Frequency-Domain Analysis of Transmission Line Circuits

(Part 2)

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Outline

- The transmission or ABCD matrix
- ABCD parameters
- Cascaded connections of 2-port networks
- Conversion between ABCD, Z, Y, and S parameters
- Analyzing interconnects with discontinuities
- Microstrip discontinuities
- Waveguide discontinuities
The Transmission Matrix (ABCD Parameters)

- Many practical high-frequency circuits consist of cascaded connections of two or more two-port networks
- The Transmission Matrix, or ABCD Parameters, or Chain Parameters, are very convenient for analyzing cascaded two-port circuits

Definition of ABCD Parameters

- The ABCD matrix is defined for two-port networks, as follows:

\[
\begin{bmatrix}
V_1 \\
I_1 \\
I_2 \\
V_2 \\
\end{bmatrix} =
\begin{bmatrix}
A & B \\
C & D \\
\end{bmatrix}
\begin{bmatrix}
V_2 \\
I_2 \\
\end{bmatrix}
\]

- \( A = \frac{V_1}{V_2_{I_1=0}} \)  
- \( B = \frac{V_1}{I_2_{V_2=0}} \)  
- \( C = \frac{I_1}{V_2_{I_2=0}} \)  
- \( D = \frac{I_1}{I_2_{V_2=0}} \)
Cascaded Connections of Two-Port Networks

- The ABCD matrix of a cascaded connection is the multiplication of the corresponding ABCD matrices:

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = \begin{bmatrix}
A_1 & B_1 \\
C_1 & D_1
\end{bmatrix} \begin{bmatrix}
V_2 \\
I_2
\end{bmatrix} = \begin{bmatrix}
A_2 & B_2 \\
C_2 & D_2
\end{bmatrix} \begin{bmatrix}
V_3 \\
I_3
\end{bmatrix}
\]

Converting ABCD to other Parameters

\[
V = ZI
\]
Properties of the ABCD Matrix

- If the 2-port network is lossless, $A$ and $D$ are real, $B$ and $C$ are imaginary.

- If the 2-port network is reciprocal, the determinant of the ABCD matrix is one
  \[ AD - BC = 1 \]

Converting Between Different Parameters

<table>
<thead>
<tr>
<th>[Z]</th>
<th>[Y]</th>
<th>[\Delta]</th>
<th>[ABCD]</th>
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<tbody>
<tr>
<td>[ Z_{11} ] [ Z_{12} ] [ Z_{21} ] [ Z_{22} ]</td>
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<td>[ \Delta Z ] [ \Delta Z ] [ \Delta Z ] [ \Delta Z ]</td>
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<td>[ \frac{Z_{11} \Delta Z}{Z_{21} \Delta Z} ] [ \frac{Z_{12} \Delta Z}{Z_{22} \Delta Z} ]</td>
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</tbody>
</table>

\[ \Delta Z = \text{Det}(Z) \]

\[ \Delta Y = \text{Det}(Y) \]

\[ \Delta h = \text{Det}(H) \]

\[ \Delta ABCD = AD - BC \]
**ABCD Parameters for Useful Circuits**

\[ A = 1 \quad B = Z \]
\[ C = 0 \quad D = 1 \]

\[ A = 1 \quad B = 0 \]
\[ C = Y \quad D = 1 \]

**ABCD Parameters for Useful Circuits (cont)**

\[ A = 1 + \frac{Z_1}{Z_3} \quad B = Z_1 + Z_2 + \frac{Z_1Z_2}{Z_3} \]
\[ C = \frac{1}{Z_3} \quad D = 1 + \frac{Z_2}{Z_3} \]

\[ A = 1 + \frac{Y_1}{Y_3} \quad B = \frac{1}{Y_3} \]
\[ C = Y_1 + Y_2 + \frac{YY_2}{Y_3} \quad D = 1 + \frac{1}{Y_3} \]
ABCD Parameters for Useful Circuits (cont)

\[ A = N \quad B = 0 \]
\[ C = 0 \quad D = 1/N \]
\[ A = \cos \beta l \quad B = jZ_o \sin \beta l \]
\[ C = jY_o \sin \beta l \quad D = \cos \beta l \]

Analyzing Interconnects

As a first-order approximation (“hand” calculations),

- Uniform interconnects can be modeled by transmission lines (usually lossless)
- Transitions or discontinuities can be modeled by equivalent lumped circuits (estimating their values by formulae or by curve-fitting), or directly by Z, Y, or ABCD matrices obtained from measured S parameters
- Each section is represented by its ABCD matrix, Z matrix or Y matrix
- Calculations are performed by combining the corresponding matrices
Modeling Interconnects with Transitions – Ex. 1

Microstrip Discontinuities

(D. M. Pozar, Microwave Engineering, Wiley, 2005)
Microstrip Discontinuities (cont)

Rectangular Waveguide Discontinuities

(D. M. Pozar, Microwave Engineering, Wiley, 2005)

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Rectangular Waveguide Discontinuities (cont)

(D. M. Pozar, Microwave Engineering, Wiley, 2005)