Transient-Domain Analysis of Transmission Line Circuits
(Part 1)

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Outline

- Quarter-wave transformer – steady state response
- Quarter-wave transformer – transient response
- Reflection coefficient revised
- Concept of “transient impedance”
- Applying DC to transmission lines
- Lattice (or bouncing or reflection) diagrams
- Building transient signals from bouncing diagrams
Quarter-Wave Transformer

\[ Z_{in} = \frac{Z_1^2}{R_L} \]

To make \( \Gamma = 0 \),

\[ Z_1 = \sqrt{R_L Z_o} \]

\( Z_1 \) must be the geometric mean of \( Z_o \) and \( R_L \)

\( \Gamma \) is the steady-state reflection coefficient

Quarter-Wave Transformer – Transient Response

\[ \Gamma_1 = \frac{Z_1 - Z_o}{Z_1 + Z_o} \]
\[ \Gamma_2 = \frac{Z_o - Z_1}{Z_o + Z_1} = -\Gamma_1 \]
\[ \Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1} \]

\[ T_1 = 1 + \Gamma_1 = \frac{2Z_1}{Z_1 + Z_o} \]
\[ T_2 = 1 + \Gamma_2 = \frac{2Z_o}{Z_o + Z_1} \]

(D. M. Pozar, Microwave Engineering, Wiley, 2005)
Initially, the incoming wave has only the incident component:

\[ V(z) = V_o^+ e^{-j\beta z} \]
Quarter-Wave Transformer – Transient Response

\[ \Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 + T_1 T_2 \Gamma_2 \Gamma_3^2 - T_1 T_2 \Gamma_2^2 \Gamma_3^3 + \ldots \]

\[ \Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2 \Gamma_3)^n \]
Quarter-Wave Transformer – Transient Response

\[ \Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2 \Gamma_3)^n \]

Since \( \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \), for \(|x|<1\)

\[ \Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} = \frac{\Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} \]

using

\[ \Gamma_1 = \frac{Z_1 - Z_o}{Z_1 + Z_o} \quad \Gamma_2 = -\Gamma_1 \quad \Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1} \quad T_1 = \frac{2Z_1}{Z_1 + Z_o} \quad T_2 = \frac{2Z_o}{Z_o + Z_1} \]

\[ \Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3 = \frac{2(Z_1^2 - Z_o R_L)}{(Z_1 + Z_o)(R_L + Z_1)} = 0 \quad \text{if} \quad Z_1 = \sqrt{Z_o R_L} \]

If \( Z_1 \) is the geometric mean of \( Z_o \) and \( R_L \), the sum of the infinite number of partial reflections is zero
Applying DC to Transmission Lines

\[ V_0 + e^c = 0 \]

\[ I^+ = \frac{V^+}{Z_0} \]

\[ v = \frac{c}{\sqrt{\varepsilon_e}} \]

Applying DC to Transmission Lines (cont)

\[ \Gamma_S = \frac{R_S - Z_o}{R_S + Z_o} \quad \Gamma_L = \frac{R_L - Z_o}{R_L + Z_o} \quad v_p = \frac{c}{\sqrt{\varepsilon_e}} \]

\[ V_o^+ = \frac{V_S Z_o}{R_S + Z_o} \quad I_o^+ = \frac{V_o^+}{Z_o} \]
Lattice (or Bouncing or Reflection) Diagrams

- Voltage Diagrams

Bouncing Diagrams for Currents

Since \( I_o^+ = \frac{V_o^+}{Z_o} \) and \( I_o^- = \frac{-V_o^-}{Z_o} \)
Transient Signals from Bouncing Diagrams

\[ v(0,t) = \begin{cases} 
V_o^+ & 0 \leq t < 2t_d \\
V_o^+ + V_o^- + V_1^- & 2t_d \leq t < 4t_d \\
V_o^+ + V_o^- + V_1^- + V_2^- & 4t_d \leq t < 6t_d \\
\vdots & \vdots 
\end{cases} \]

\[ t_d = \frac{l}{v_p} \]

\[ v(l,t) = \begin{cases} 
0 & 0 \leq t < t_d \\
V_o^+ + V_o^- & t_d \leq t < 3t_d \\
V_o^+ + V_o^- + V_1^- + V_1^- & 3t_d \leq t < 5t_d \\
\vdots & \vdots 
\end{cases} \]

Bouncing Diagrams – Other Representation

(M. Leddige, 2003)
Example – Underdriven Transmission Line

Assume $Z_s = 75$ ohms
$Z_0 = 50$ ohms
$V_S = 0.2$ volts

\[
\rho_{\text{source}} = \frac{Z_s - Z_0}{Z_s + Z_0} = \frac{75 - 50}{75 + 50} = 0.2
\]
\[
\rho_{\text{load}} = \frac{Z_l - Z_0}{Z_l + Z_0} = \frac{\infty - 50}{\infty + 50} = 1
\]

\[
V_{\text{load}} = V_S \left( 1 + \frac{Z_0}{Z_s} \right) = \frac{Z_0 V_S}{Z_s + Z_0}
\]

\[
\text{Delay} = 250 \text{ ps}
\]

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Example – Underdriven Transmission Line (cont)

Response from lattice diagram

\[
\text{Response from lattice diagram}
\]

Example – Overdriven Transmission Line

Assume \( Z_s = 25 \) ohms
\( Z_0 = 50 \) ohms
\( V_s = 0 \) to 2 volts

\[
V_{\text{init}} = V_s \frac{Z_0}{Z_s + Z_0} = (2) \frac{50}{25 + 50} = 1.333 \\
\rho_{\text{source}} = \frac{Z_s - Z_0}{Z_s + Z_0} = \frac{25 - 50}{25 + 50} = -0.33333 \\
\rho_{\text{load}} = \frac{Z_l - Z_0}{Z_l + Z_0} = \frac{\infty - 50}{\infty + 50} = 1
\]

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Using the Bouncing Diagrams – Example

\[ R_S = 25\Omega; \quad R_L = 150\Omega; \]
\[ V_S = 3V; \quad Z_0 = 50\Omega; \]
\[ \varepsilon_c = 3; \quad l = 15\text{cm} \]

\[ v(x = 0, t) = ?; \quad v(x = l, t) = ?; \quad v(x = \frac{3}{4}l, t) = ? \]

\[ i(x = 0, t) = ?; \quad i(x = l, t) = ?; \quad i(x = \frac{3}{4}l, t) = ? \]

Using the Bouncing Diagrams – Example (cont)

\[ \Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{1}{2} \]
\[ \Gamma_S = \frac{R_S - Z_0}{R_S + Z_0} = -\frac{1}{3} \]
\[ V_o^+ = \frac{V_S Z_0}{R_S + Z_0} = 2V \]

\[ v_p = \frac{c}{\sqrt{\varepsilon_c}} = 173.2\text{Mm/s} \]
\[ t_d = \frac{l}{v_p} = 866.02\text{ps} \]
Example Simulated with APLAC

Length=0.15 $ meters
L=288.68nH $ nH/m
C=115.47pF $ pF/m
$ Z=50ohms, epse=3

Sweep
"Lossless Transmission Line"
LOOP 300 TIME LIN 0 8ns
W=0 grid Y *** 0 3.1
W=1 grid Y *** 0 45m
Show W=0 Y=Vtran(S) Y=Vtran(in) Y=Vtran(L)
Show W=1 Y=(Vtran(S)-Vtran(in))/25 Y=Vtran(L)/150
EndSweep

Example Simulated with APLAC (cont)

Length=0.15 $ meters
Z=50ohms
er=3
$L=288.68nH $ nH/m
$C=115.47pF $ pF/m
$ Zo=50ohms, epse=3

Sweep
"Lossless Transmission Line"
LOOP 300 TIME LIN 0 8ns
W=0 grid Y *** 3 0 0.01ps 0.01ps 8ns 9ns
W=1 grid Y *** 0 45m
Show W=0 Y=Vtran(S) Y=Vtran(in) Y=Vtran(L)
Show W=1 Y=(Vtran(S)-Vtran(in))/25 Y=Vtran(L)/150
EndSweep
Example Simulated with APLAC (cont)

![Graph 1](image1)

Example Simulated with APLAC (cont)

![Graph 2](image2)
Example Simulated with APLAC (from Matlab)

![Graph showing voltage and current over time](image-url)
Example Simulated with ADS

Vf_Pulse
SRC1
Vpeak=3 V
Vdc=0 V
Freq=0.1 GHz
Width=10 nsec
Rise=1 psec
Fall=1 psec
Delay=0 psec
Weight=no
Harmonics=16

R
RS
R=25 Ohm

TLINP
TL1
Z=50.0 Ohm
L=15 cm
K=3
A=0
F=1 GHz
TanD=0
Mur=1
TanM=0
Sigma=0

TRANSPORT
Tran
Tran1
StopTime=8 nsec
MaxTimeStep=10 psec

Example Simulated with ADS (cont)

Voltages

(3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0

out
s

0
1
2
3
4
5
6
7
8

time, nsec

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Example Simulated with ADS (Currents)

Example Simulated with ADS (cont)
**Ideal Case**

\[ V_S = 1\, \text{V}, \; Z_o = 50\, \Omega, \; R_S = 50\, \Omega, \; R_L = 50\, \Omega \]

**Source Matched**

\[ V_S = 1\, \text{V}, \; Z_o = 50\, \Omega, \; R_S = 50\, \Omega, \; R_L = 1\, \text{M\Omega} \]
**Load Matched**

$$V_s = 1V, \ Z_o = 50\Omega, \ R_S = 0.1\Omega, \ R_L = 50\Omega$$

**Load and Source Un-Matched**

$$V_s = 1V, \ Z_o = 50\Omega, \ R_S = 0.1\Omega, \ R_L = 1M\Omega$$
Load and Source Un-Matched (cont.)

\[ V_S = 1V, Z_o = 50\Omega, R_S = 0.1\Omega, R_L = 1M\Omega \]