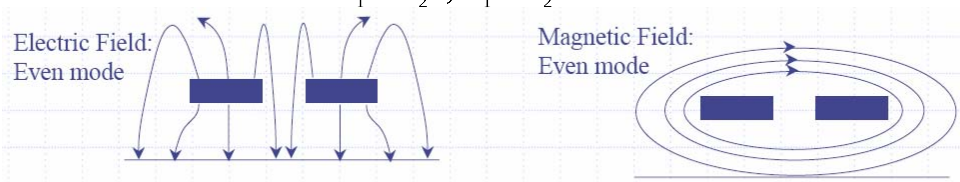
## Even Mode in 2-Coupled Symmetrical TLs (cont)

$$V_1 = V_2 \; ; \; I_1 = I_2$$




*(H. Mellitz 2003)*

$$Z_{o-even} = \sqrt{\frac{L_s + L_m}{C_s}} \qquad v_{p-even} = \frac{1}{\sqrt{(L_s + L_m)C_s}}$$

$Z_{o\text{-}even}$ is the characteristic impedance of one of the conductors when the coupled line is operated in even mode



## Odd Mode in 2-Coupled Symmetrical TLs

$$\frac{dV_1}{dz} = -j\omega L_s I_1 - j\omega L_m I_2 \qquad \frac{dV_2}{dz} = -j\omega L_m I_1 - j\omega L_s I_2$$

$$\frac{dI_1}{dz} = -j\omega(C_s + C_m)V_1 + j\omega C_m V_2 \qquad \frac{dI_2}{dz} = j\omega C_m V_1 - j\omega(C_s + C_m)V_2$$

Since $V_1 = -V_2$ and $I_1 = -I_2$

$$\frac{dV_1}{dz} = -j\omega(L_s - L_m)I_1 \qquad \frac{dI_1}{dz} = -j\omega(C_s + 2C_m)V_1$$

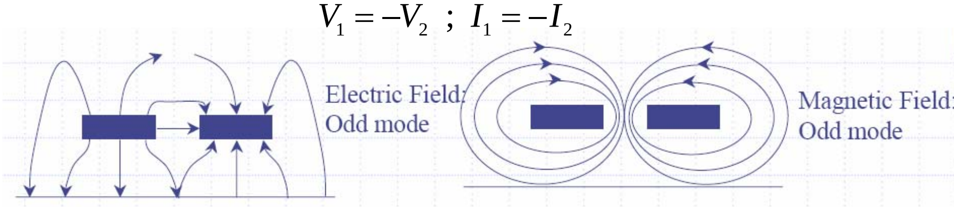
The effective $L$ and $C$ are

$$L_{eff} = L_s - L_m \qquad C_{eff} = C_s + 2C_m$$

Hence $$Z_{o-odd} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \qquad v_{p-odd} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}$$

## Odd Mode in 2-Coupled Symmetrical TLs (cont)

$$V_1 = -V_2 \;;\; I_1 = -I_2$$



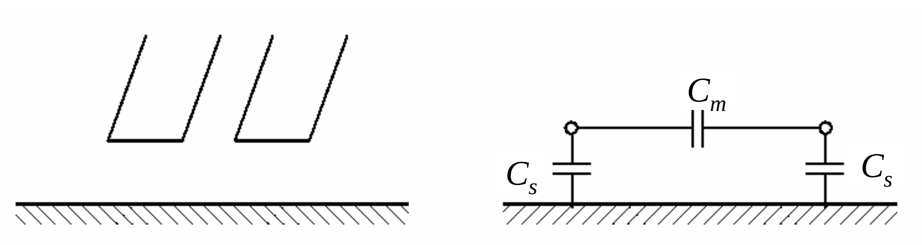

*(H. Mellitz 2003)*

$$Z_{o-odd} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \qquad v_{p-odd} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}$$

$Z_{o\text{-}odd}$ is the characteristic impedance of one of the conductors when the coupled line is operated in odd mode

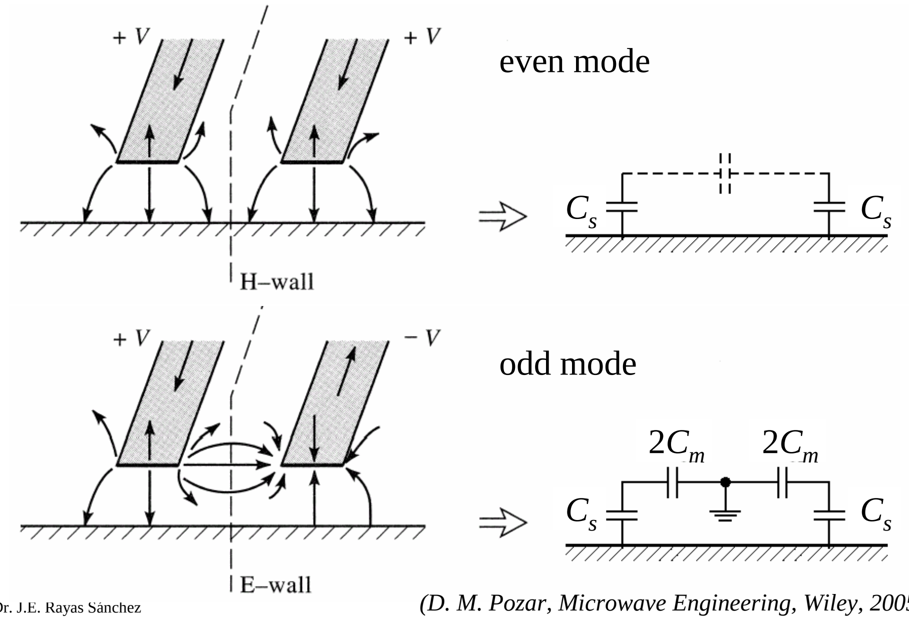


## Distributed Capacitances in Coupled Lines




*(D. M. Pozar, Microwave Engineering, Wiley, 2005)*

## Distributed Capacitances in Coupled Lines (cont)




(D. M. Pozar, Microwave Engineering, Wiley, 2005)



## $Z_o$ and $v_p$ for Even and Odd Modes

- If $Z_o$ is the characteristic impedance of each isolated conductor, and $v_p$ is the propagation velocity or wave speed in each isolated conductor

$$Z_o = \sqrt{\frac{L_s}{C_s}} \qquad v_p = \frac{1}{\sqrt{L_s C_s}}$$

- Since

$$Z_{o-even} = \sqrt{\frac{L_s + L_m}{C_s}} \qquad v_{p-even} = \frac{1}{\sqrt{(L_s + L_m)C_s}}$$

$$Z_{o-odd} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \qquad v_{p-odd} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}$$

- then

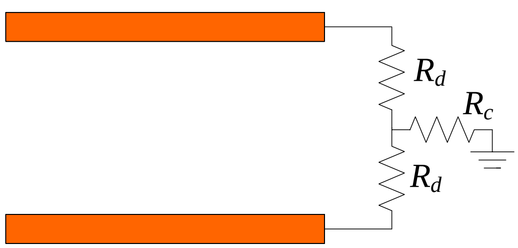
$$Z_{o-odd} < Z_o < Z_{o-even} \qquad v_{p-even} < v_p$$

# Termination Techniques

- A single-resistor termination for each conductor is not enough for coupled lines
- Proper terminations are needed to avoid reflections in both even and odd modes
- The most common termination networks are the T and Pi configurations



# T-Termination




$$R_c = \frac{1}{2}(Z_{o-even} - Z_{o-odd})$$

$$R_d = Z_{o-odd}$$



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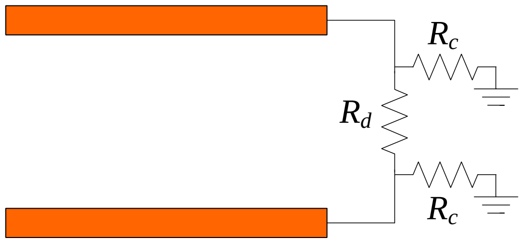
## Pi-Termination

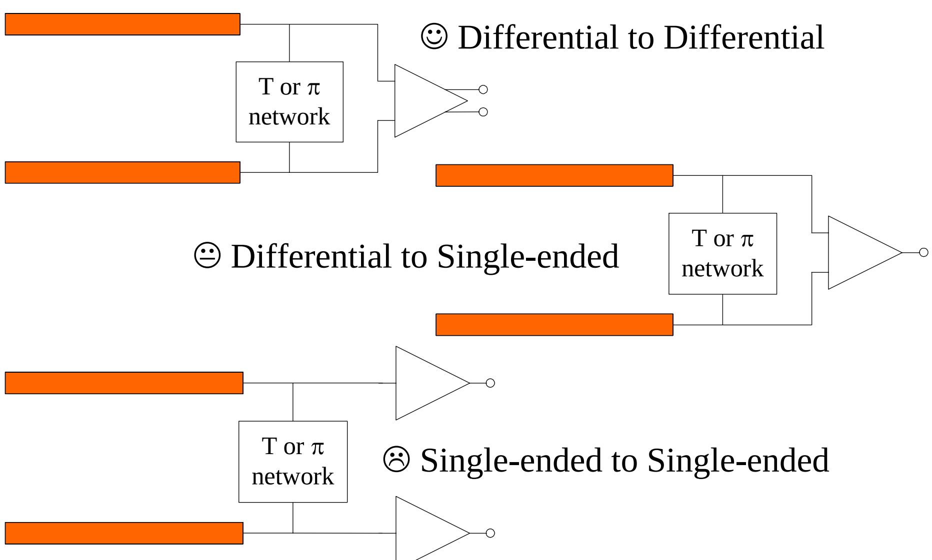



$$R_c = Z_{o-even}$$

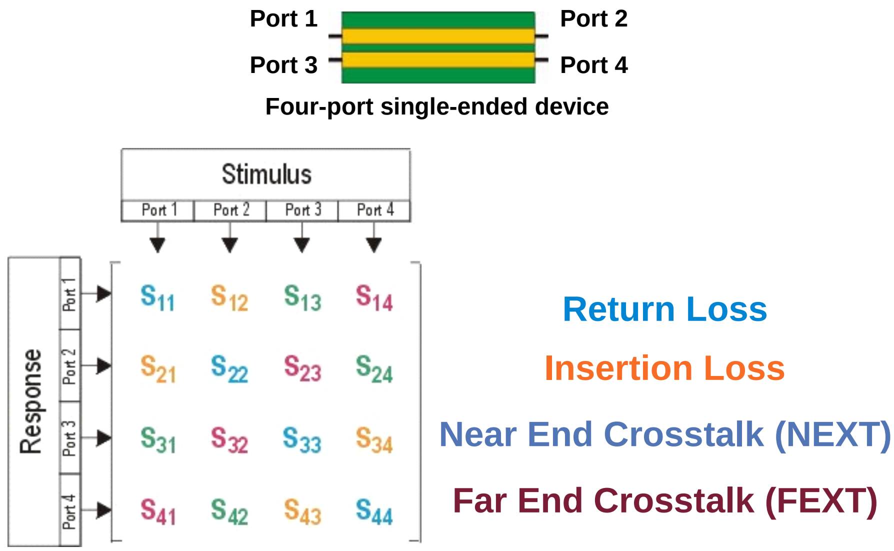
$$R_d = \frac{2Z_{o-even}Z_{o-odd}}{Z_{o-even} - Z_{o-odd}}$$



## Adding Buffers for Differential Signaling



## S-Parameters for Two-Coupled Lines

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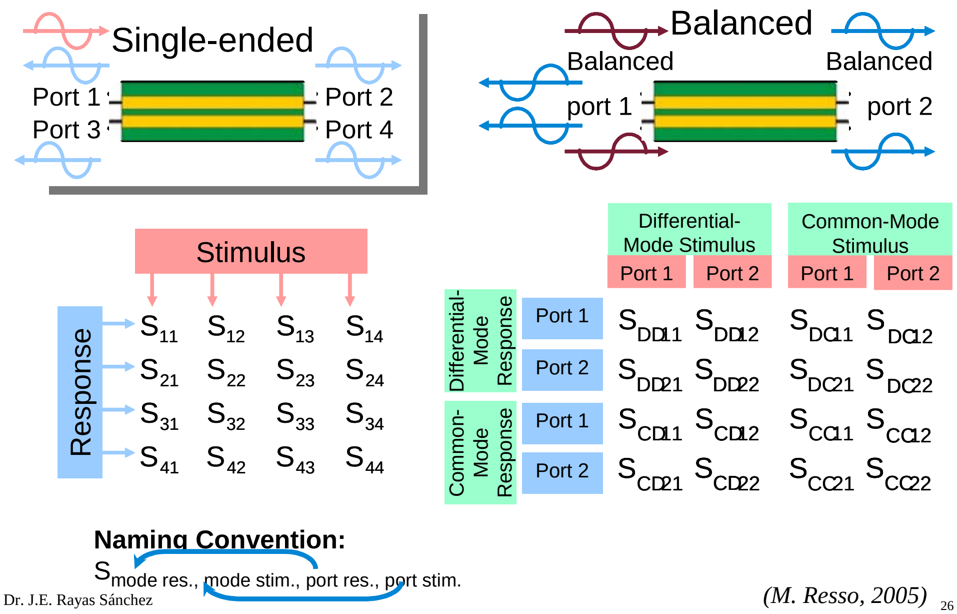
## Single-ended to Balanced S-Parameters

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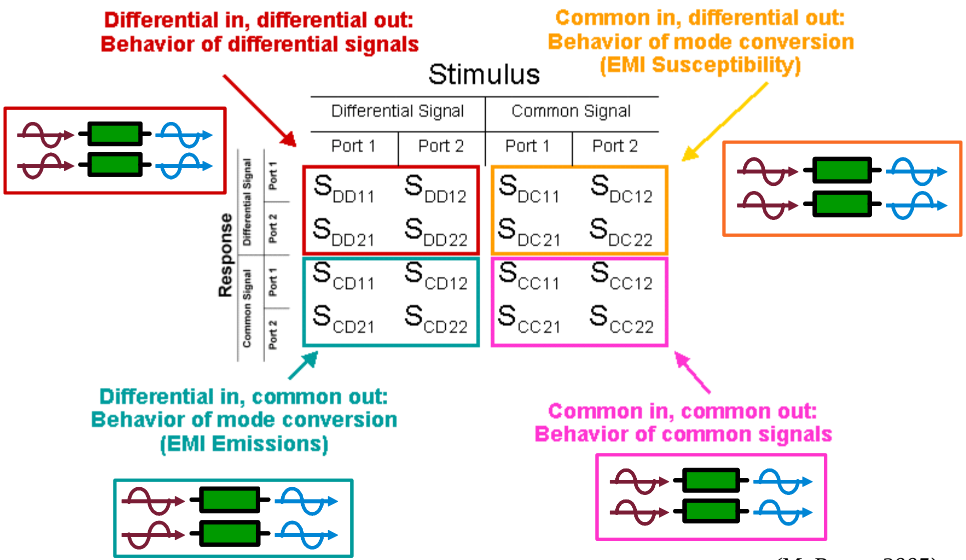
# Balanced, Differential or Mixed-Mode S-Param.

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