## Signal Integrity and High-Speed Interconnects Assignment 1

February 2006
Dr. J. E. Rayas Sánchez

## Introduction

The following problems are using sections of microstrip lines whose physical structure is illustrated below. We will assume that this microstrip line is on an FR-4 substrate (Nelco 4000-13, with $\varepsilon_{r}=3.9$ and $\tan \gamma_{d}=0.01$, measured at 1 GHz ). We will also assume that the substrate height is fixed, with $H=40 \mathrm{mil}$ ${ }^{(1)}$. We will neglect the thickness of the metallic strip, as well as its conductive losses.


The characteristic impedance of this microstrip line, $Z_{o}$, can be estimated using the following plot (we will study how to obtain this impedance function later in the course, for different dielectric constants, neglecting losses):


For instance, if $W / H=2.1073, Z_{o}=50 \Omega$; if $W / H=0.51032, Z_{o}=100 \Omega$.
(1) $1 \mathrm{mil}=10^{-3}$ inches. This unit is commonly used in the design of printed circuit boards (PCBs), as well as in microstrip RF circuits.

The propagation velocity $v_{p}$ or wave speed on the microstrip line is calculated using

$$
v_{p}=\frac{c}{\sqrt{\varepsilon_{e}}}
$$

where $c$ is the speed of light in free-space, and $\varepsilon_{e}$ is the effective dielectric constant given by

$$
\varepsilon_{e}=\frac{\varepsilon_{r}+1}{2}+\frac{\varepsilon_{r}-1}{2} \frac{1}{\sqrt{1+\frac{12 H}{W}}}
$$

You will solve the following problems using simple transmission line theory, approximating the microstrip line sections by lossless transmission lines. Later in the course, you can solve these same problems using much more accurate models, including electromagnetic models. You will verify that transmission line theory yields quite acceptable accuracy up to these frequencies and physical dimensions.

## Problem 1

Consider a microstrip line with a physical length $l=549.23 \mathrm{mil}$ and width $W=84.3 \mathrm{mil}$. The microstrip line is terminated with a short circuit, and it is operating at 3 GHz .
a) Calculate the magnitude of the input impedance of the line, $\left|Z_{\text {in }}(l=549.23 \mathrm{mil})\right|$.
b) Calculate the inductance per-unit length, $L$, and the capacitance per-unit length, $C$.
c) Approximating the line by a lumped circuit made up of cascaded LC sections, as shown below, find the minimum number of sections needed to calculate the input impedance with a $5 \%$ accuracy with respect to that one calculated in step a).


## Problem 2

Consider a microstrip line with a physical length $l=10,210.94 \mathrm{mil}$ (aprox. 26 cm ) and width $W=84.3 \mathrm{mil}$. The microstrip line is being excited by a sinusoidal source of 3 V of amplitude and 1 GHz of frequency, in series with a source resistor of $100 \Omega$. The line is terminated with a load impedance consisting of an $80 \Omega$ resistor in series with a 3.98 pF -capacitance.
a) Calculate the input impedance of the line, $Z_{i n}(l=10,210.94 \mathrm{mil})$.
b) Calculate the voltage at the input of the line.
c) Calculate the voltage at the load.
d) Calculate the steady-state incident wave at the load $\left(V_{o}^{+}\right)$.
e) Calculate the reflection coefficient at the load, $\Gamma$.
f) Plot the magnitude of the voltage along the line, from the source to the load.

## Problem 3

Consider a microstrip line with a physical length $l=1,361.46$ mil (approx. 3.5 cm ) and width $W=84.3 \mathrm{mil}$, operating at 2 GHz . The microstrip line is terminated with a load impedance consisting of an $60 \Omega$-resistor in series with a 3.98 nH -inductance. Using the Smith chart, calculate:
a) The SWR on the line.
b) The reflection coefficient at the load.
c) The equivalent load admittance.
d) The input impedance of the line.
e) The distance from the load to the first voltage minimum.
f) The distance from the load to the first voltage maximum.

## Problem 4

A microstrip line operating at 3 GHz using a width $W=20.41 \mathrm{mil}$ has to be matched to a $150 \Omega$ load. Using a quarter-wavelength microstrip line to avoid reflections at the operating frequency,
a) Calculate the width and length of the inserted quarter-wave microstrip line.
b) Plot the reflection coefficient at the input of the inserted quarter-wavelength microstrip line, from 1 GHz to 5 GHz (if necessary, write a computer program using Matlab, MathCad, or a similar tool).

## Problem 5

A microstrip line operating at 3 GHz using a width $W=84.3 \mathrm{mil}$ has to be matched to a complex load consisting of a $100 \Omega$ load resistor in series with a 4.77 nH inductor. Using two sections of microstrip lines inserted between the original line and the load,
a) Calculate the width and length of each of the two inserted microstrip lines to avoid reflections in the original line at the operating frequency.
b) Plot the reflection coefficient at the input of the inserted microstrip lines (or at the output of the original line), from 1 GHz to 5 GHz .

Submission deadline: Thursday March 2, 2006

