Frequency-Domain Analysis of Transmission Line Circuits

(Part 2)

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Outline

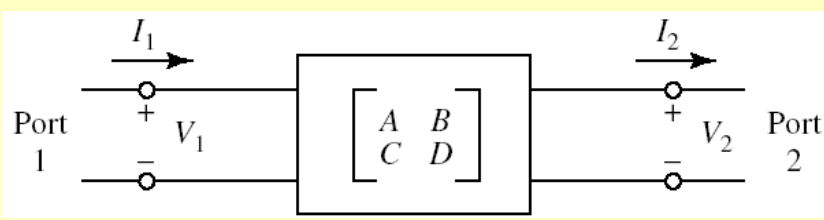
- The transmission or ABCD matrix
- ABCD parameters
- Cascaded connections of 2-port networks
- Conversion between ABCD, Z, Y, and S parameters
- Analyzing interconnects with discontinuities
- Microstrip discontinuities
- Waveguide discontinuities

The Transmission Matrix (ABCD Parameters)

- Many practical high-frequency circuits consists of cascaded connections of two o more two-port networks
- The Transmission Matrix, or ABCD Parameters, or Chain Parameters, or Cascading Parameters, are very convenient for analyzing cascaded two-port circuits

Definition of ABCD Parameters

- The ABCD matrix is defined only for two-port networks:



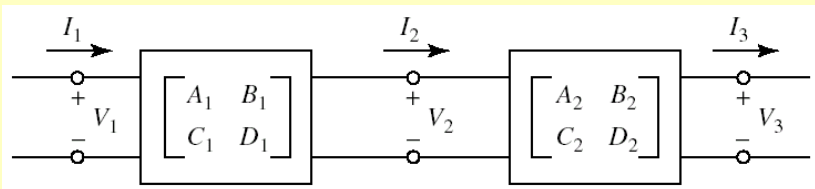
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

Cascaded Connections of Two-Port Networks

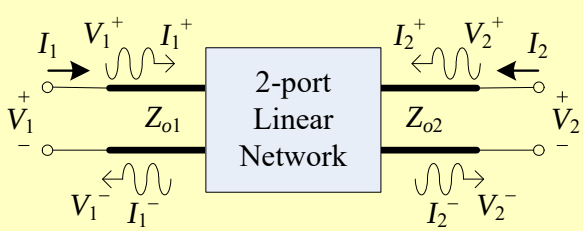
- The ABCD matrix of a cascaded connection is the multiplication of the corresponding ABCD matrices:



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

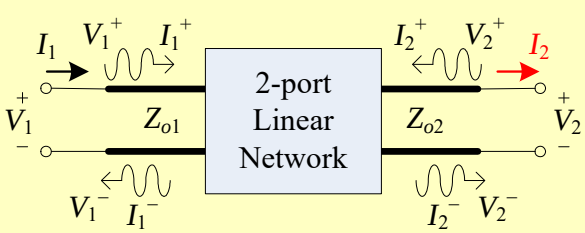
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

Converting ABCD to other Parameters



$$\boldsymbol{V} = \boldsymbol{Z}\boldsymbol{I}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$A = Z_{11} / Z_{21}$$

$$B = (Z_{11}Z_{22} - Z_{12}Z_{21}) / Z_{21}$$

$$C = 1 / Z_{21}$$

$$D = Z_{22} / Z_{21}$$

Properties of the ABCD Matrix

- If the 2-port network is lossless, *A* and *D* are real, *B* and *C* are imaginary
- If the 2-port network is reciprocal, the determinant of the ABCD matrix is one

$$AD - BC = 1$$

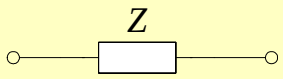
Converting Between Different Parameters

	[Z]	[Y]	[h]	[ABCD]
[Z]	$\begin{matrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{matrix}$	$\begin{matrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{matrix}$	$\begin{matrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{matrix}$	$\begin{matrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{matrix}$
[Y]	$\begin{matrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{matrix}$	$\begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix}$	$\begin{matrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{matrix}$	$\begin{matrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{\Delta Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{matrix}$
[h]	$\begin{matrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$	$\begin{matrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{matrix}$
[ABCD]	$\begin{matrix} \frac{A}{C} & \frac{\Delta ABCD}{C} \\ \frac{1}{C} & \frac{D}{C} \end{matrix}$	$\begin{matrix} \frac{D}{B} & -\frac{\Delta ABCD}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{matrix}$	$\begin{matrix} \frac{B}{D} & \frac{\Delta ABCD}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{matrix}$	$\begin{matrix} A & B \\ C & D \end{matrix}$

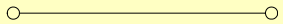
$$\Delta Z = \mathrm{Det}(\boldsymbol{Z})$$
$$\Delta Y = \mathrm{Det}(\boldsymbol{Y})$$
$$\Delta h = \mathrm{Det}(\boldsymbol{H})$$
$$\Delta ABCD = AD - BC$$

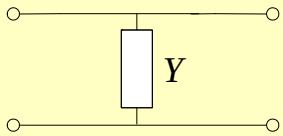
(R. Ludwig and P. Bretchko, RF Circuit Design, Prentice Hall, 2000)

ABCD Parameters for Useful Circuits



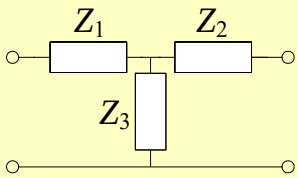
$$A = 1 \quad B = Z$$
$$C = 0 \quad D = 1$$



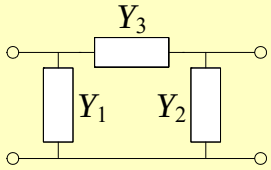


$$A = 1 \quad B = 0$$
$$C = Y \quad D = 1$$

ABCD Parameters for Useful Circuits (cont.)

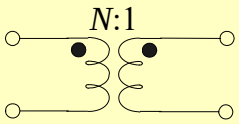


$$A = 1 + \frac{Z_1}{Z_3} \quad B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$
$$C = \frac{1}{Z_3} \quad D = 1 + \frac{Z_2}{Z_3}$$

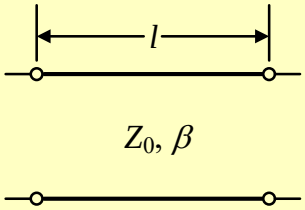


$$A = 1 + \frac{Y_2}{Y_3} \quad B = \frac{1}{Y_3}$$
$$C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3} \quad D = 1 + \frac{Y_1}{Y_3}$$

ABCD Parameters for Useful Circuits (cont.)

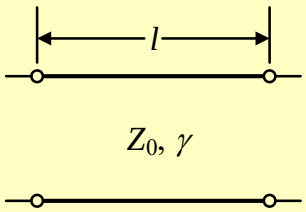


$$A = N \qquad B = 0$$
$$C = 0 \qquad D = 1/N$$



$$A = \cos\beta l \qquad B = jZ_0 \sin\beta l$$
$$C = jY_0 \sin\beta l \qquad D = \cos\beta l$$

ABCD Parameters for Useful Circuits (cont.)



$$A = \cosh\gamma l \qquad B = Z_0 \sinh\gamma l$$
$$C = Y_0 \sinh\gamma l \qquad D = \cosh\gamma l$$

Analyzing Interconnects

As a first-order approximation ("hand" calculations),

- Uniform interconnects can be modeled by transmission lines (in many cases lossless)
- Transitions or discontinuities can be modeled by equivalent lumped circuits (estimating their values by formulae or by curve-fitting), or directly by Z, Y, or ABCD matrices obtained from measured S parameters
- Each section is represented by its ABCD matrix, Z matrix or Y matrix
- Calculations are performed by combining the corresponding matrices

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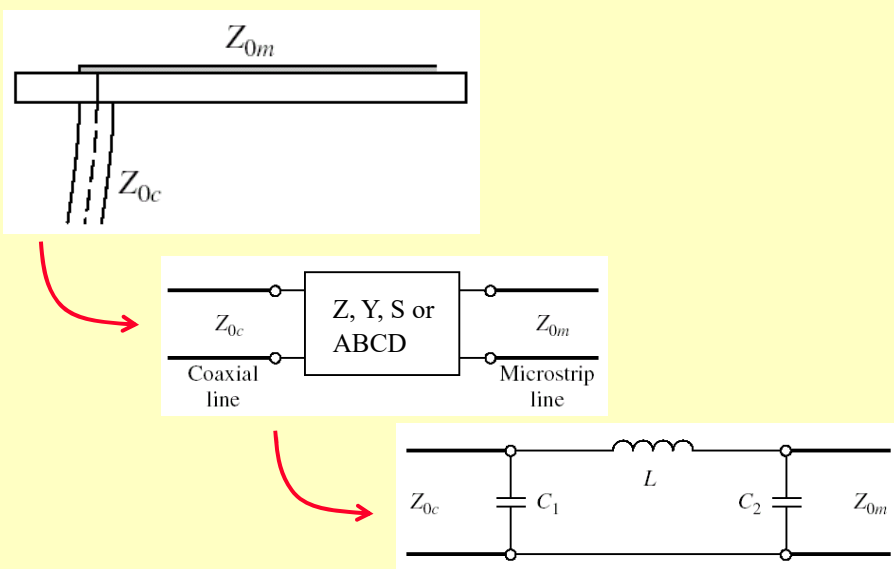
Modeling Interconnects with Transitions – Ex. 1



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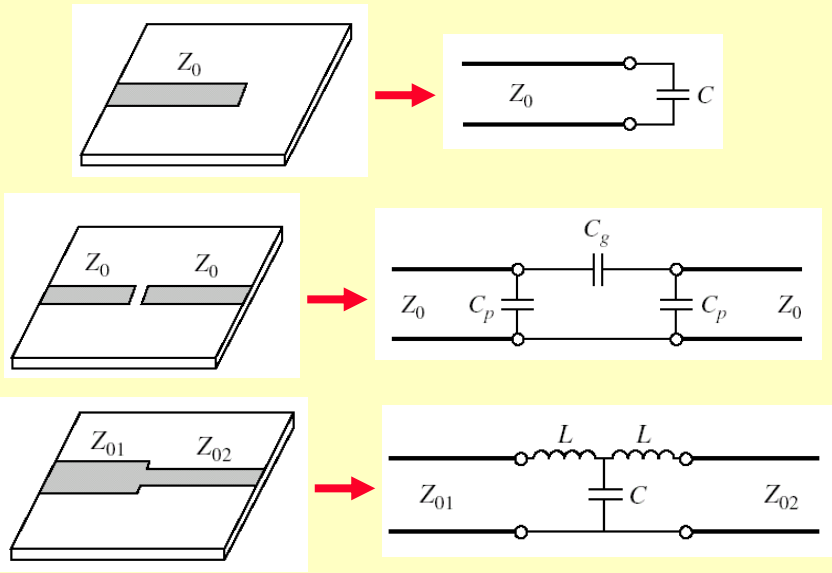
Microstrip Discontinuities



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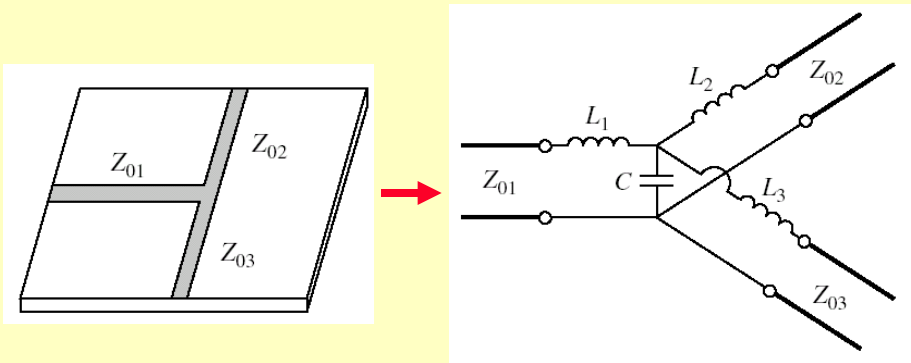
(D. M. Pozar, Microwave Engineering, Wiley, 2005) 15

Microstrip Discontinuities (cont.)



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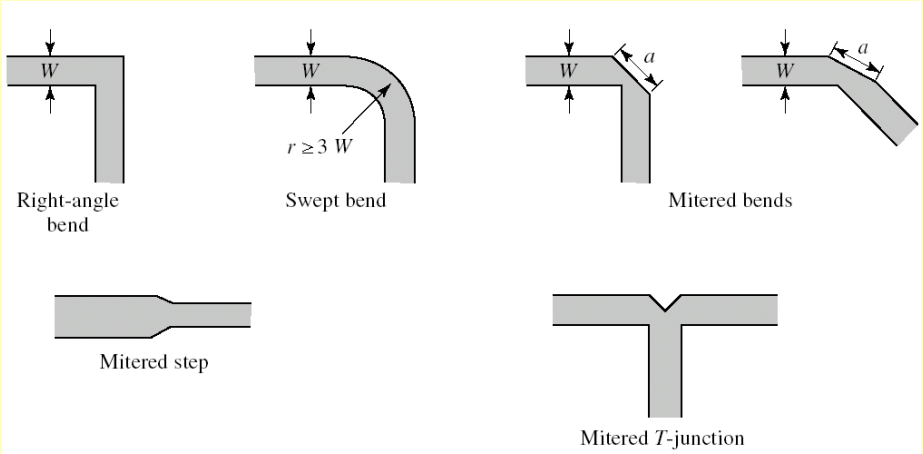
Microstrip Discontinuities (cont.)



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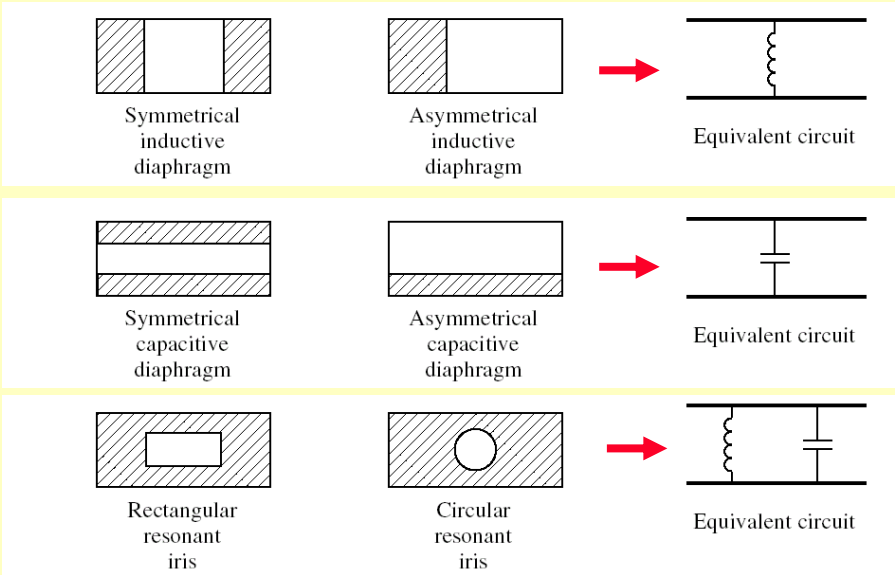
Rectangular Waveguide Discontinuities



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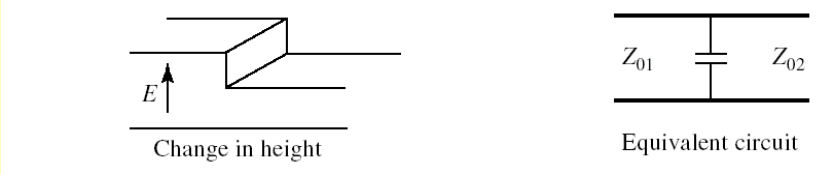
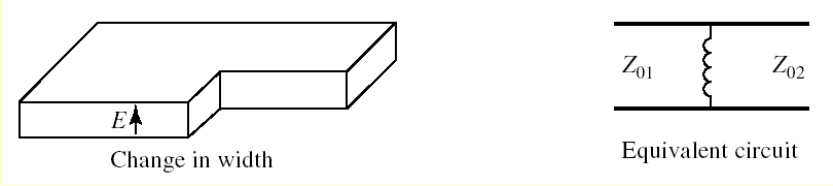
Rectangular Waveguide Discontinuities (cont.)





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