TRANSMISSION LINE INPUT IMPEDANCE

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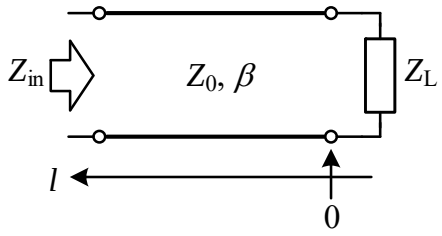
## VARIATION OF THE TRANSMISSION LINE INPUT IMPEDANCE WITH FREQUENCY AND LENGTH

Consider the following lossless transmission line circuit.



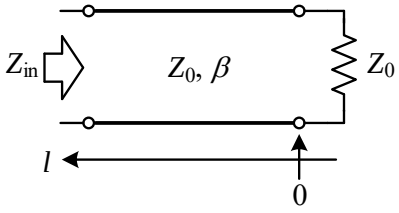

The input impedance is given by

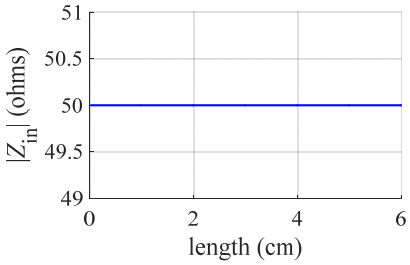
$$Z_{\text{in}}(l) = Z_0 \frac{Z_{\text{L}} + jZ_0 \tan(\beta l)}{Z_0 + jZ_{\text{L}} \tan(\beta l)}$$

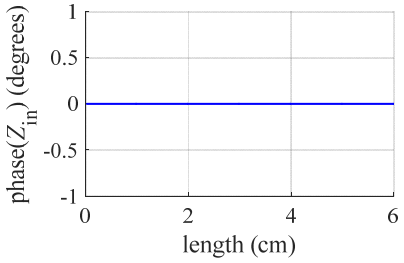
where $Z_0$ is the characteristic impedance of the transmission line, $Z_{\text{L}}$ is the load impedance, $\beta$ is the propagation constant, and $l$ is the physical length. The effective permittivity is $\varepsilon_{\text{e}}$. Let $Z_0 = 50\ \Omega$, and $\varepsilon_{\text{e}} = 4$.

Fixed frequency, $f = 2$ GHz, varying length:

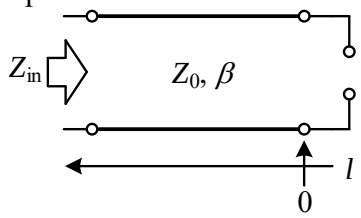
Matched Load

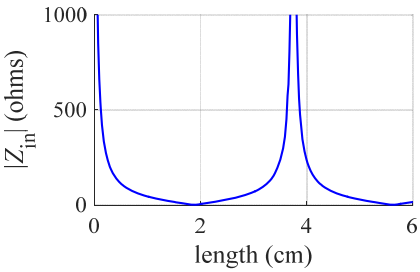


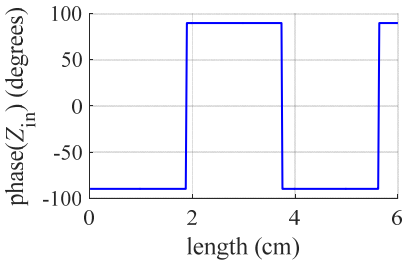



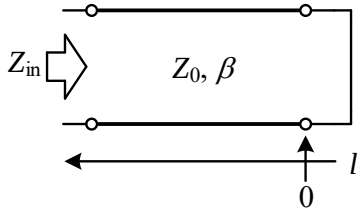

Open-Circuited Load

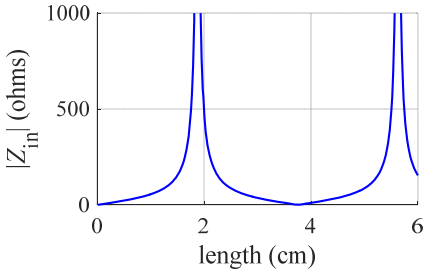


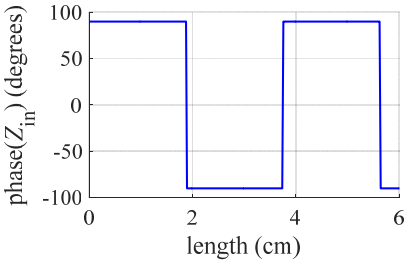




Short-Circuited Load



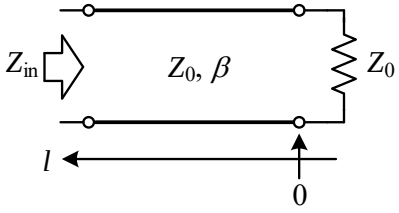



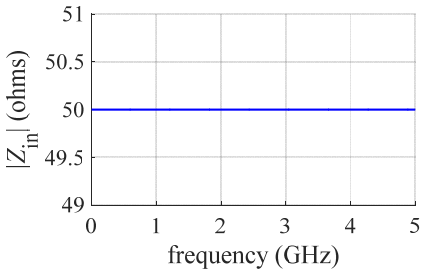



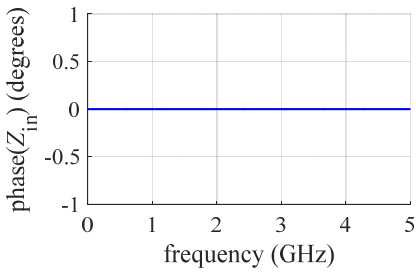

Notice that, in this case, $\dfrac{\lambda}{4} = \dfrac{v_{\text{p}}}{4f} = \dfrac{c}{4f\sqrt{\varepsilon_{\text{e}}}} = \dfrac{0.3\text{Gm/s}}{4(2\text{GHz})\sqrt{4}} = 1.875$ cm.

Fixed length, $l = 2.5$ cm, varying frequency:

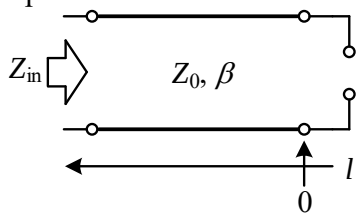
Matched Load

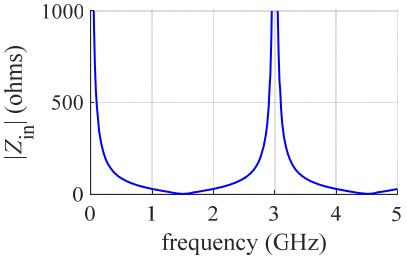


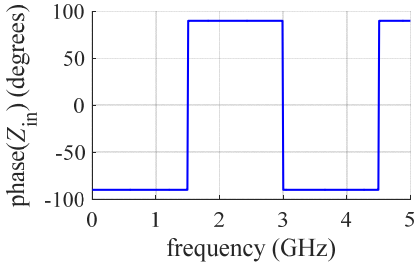



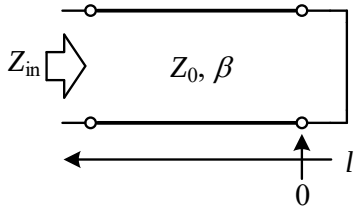

Open-Circuited Load

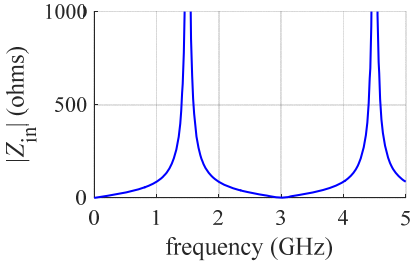


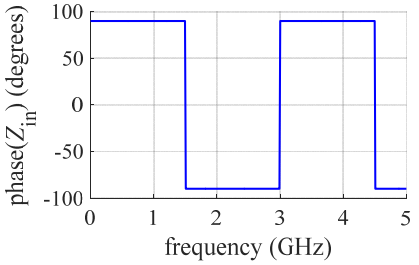




Short-Circuited Load




Since $\frac{\lambda}{4} = \frac{v_p}{4f} = \frac{c}{4f\sqrt{\varepsilon_e}}$, then the frequency at which $l = \lambda/4$ is $f = \frac{c}{4l\sqrt{\varepsilon_e}} = \frac{0.3\text{Gm/s}}{4(2.5\text{cm})\sqrt{4}} = 1.5$ GHz.