Frequency-Domain Analysis of Transmission Line Circuits

(Part 3)

Dr. José Ernesto Rayas-Sánchez

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Outline

- Differential transmission lines
- Common mode signaling
- Differential mode signaling
- Mode conversion
- Even and odd modes
- 2-coupled lossless transmission line theory
- Termination techniques
- Differential or Mixed-Mode S-parameters

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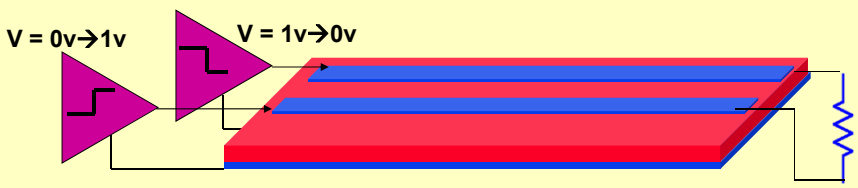
Differential Transmission Lines

For high data rates, differential signaling is more used due to:

- Radiation is reduced (cancellation of fields)
- Receiver rejects signals that are common to both lines (high CMRR at the receiver)
- Signal voltage amplitudes can be smaller



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(M. Resso, 2005)

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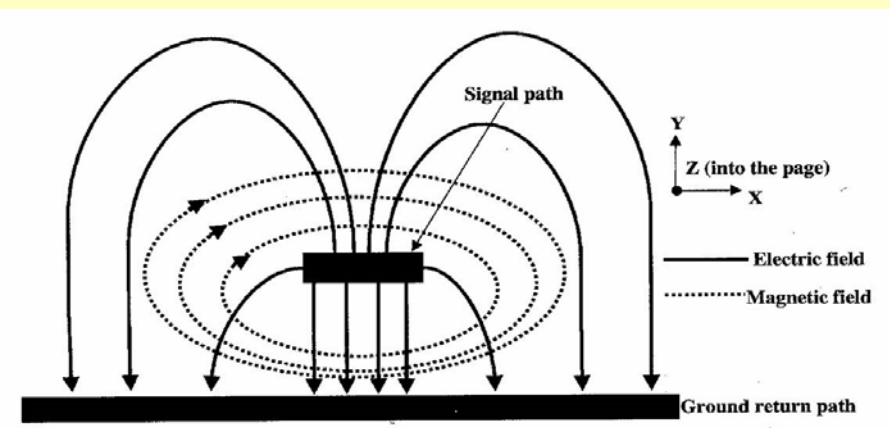
Electromagnetic Fields in a Microstrip Line



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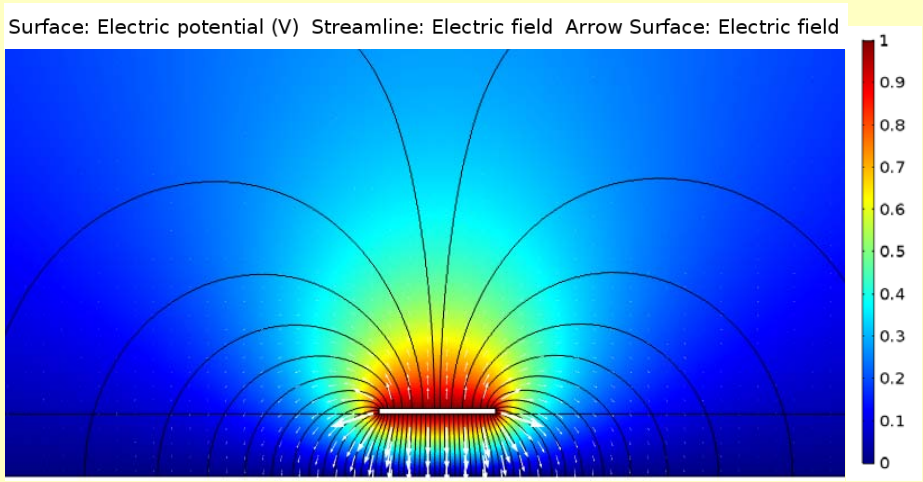
Electric Field Distribution

Substrate: Air ($\varepsilon_r = 1$)



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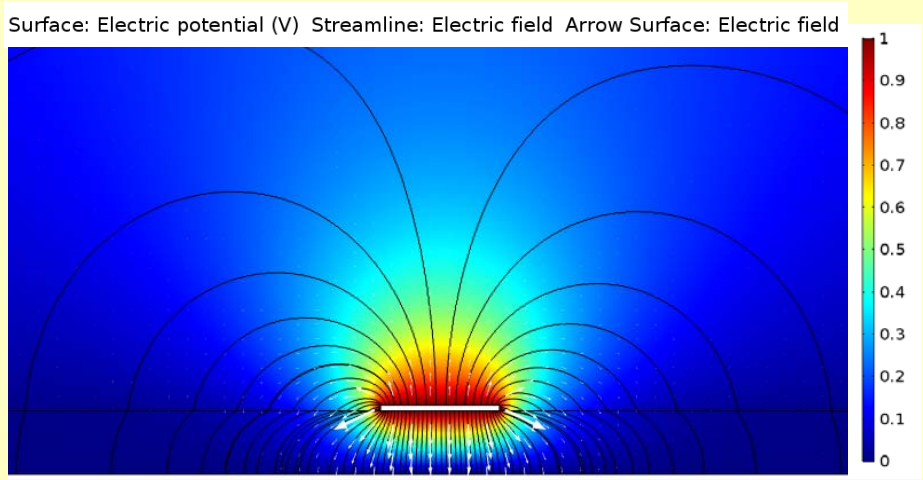
Electric Field Distribution (cont.)

Substrate: FR4 ($\varepsilon_r = 4.5$)



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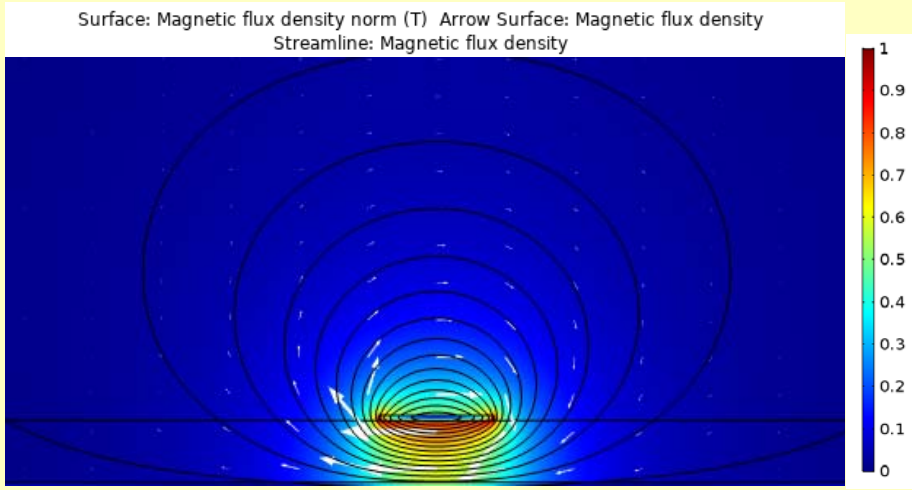
Magnetic Field Distribution

Substrate: Air or FR4



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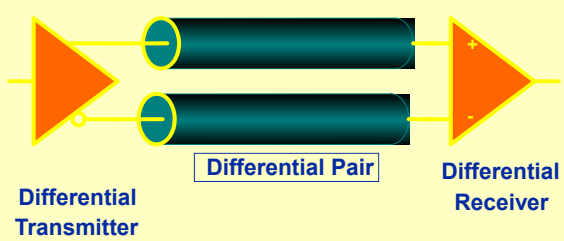
Differential Signaling for High-Speed Links

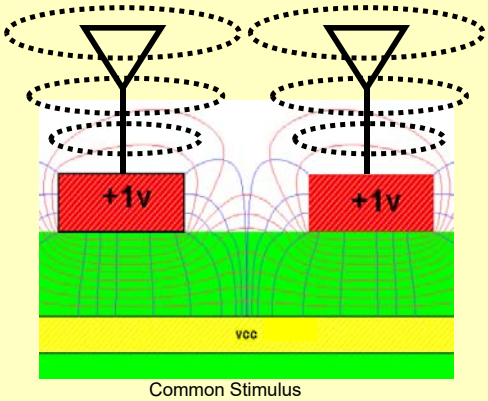
- Differential signaling can operate at much higher data rates
- High speed links operating in excess of ~1 Gb/s use differential signaling (e.g. Infiniband, PCI-Express).
- Differential signals are already used for high speed clocks



(H. Heck 2002)

Common Mode Signaling

As data rates go up, frequencies increase, lines become antennas (both send and receive) and corrupt the communication (BER, crosstalk, etc)



Common Stimulus

E and H fields addition outside

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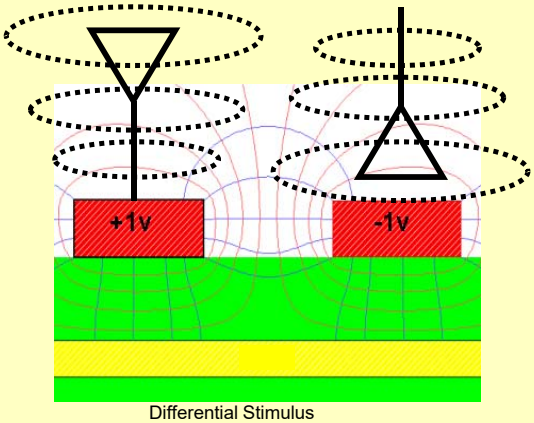
(M. Resso, 2005) 9

Differential Mode Signaling

Using differential excitations (differential transmission lines), most of the outside electromagnetic field cancels



Differential Stimulus

E and H fields cancellation outside

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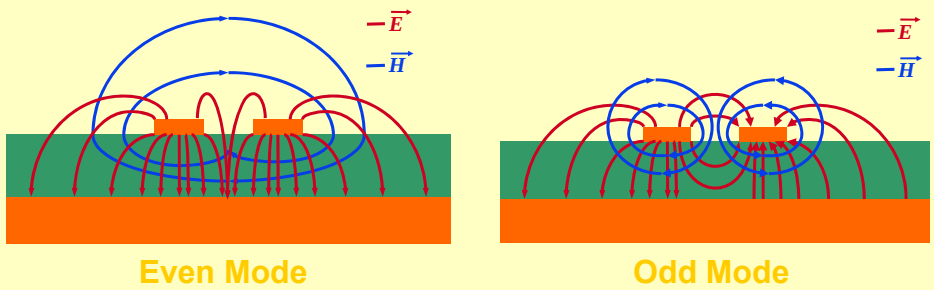
(M. Resso, 2005) 10

Even Mode and Odd Mode

- Practical differential pairs operate at even and odd modes simultaneously
- Even mode – excited in phase with equal amplitudes
- Odd mode – driven 180º out of phase with equal amplitudes



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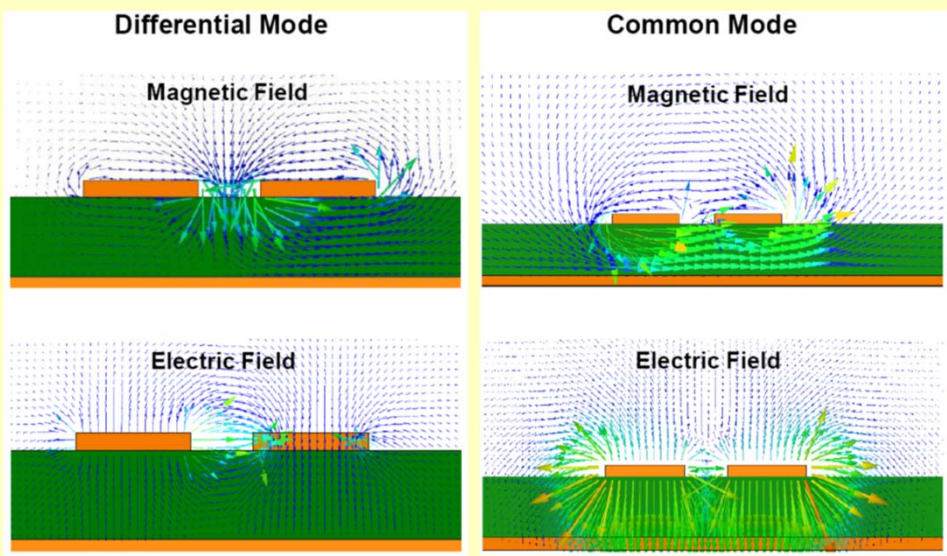
(H. Heck 2002) 11

Even Mode and Odd Mode (cont.)



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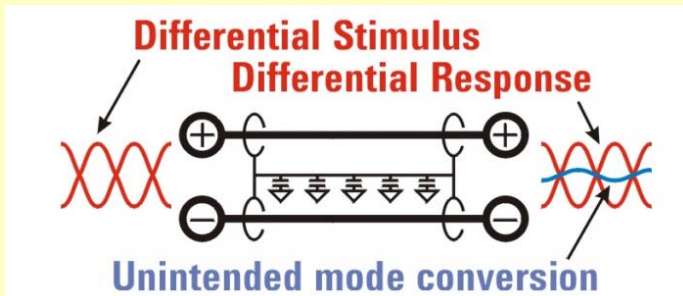
(P. Huray and S. Pytel, 2009) 12

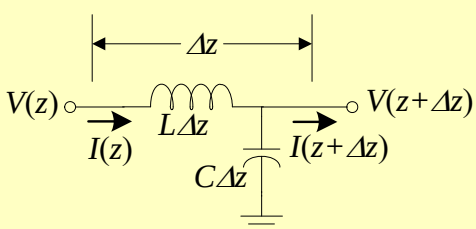
Mode Conversion

- Is produced by asymmetries in the differential pairs
- Can cause a differential signal to be converted to a common mode signal (radiation, crosstalk, etc.)



(M. Resso, 2005)

Lossless Transmission Lines



$$\frac{dV(z)}{dz} = -(j\omega L)I(z)$$

$$\frac{dI(z)}{dz} = -(j\omega C)V(z)$$

$$\frac{dV}{dz} = -j\omega LI$$

$$\frac{dI}{dz} = -j\omega CV$$

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z}$$

$$I(z) = I_0^+ e^{-j\beta z} + I_0^- e^{+j\beta z}$$

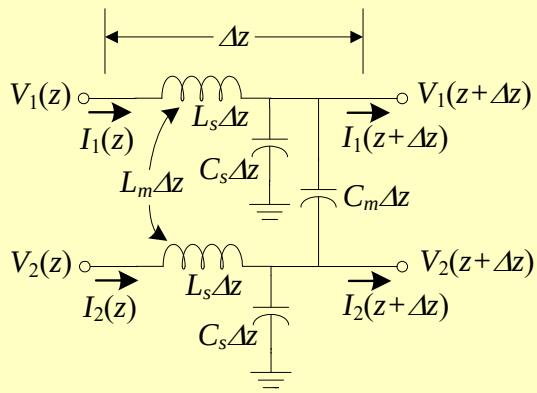
$$\beta = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{L}{C}} \qquad v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

$$-\frac{d}{dz}\begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} 0 & Z_L \\ Y_C & 0 \end{bmatrix}\begin{bmatrix} V \\ I \end{bmatrix}$$

$$Z_L = j\omega L \qquad Y_C = j\omega C$$

2-Coupled Lossless Symmetrical TLs



$$\frac{dV_1}{dz} = -j\omega L_s I_1 - j\omega L_m I_2 \qquad \frac{dI_1}{dz} = -j\omega(C_s + C_m)V_1 + j\omega C_m V_2$$

$$\frac{dV_2}{dz} = -j\omega L_m I_1 - j\omega L_s I_2 \qquad \frac{dI_2}{dz} = +j\omega C_m V_1 - j\omega(C_s + C_m)V_2$$

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2-Coupled Lossless Symmetrical TLs (cont.)

$$\frac{dV_1}{dz} = -j\omega L_s I_1 - j\omega L_m I_2$$

$$\frac{dV_2}{dz} = -j\omega L_m I_1 - j\omega L_s I_2$$

$$\frac{dI_1}{dz} = -j\omega(C_s + C_m)V_1 + j\omega C_m V_2$$

$$\frac{dI_2}{dz} = +j\omega C_m V_1 - j\omega(C_s + C_m)V_2$$

$$\boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \boldsymbol{I} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\boldsymbol{Y}_C = j\omega \begin{bmatrix} (C_s + C_m) & -C_m \\ -C_m & (C_s + C_m) \end{bmatrix}$$

$$\boldsymbol{Z}_L = j\omega \begin{bmatrix} L_s & L_m \\ L_m & L_s \end{bmatrix}$$

$$-\frac{d}{dz}\begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{I} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{Z}_L \\ \boldsymbol{Y}_C & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{I} \end{bmatrix}$$

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LC Matrices of 2-Coupled TLs

$$\boldsymbol{C} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} (C_s + C_m) & -C_m \\ -C_m & (C_s + C_m) \end{bmatrix}$$

$$\boldsymbol{L} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} = \begin{bmatrix} L_s & L_m \\ L_m & L_s \end{bmatrix}$$

$$Z_0 = ?$$

$$v_{\mathrm{p}} = ?$$

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Even Mode in 2-Coupled Symmetrical TLs

$$\frac{dV_1}{dz} = -j\omega L_s I_1 - j\omega L_m I_2 \qquad \frac{dV_2}{dz} = -j\omega L_m I_1 - j\omega L_s I_2$$

$$\frac{dI_1}{dz} = -j\omega(C_s + C_m)V_1 + j\omega C_m V_2 \qquad \frac{dI_2}{dz} = j\omega C_m V_1 - j\omega(C_s + C_m)V_2$$

Since $V_1 = V_2$ and $I_1 = I_2$

$$\frac{dV_1}{dz} = -j\omega(L_s + L_m)I_1 \qquad \frac{dI_1}{dz} = -j\omega C_s V_1$$

The effective L and C are

$$L_{\mathrm{eff}} = L_s + L_m \qquad C_{\mathrm{eff}} = C_s$$

Hence

$$Z_{0\text{-even}} = \sqrt{\frac{L_s + L_m}{C_s}} \qquad v_{\text{p-even}} = \frac{1}{\sqrt{(L_s + L_m)C_s}}$$

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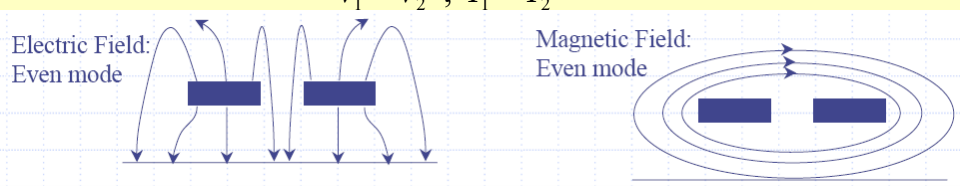
Even Mode in 2-Coupled Symmetrical TLs (cont.)

$$V_1 = V_2 \;\; ; \;\; I_1 = I_2$$



(R. Mellitz 2003)

$$Z_{0\text{-even}} = \sqrt{\frac{L_s + L_m}{C_s}} \qquad v_{\text{p-even}} = \frac{1}{\sqrt{(L_s + L_m)C_s}}$$

$Z_{0\text{-even}}$ is the characteristic impedance of one of the conductors when the coupled line is operated in even mode

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Odd Mode in 2-Coupled Symmetrical TLs

$$\frac{dV_1}{dz} = -j\omega L_s I_1 - j\omega L_m I_2 \qquad \frac{dV_2}{dz} = -j\omega L_m I_1 - j\omega L_s I_2$$

$$\frac{dI_1}{dz} = -j\omega(C_s + C_m)V_1 + j\omega C_m V_2 \qquad \frac{dI_2}{dz} = j\omega C_m V_1 - j\omega(C_s + C_m)V_2$$

Since $V_1 = -V_2$ and $I_1 = -I_2$

$$\frac{dV_1}{dz} = -j\omega(L_s - L_m)I_1 \qquad \frac{dI_1}{dz} = -j\omega(C_s + 2C_m)V_1$$

The effective L and C are

$$L_{\text{eff}} = L_s - L_m \qquad C_{\text{eff}} = C_s + 2C_m$$

Hence

$$Z_{0\text{-odd}} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \qquad v_{\text{p-odd}} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}$$

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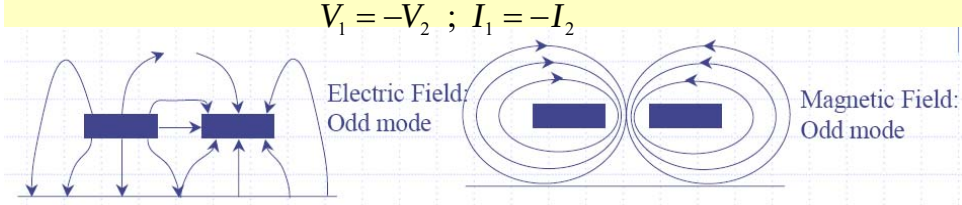
Odd Mode in 2-Coupled Symmetrical TLs (cont.)

$$V_1 = -V_2 \;\; ; \;\; I_1 = -I_2$$

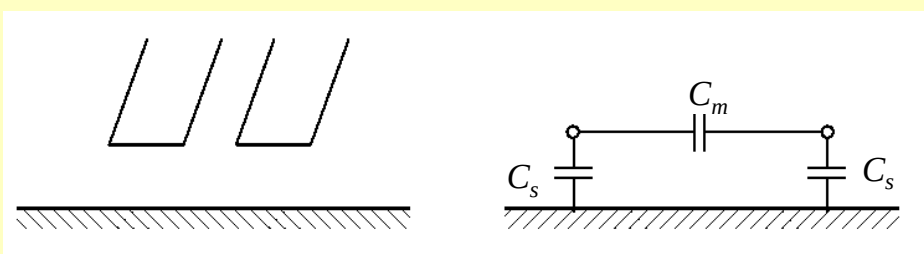


(R. Mellitz 2003)

$$Z_{0\text{-odd}} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \qquad v_{\text{p-odd}} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}$$

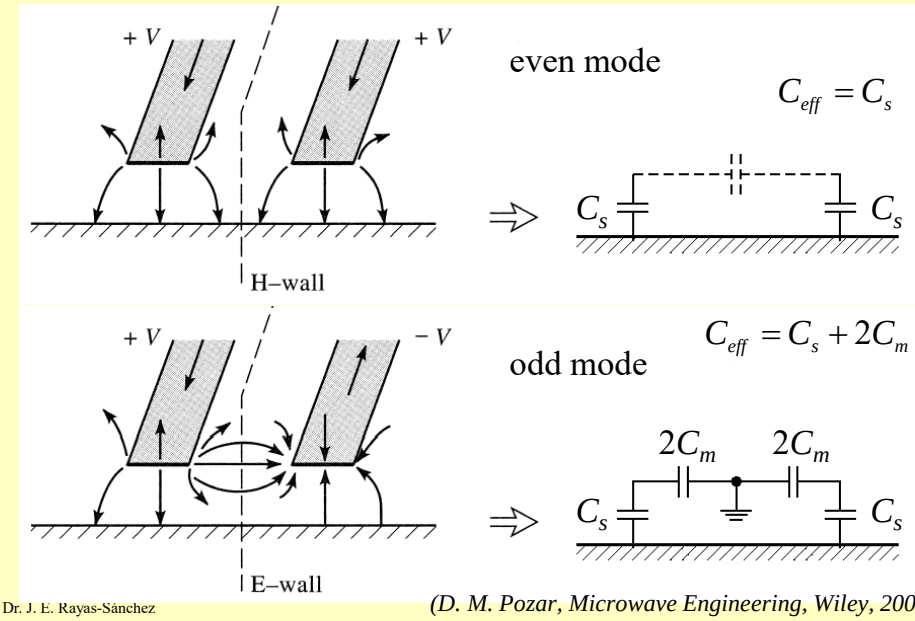
$Z_{0\text{-odd}}$ is the characteristic impedance of one of the conductors when the coupled line is operated in odd mode

Distributed Capacitances in Coupled Lines



(D. M. Pozar, Microwave Engineering, Wiley, 2005)

Distributed Capacitances in Coupled Lines (cont.)



(D. M. Pozar, Microwave Engineering, Wiley, 2005)

Z_0 and v_p for Even and Odd Modes

- If Z_0 is the characteristic impedance of each isolated conductor, and v_p is the propagation velocity or wave speed in each isolated conductor

$$Z_0 = \sqrt{\frac{L_s}{C_s}} \qquad v_p = \frac{1}{\sqrt{L_s C_s}}$$

- Since

$$Z_{0\text{-even}} = \sqrt{\frac{L_s + L_m}{C_s}} \qquad v_{p\text{-even}} = \frac{1}{\sqrt{(L_s + L_m)C_s}}$$

$$Z_{0\text{-odd}} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \qquad v_{p\text{-odd}} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}$$

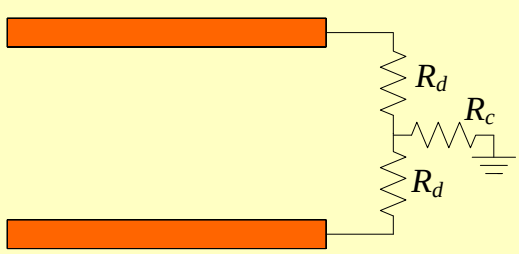
- then

$$Z_{0\text{-odd}} < Z_0 < Z_{0\text{-even}} \qquad v_{p\text{-even}} < v_{p\text{-odd}} < v_p$$

Termination Techniques

- A single-resistor termination for each conductor is not enough for coupled lines
- Proper terminations are needed to avoid reflections in both even and odd modes
- The most common termination networks are the T and Pi configurations

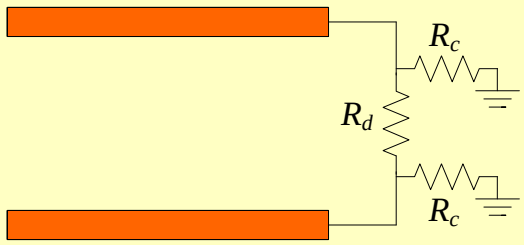
T-Termination



$$R_c = \frac{1}{2}(Z_{0\text{-even}} - Z_{0\text{-odd}})$$

$$R_d = Z_{0\text{-odd}}$$

Pi-Termination



$$R_c = Z_{0\text{-even}}$$

$$R_d = \frac{2Z_{0\text{-even}}Z_{0\text{-odd}}}{Z_{0\text{-even}} - Z_{0\text{-odd}}}$$

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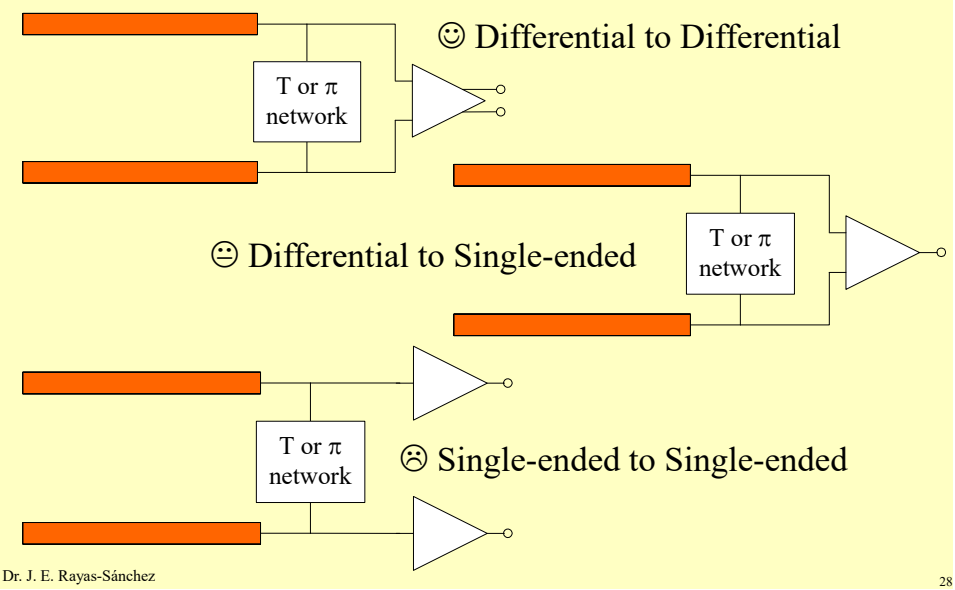
Adding Buffers for Differential Signaling



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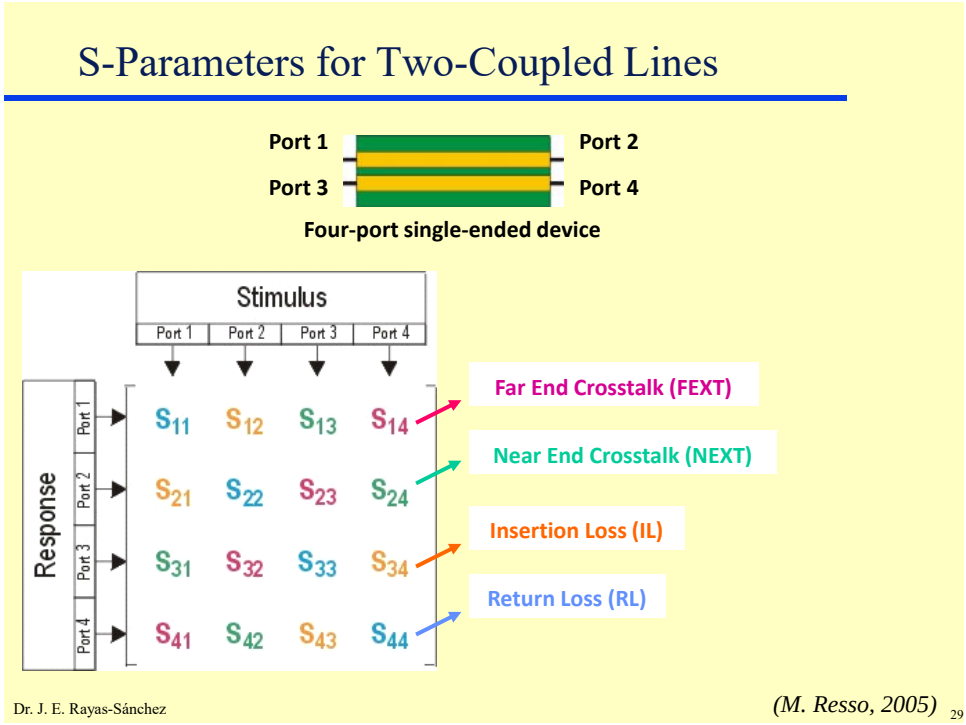
S-Parameters for Two-Coupled Lines
Port 1
Port 2
Port 3
Port 4
Four-port single-ended device
Stimulus
Port 1
Port 2
Port 3
Port 4
Response
Port 1
Port 2
Port 3
Port 4
S11 S12 S13 S14
S21 S22 S23 S24
S31 S32 S33 S34
S41 S42 S43 S44
Far End Crosstalk (FEXT)
Near End Crosstalk (NEXT)
Insertion Loss (IL)
Return Loss (RL)
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(M. Resso, 2005)
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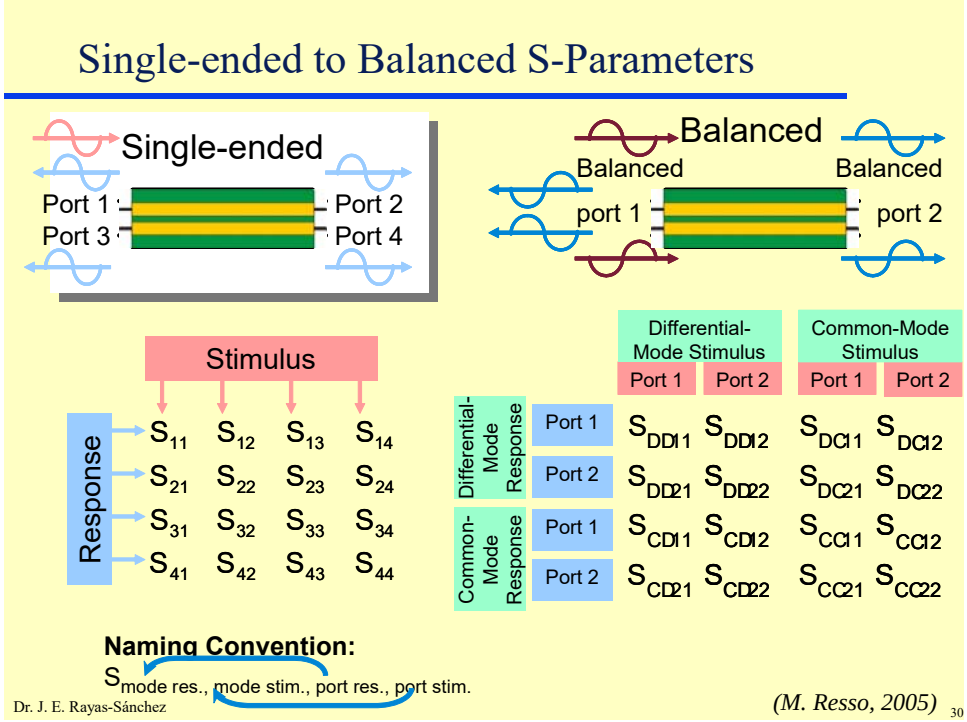
Single-ended to Balanced S-Parameters
Single-ended
Port 1
Port 3
Port 2
Port 4
Balanced
Balanced
Balanced
port 1
port 2
Stimulus
Response
S11 S12 S13 S14
S21 S22 S23 S24
S31 S32 S33 S34
S41 S42 S43 S44
Differential-Mode Stimulus
Common-Mode Stimulus
Port 1
Port 2
Port 1
Port 2
Differential-Mode Response
Common-Mode Response
Port 1
Port 2
Port 1
Port 2
SDD11 SDD12 SDC11 SDC12
SDD21 SDD22 SDC21 SDC22
SCD11 SCD12 SCC11 SCC12
SCD21 SCD22 SCC21 SCC22
Naming Convention:
Smode res., mode stim., port res., port stim.
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(M. Resso, 2005)
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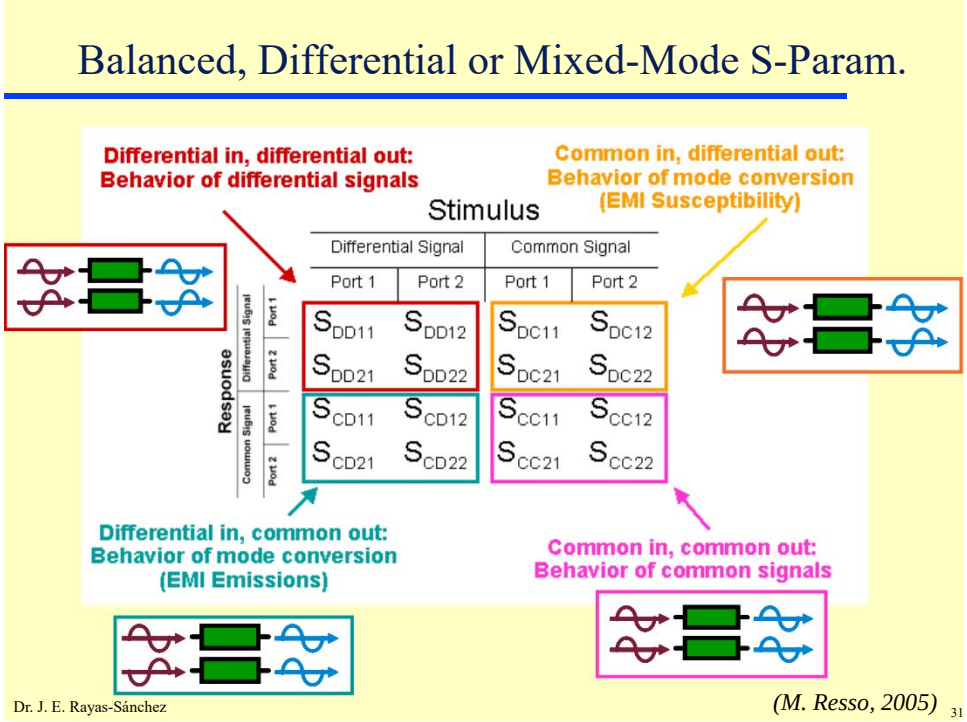
Balanced, Differential or Mixed-Mode S-Param.
Differential in, differential out:
Behavior of differential signals
Common in, differential out:
Behavior of mode conversion
(EMI Susceptibility)
Stimulus
Differential Signal
Common Signal
Port 1
Port 2
Port 1
Port 2
Response
Differential Signal
Common Signal
Port 1
Port 2
Port 1
Port 2
S_{DD11} S_{DD12}
S_{DD21} S_{DD22}
S_{DC11} S_{DC12}
S_{DC21} S_{DC22}
S_{CD11} S_{CD12}
S_{CD21} S_{CD22}
S_{CC11} S_{CC12}
S_{CC21} S_{CC22}
Differential in, common out:
Behavior of mode conversion
(EMI Emissions)
Common in, common out:
Behavior of common signals
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(M. Resso, 2005)
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