

# **The Smith Chart**

(Part 2)

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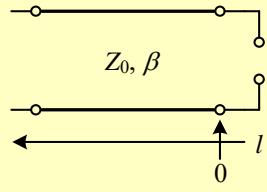
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## Outline

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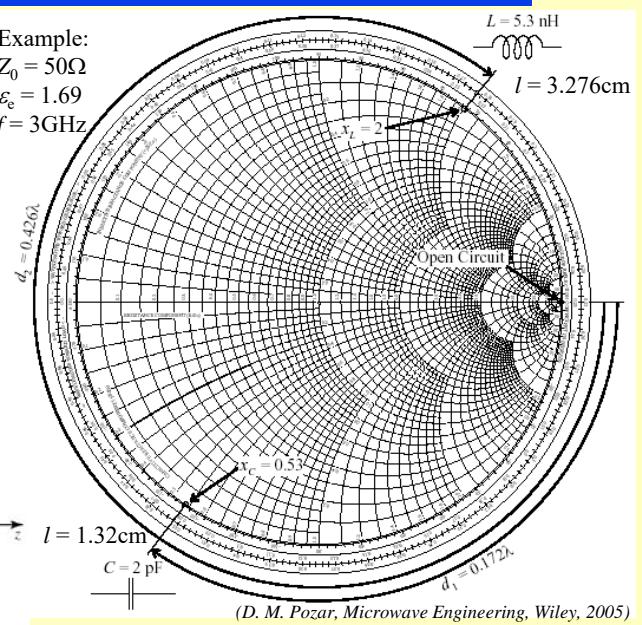
- Open circuit transformations using the Smith Chart
- Short circuit transformations
- Impedance-admittance transformations
- Impedance/admittance Smith Chart
- The quarter-wave transformer

## Open Circuit Transformations

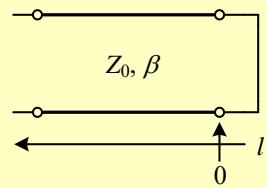


Given  $Z_0$ ,  $\beta$  (or  $\epsilon_e$ ), and  $f$ , we can calculate the lengths for a desired  $C$  or  $L$

Example:  
 $Z_0 = 50\Omega$   
 $\epsilon_e = 1.69$   
 $f = 3\text{GHz}$

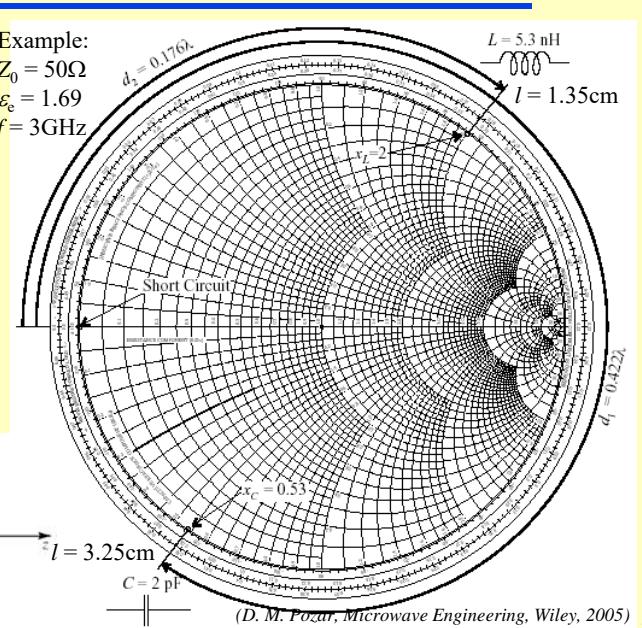


## Short Circuit Transformations

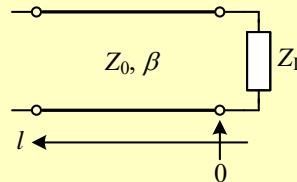


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## Impedance-Admittance Transformations



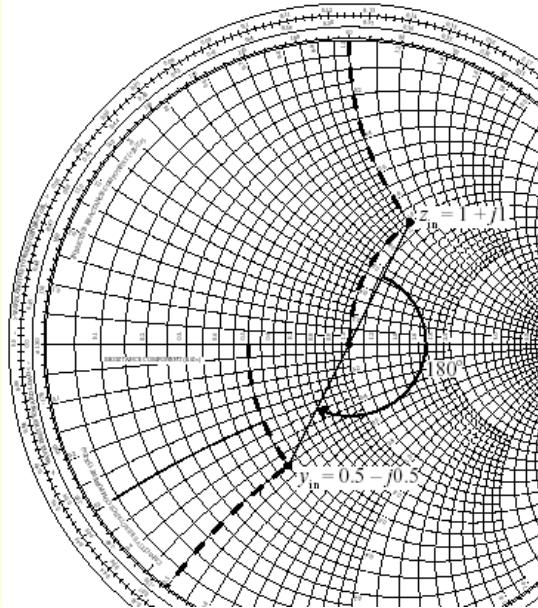
$$Z_{in}(l) = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

$$Z_{in}(l = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots) = \frac{Z_0^2}{Z_L}$$

$$z_{in}(l = \frac{\lambda}{4}) = \frac{Z_0}{Z_L} = \frac{1}{z_L} = y_{in}$$

$$Y_{in} = \frac{1}{Z_L}$$

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## Impedance-Admittance Transf. – Example 1

$$Z = 25 + j75 \Omega$$

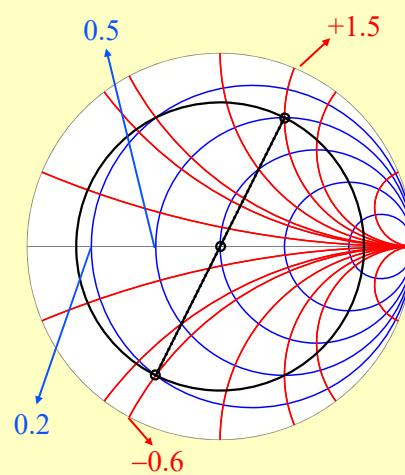
$$Z_0 = 50 \Omega$$

$$z = Z / Z_0 = 0.5 + j1.5$$

$$y = 0.2 - j0.6$$

$$Y = y(1/Z_0)$$

$$Y = 0.004 - j0.012 \text{ S}$$



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## Impedance-Admittance Transf. – Example 2

$$Z = 150 - j50 \Omega$$

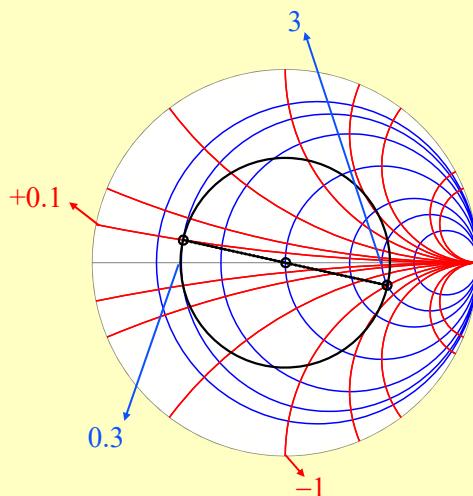
$$Z_0 = 50 \Omega$$

$$z = Z / Z_0 = 3 - j$$

$$y = 0.3 + j0.1$$

$$Y = y(1 / Z_0)$$

$$Y = 0.006 + j0.002 \text{ S}$$

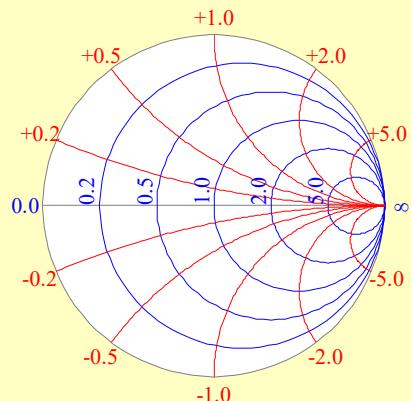


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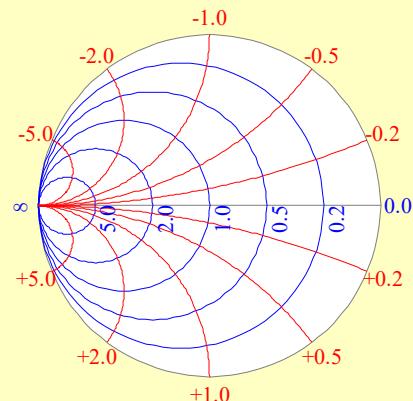
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## Impedance-Admittance Smith Charts

Impedance Smith Chart



Admittance Smith Chart

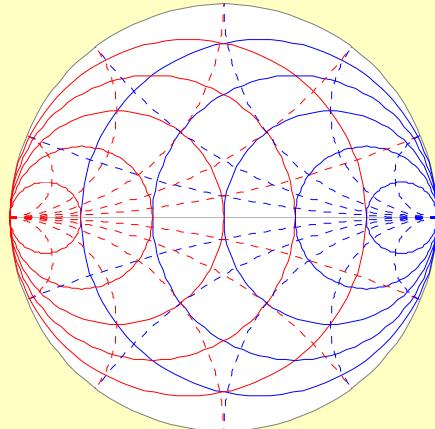


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## Impedance-Admittance Smith Charts (cont.)

Impedance/Admittance Smith Chart

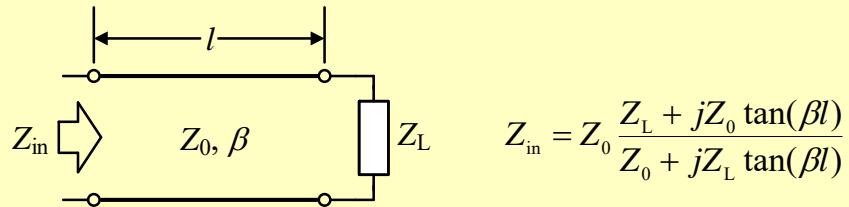


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## Quarter-Wave Transformer

- Simple technique for impedance matching
- It can achieve perfect match at a single frequency

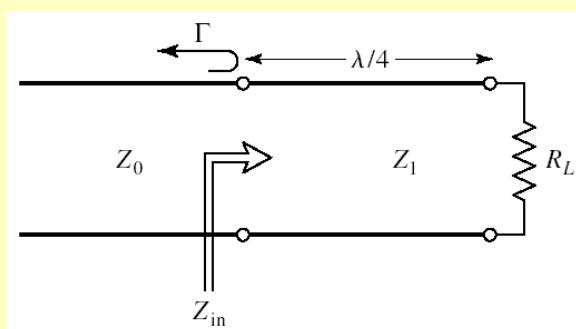


$$\text{If } l = \frac{\lambda}{4} \Rightarrow Z_{in} = \frac{Z_0^2}{Z_L}$$

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## Quarter-Wave Transformer (cont.)



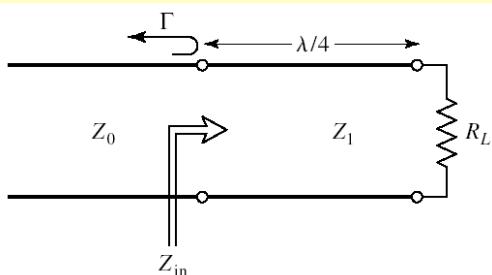
$$Z_{in} = \frac{Z_1^2}{R_L}$$

To make  $\Gamma = 0$ ,

$$Z_1 = \sqrt{R_L Z_0}$$

$Z_1$  must be the geometric mean of  $Z_0$  and  $R_L$

## Quarter-Wave Transformer – Example



$$Z_0 = 50\Omega$$

$$R_L = 100\Omega$$

$$Z_1 = ?$$

$$\Gamma(f) = ?$$

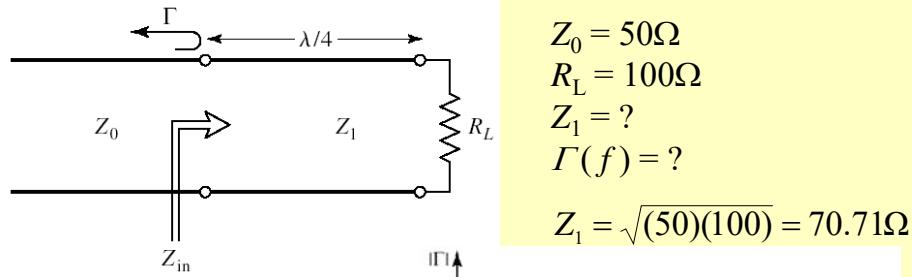
$$Z_1 = \sqrt{(50)(100)} = 70.71\Omega$$

$\Gamma = 0$  only at the frequencies at which the electrical length of the matching section is  $90^\circ, 270^\circ, \dots$

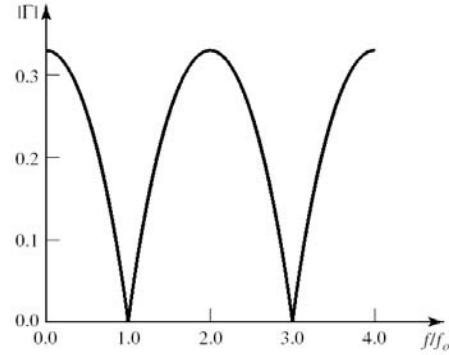
$$\text{Since } \Gamma_l = \frac{Z_{in}(l) - Z_0}{Z_{in}(l) + Z_0} \quad \text{and} \quad Z_{in}(l) = Z_1 \frac{Z_L + jZ_1 \tan(\beta l)}{Z_1 + jZ_L \tan(\beta l)}$$

If  $f_0$  is the tuning frequency, and  $\lambda_0$  the wavelength at  $f_0$   $\rightarrow \beta l = \left( \frac{2\pi}{\lambda} \right) \left( \frac{\lambda_0}{4} \right) = \left( \frac{2\pi f}{v_p} \right) \left( \frac{v_p}{4f_0} \right) = \frac{\pi}{2} \frac{f}{f_0}$

## Quarter-Wave Transformer – Example (cont.)



$$\beta l = \frac{\pi}{2} \frac{f}{f_0}$$



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