Frequency-Domain Analysis of Transmission Line Circuits

(Part 2)

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Outline

- The transmission or ABCD matrix
- ABCD parameters
- Cascaded connections of 2-port networks
- Conversion between ABCD, Z, Y, and S parameters
- Analyzing interconnects with discontinuities
- Microstrip discontinuities
- Waveguide discontinuities

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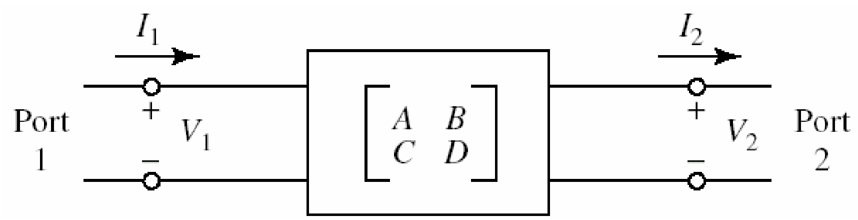
The Transmission Matrix (ABCD Parameters)

- Many practical high-frequency circuits consists of cascaded connections of two o more two-port networks
- The Transmission Matrix, or ABCD Parameters, or Chain Parameters, are very convenient for analyzing cascaded two-port circuits

Definition of ABCD Parameters

- The ABCD matrix is defined for two-port networks, as follows:



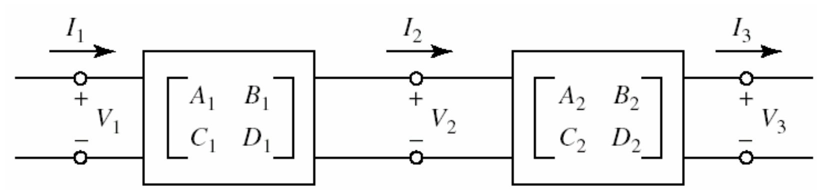
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

Cascaded Connections of Two-Port Networks

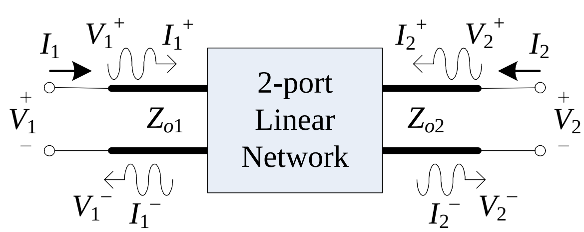
- The ABCD matrix of a cascaded connection is the multiplication of the corresponding ABCD matrices:



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

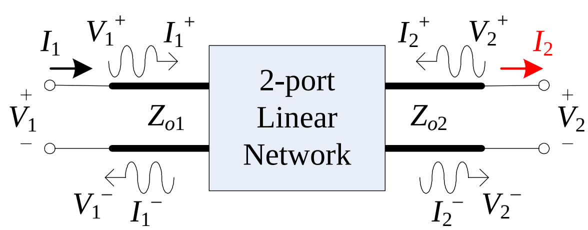
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

Converting ABCD to other Parameters



$$\boldsymbol{V} = \boldsymbol{ZI}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$A = Z_{11} / Z_{21}$$

$$B = (Z_{11}Z_{22} - Z_{12}Z_{21}) / Z_{21}$$

$$C = 1 / Z_{21}$$

$$D = Z_{22} / Z_{21}$$

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Properties of the ABCD Matrix

- If the 2-port network is lossless, A and D are real, B and C are imaginary
- If the 2-port network is reciprocal, the determinant of the ABCD matrix is one

$$AD - BC = 1$$

Converting Between Different Parameters

	[Z]	[Y]	[h]	[ABCD]
[Z]	$\begin{matrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{matrix}$	$\begin{matrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{matrix}$	$\begin{matrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{matrix}$	$\begin{matrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{matrix}$
[Y]	$\begin{matrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{matrix}$	$\begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix}$	$\begin{matrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{matrix}$	$\begin{matrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{\Delta Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{matrix}$
[h]	$\begin{matrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$	$\begin{matrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{matrix}$
[ABCD]	$\begin{matrix} \frac{A}{C} & \frac{\Delta ABCD}{C} \\ \frac{1}{C} & \frac{D}{C} \end{matrix}$	$\begin{matrix} \frac{D}{B} & -\frac{\Delta ABCD}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta ABCD}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$

(R. Ludwig and P. Bretchko, RF Circuit Design, Prentice Hall, 2000)

$$\Delta Z = \text{Det}(\boldsymbol{Z})$$
$$\Delta Y = \text{Det}(\boldsymbol{Y})$$
$$\Delta h = \text{Det}(\boldsymbol{H})$$
$$\Delta ABCD = AD - BC$$

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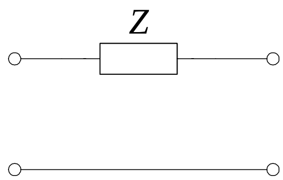
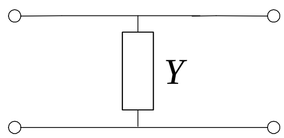
ABCD Parameters for Useful Circuits

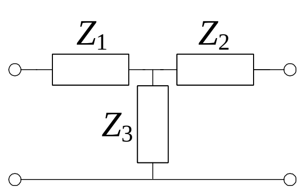


$$A = 1 \quad B = Z$$
$$C = 0 \quad D = 1$$

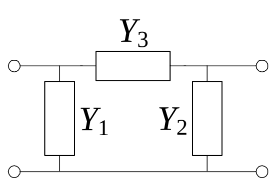


$$A = 1 \quad B = 0$$
$$C = Y \quad D = 1$$

ABCD Parameters for Useful Circuits (cont)



$$A = 1 + \frac{Z_1}{Z_3} \quad B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$
$$C = \frac{1}{Z_3} \quad D = 1 + \frac{Z_2}{Z_3}$$



$$A = 1 + \frac{Y_2}{Y_3} \quad B = \frac{1}{Y_3}$$
$$C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3} \quad D = 1 + \frac{1}{Y_3}$$

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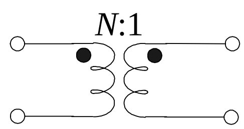
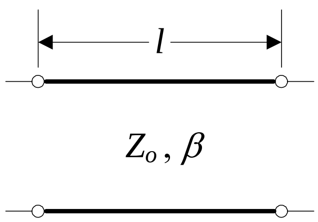
ABCD Parameters for Useful Circuits (cont)



$$A = N \qquad B = 0$$
$$C = 0 \qquad D = 1/N$$



$$A = \cos\beta l \qquad B = jZ_o \sin\beta l$$
$$C = jY_o \sin\beta l \qquad D = \cos\beta l$$

Analyzing Interconnects

As a first-order approximation (“hand” calculations),

- Uniform interconnects can be modeled by transmission lines (usually lossless)
- Transitions or discontinuities can be modeled by equivalent lumped circuits (estimating their values by formulae or by curve-fitting), or directly by Z, Y, or ABCD matrices obtained from measured S parameters
- Each section is represented by its ABCD matrix, Z matrix or Y matrix
- Calculations are performed by combining the corresponding matrices

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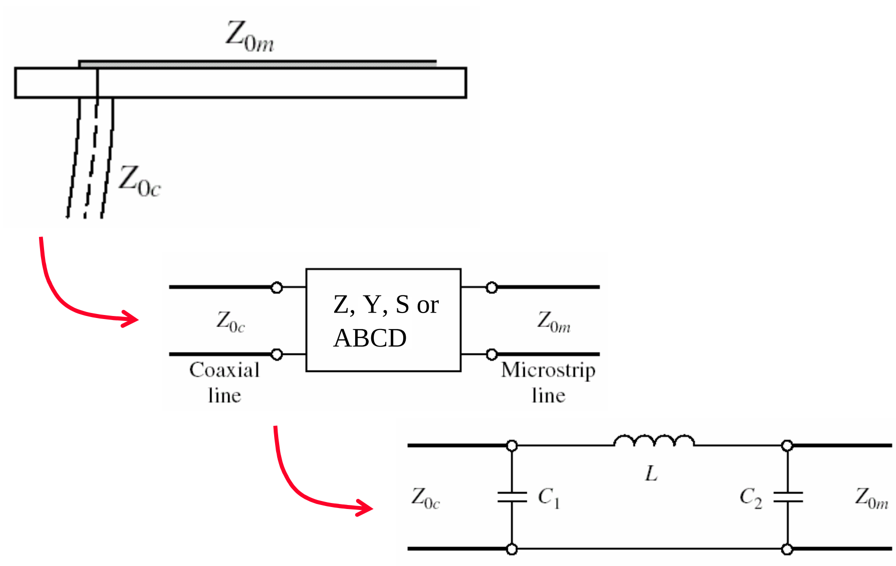
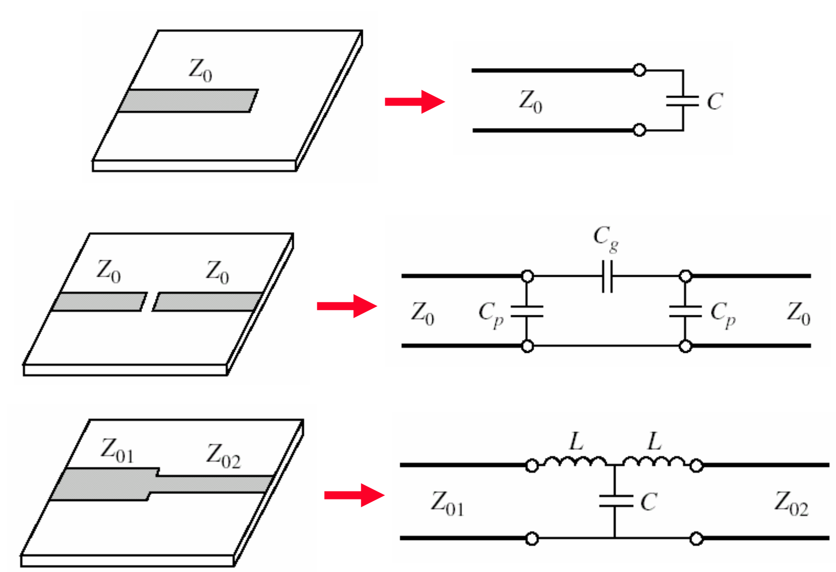
Modeling Interconnects with Transitions – Ex. 1



Microstrip Discontinuities



(D. M. Pozar, Microwave Engineering, Wiley, 2005)

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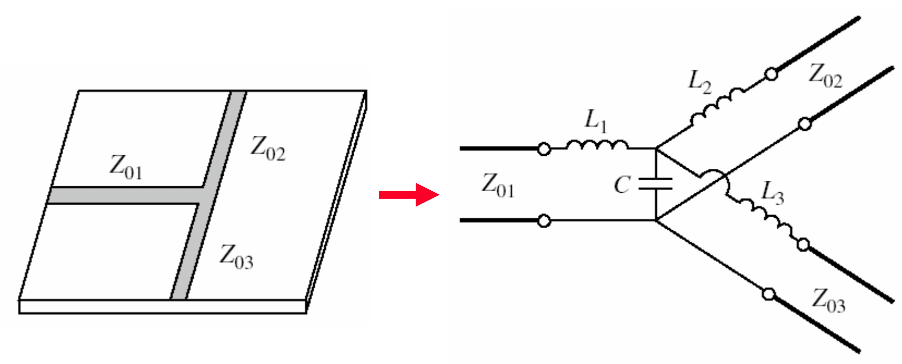
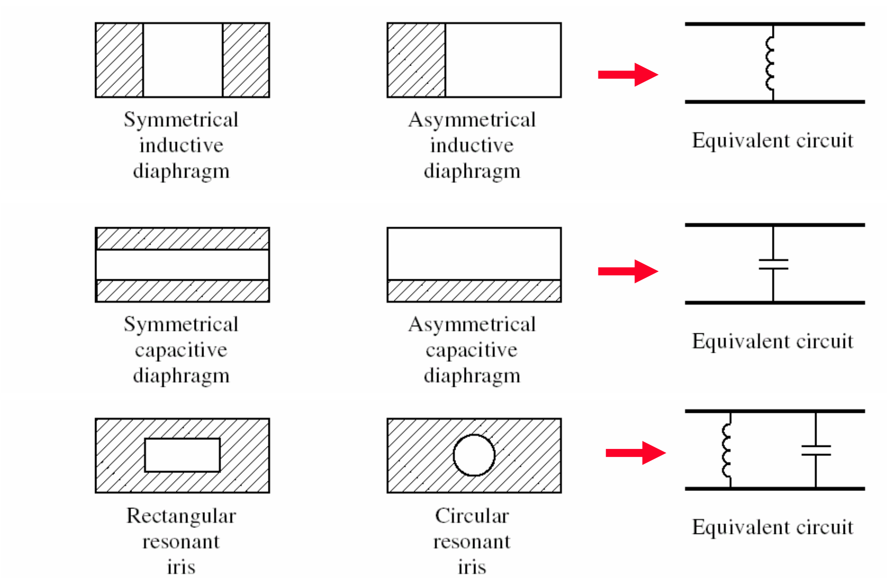
Microstrip Discontinuities (cont)



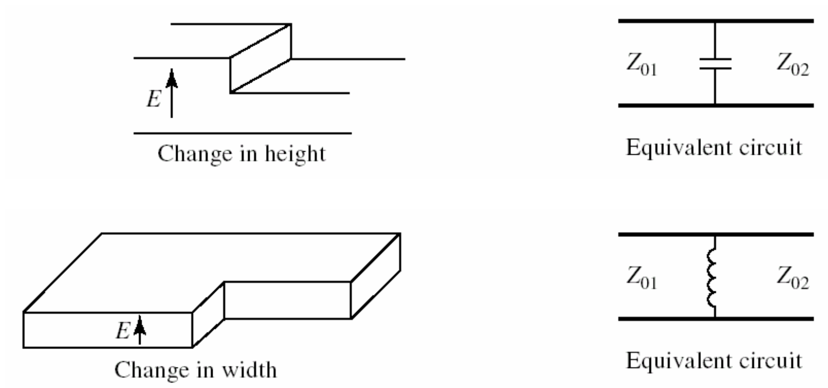
(D. M. Pozar, Microwave Engineering, Wiley, 2005)

Rectangular Waveguide Discontinuities



(D. M. Pozar, Microwave Engineering, Wiley, 2005)

Rectangular Waveguide Discontinuities (cont)



(D. M. Pozar, Microwave Engineering, Wiley, 2005)