Frequency-Domain Analysis of Transmission Line Circuits
(Part 3)

Dr. José Ernesto Rayas Sánchez

Outline

- Differential transmission lines
- Common mode signaling
- Differential mode signaling
- Mode conversion
- Even and odd modes
- 2-coupled lossless transmission line theory
- Termination techniques
- Differential or Mixed-Mode S-parameters
Differential Transmission Lines

For high data rates, differential signaling is more used due to:

- Radiation is reduced (cancellation of fields)
- Receiver rejects signals that are common to both lines (high CMRR at the receiver)
- Signal voltage amplitudes can be smaller

Electromagnetic Fields in a Microstrip Line
Common Mode Signaling

As data rates go up, frequencies increase, lines become antennas (both send and receive) and corrupt the communication (BER, crosstalk, etc).

Differential Mode Signaling

Using differential excitations (differential transmission lines), most of the outside electromagnetic field cancels.
**Mode Conversion**

- Is produced by asymmetries in the differential pairs
- Can cause a differential signal to be converted to a common mode signal (radiation, crosstalk, etc.)

(M. Resso, 2005)

---

**Even Mode and Odd Mode**

- Practical differential pairs operate at even and odd modes simultaneously
- Even mode – excited in phase with equal amplitudes
- Odd mode – driven 180° out of phase with equal amplitudes

(H. Heck, 2002)
Differential Signaling for High-Speed Links

- Differential signaling can operate at much higher data rates
- High speed links operating in excess of ~1 Gb/s use differential signaling (e.g. Infiniband, PCI-Express).
- In fact, differential signals are already used for high speed clocks in desktop

Lossless Transmission Lines

\[
\begin{align*}
V(z) &= V_o^+ e^{-j \beta \Delta z} + V_o^- e^{+j \beta \Delta z} \\
I(z) &= I_o^+ e^{-j \beta \Delta z} + I_o^- e^{+j \beta \Delta z} \\
\beta &= \omega \sqrt{LC} \\
Z_o &= \sqrt{\frac{L}{C}} \\
v_p &= \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}
\end{align*}
\]

\[
\begin{align*}
\frac{dV(z)}{dz} &= -(j \omega L)I(z) \\
\frac{dI(z)}{dz} &= -(j \omega C)V(z)
\end{align*}
\]

Dr. J.E. Rayas Sánchez

http://iteso.mx/~erayas  erayas@iteso.mx
2-Coupled Lossless Symmetrical TLs

\[ \frac{dV_1}{dz} = -j \omega L_s I_1 - j \omega L_m I_2 \]
\[ \frac{dV_2}{dz} = -j \omega L_m I_1 - j \omega L_s I_2 \]
\[ \frac{dI_1}{dz} = -j \omega (C_s + C_m) V_1 + j \omega C_m V_2 \]
\[ \frac{dI_2}{dz} = +j \omega C_m V_1 - j \omega (C_s + C_m) V_2 \]

\[ Y_C = j \omega \begin{bmatrix} (C_s + C_m) & -C_m \\ -C_m & (C_s + C_m) \end{bmatrix} \]
\[ Z_L = j \omega \begin{bmatrix} L_s & L_m \\ L_m & L_s \end{bmatrix} \]

\[ -\frac{d}{dz} \begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} 0 & Z_L \\ Y_C & 0 \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix} \]
LC Matrices of 2-Coupled TLs

\[ C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} (C_s + C_m) & -C_m \\ -C_m & (C_s + C_m) \end{bmatrix} \]

\[ L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} = \begin{bmatrix} L_s & L_m \\ L_m & L_s \end{bmatrix} \]

\[ Z_o = ? \]
\[ V_p = ? \]

Even Mode in 2-Coupled Symmetrical TLs

\[ \frac{dV_1}{dz} = -j \omega L_s I_1 - j \omega L_m I_2 \]
\[ \frac{dV_2}{dz} = -j \omega L_m I_1 - j \omega L_s I_2 \]

\[ \frac{dI_1}{dz} = -j \omega (C_s + C_m)V_1 + j \omega C_m V_2 \]
\[ \frac{dI_2}{dz} = j \omega C_m V_1 - j \omega (C_s + C_m)V_2 \]

Since \( V_1 = V_2 \) and \( I_1 = I_2 \),

\[ \frac{dV_1}{dz} = -j \omega (L_s + L_m)I_1 \]
\[ \frac{dI_1}{dz} = -j \omega C_s V_1 \]

The effective \( L \) and \( C \) are

\[ L_{\text{eff}} = L_s + L_m \]
\[ C_{\text{eff}} = C_s \]

Hence

\[ Z_{o,\text{even}} = \sqrt{\frac{L_s + L_m}{C_s}} \]
\[ V_{p,\text{even}} = \frac{1}{\sqrt{(L_s + L_m)C_s}} \]
Even Mode in 2-Coupled Symmetrical TLs (cont)

\[ Z_{o-even} = \sqrt{\frac{L_s + L_m}{C_s}} \quad \nu_{p-even} = \frac{1}{\sqrt{(L_s + L_m)C_s}} \]

\( Z_{o-even} \) is the characteristic impedance of one of the conductors when the coupled line is operated in even mode.

Odd Mode in 2-Coupled Symmetrical TLs

\[ \frac{dV_1}{dz} = -j\omega L_s I_1 - j\omega L_m I_2 \quad \frac{dV_2}{dz} = -j\omega L_m I_1 - j\omega L_s I_2 \]

\[ \frac{dI_1}{dz} = -j\omega(C_s + C_m)V_1 + j\omega C_m V_2 \quad \frac{dI_2}{dz} = j\omega C_m V_1 - j\omega(C_s + C_m)V_2 \]

Since \( V_1 = -V_2 \) and \( I_1 = -I_2 \)

\[ \frac{dV_1}{dz} = -j\omega(L_s - L_m)I_1 \quad \frac{dI_1}{dz} = -j\omega(C_s + 2C_m)V_1 \]

The effective \( L \) and \( C \) are

\[ L_{eff} = L_s - L_m \quad C_{eff} = C_s + 2C_m \]

Hence

\[ Z_{o-odd} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \quad \nu_{p-odd} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}} \]
Odd Mode in 2-Coupled Symmetrical TLs (cont)

\[ V_1 = -V_2 \; ; \; I_1 = -I_2 \]

\[ Z_{o-odd} = \frac{L_s - L_m}{\sqrt{C_s + 2C_m}} \]

\[ v_{p-odd} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}} \]

\[ Z_{o-odd} \] is the characteristic impedance of one of the conductors when the coupled line is operated in odd mode.

Distributed Capacitances in Coupled Lines

(D. M. Pozar, Microwave Engineering, Wiley, 2005)
Distributed Capacitances in Coupled Lines (cont)

\[ Z_o \text{ and } v_p \text{ for Even and Odd Modes} \]

- If \( Z_o \) is the characteristic impedance of each isolated conductor, and \( v_p \) is the propagation velocity or wave speed in each isolated conductor

\[
Z_o = \sqrt{\frac{L_s}{C_s}} \quad v_p = \frac{1}{\sqrt{L_s C_s}}
\]

- Since

\[
Z_{o-even} = \sqrt{\frac{L_s + L_m}{C_s}} \quad v_{p-even} = \frac{1}{\sqrt{(L_s + L_m)C_s}}
\]

\[
Z_{o-odd} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \quad v_{p-odd} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}
\]

- then

\[ Z_{o-odd} < Z_o < Z_{o-even} \quad v_{p-even} < v_p \]
Termination Techniques

- A single-resistor termination for each conductor is not enough for coupled lines
- Proper terminations are needed to avoid reflections in both even and odd modes
- The most common termination networks are the T and Pi configurations

T-Termination

\[
R_c = \frac{1}{2} (Z_{o-even} - Z_{o-odd})
\]

\[
R_d = Z_{o-odd}
\]
Pi-Termination

\[ R_c = Z_{o-even} \]
\[ R_d = \frac{2Z_{o-even}Z_{o-odd}}{Z_{o-even} - Z_{o-odd}} \]

Adding Buffers for Differential Signaling

😊 Differential to Differential

😊 Differential to Single-ended

😊 Single-ended to Single-ended
**S-Parameters for Two-Coupled Lines**

Dr. J.E. Rayas Sánchez

**Return Loss**

**Insertion Loss**

Near End Crosstalk (NEXT)

Far End Crosstalk (FEXT)

(M. Resso, 2005)

---

**Single-ended to Balanced S-Parameters**

Dr. J.E. Rayas Sánchez

(M. Resso, 2005)
Balanced, Differential or Mixed-Mode S-Param.

(M. Resso, 2005)