The Smith Chart

(Part 1)

Dr. José Ernesto Rayas-Sánchez

Outline

- Origins of the Smith Chart
- Mathematical basis of the Smith Chart
- Smith Chart interpretation
- Basic Smith Chart applications
  - Calculating Γ, RL, SWR
  - Calculating Γ_l and Z_{in}
  - Circles of SWR, V_{max} and V_{min}

The Smith Chart

- Developed in 1939 by P. H. Smith at Bell Labs
- Very useful for visualizing transmission line phenomena and impedance matching problems
- It is part of most current CAD tools and modern measurement equipments
- It is essentially a plot of Γ in polar coordinates
- Any impedance can be mapped in the Γ plane

The Smith Chart (cont.)

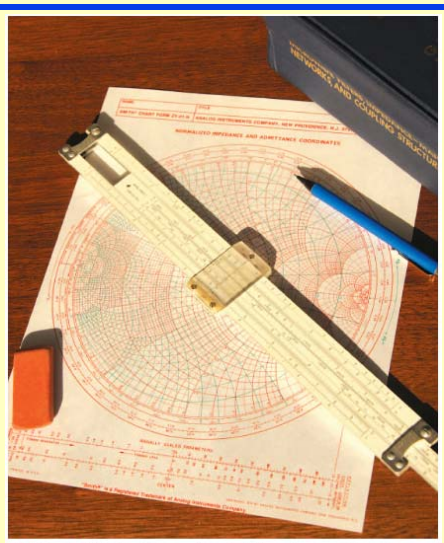
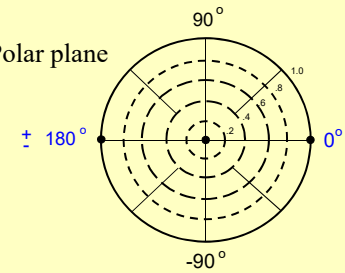


Figure 1. *The Smith chart and slide rule were the primary microwave design tools in the 1970s and before.*

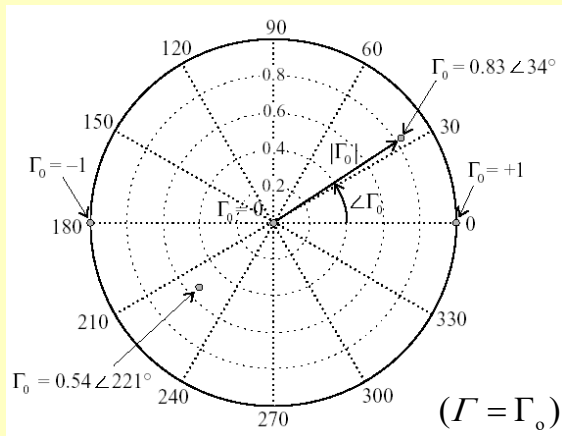
(J. Rautio, IEEE Microwave Magazine, 2007)

The Smith Chart (cont.)

Polar plane



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta}$$



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(R. Ludwig and P. Bretchko, RF Circuit Design, Prentice Hall, 2000)

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The Smith Chart (cont.)

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = |\Gamma| e^{j\theta}$$

Normalizing Z w.r.t Z_o

$$\frac{z-1}{z+1} = |\Gamma| e^{j\theta} \quad \text{where} \quad z = \frac{Z}{Z_o}$$

$$z = \frac{1 + |\Gamma| e^{j\theta}}{1 - |\Gamma| e^{j\theta}}$$

$$r + jx = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i}$$

Solving for r and x

$$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$x = \frac{2\Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

Rearranging

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

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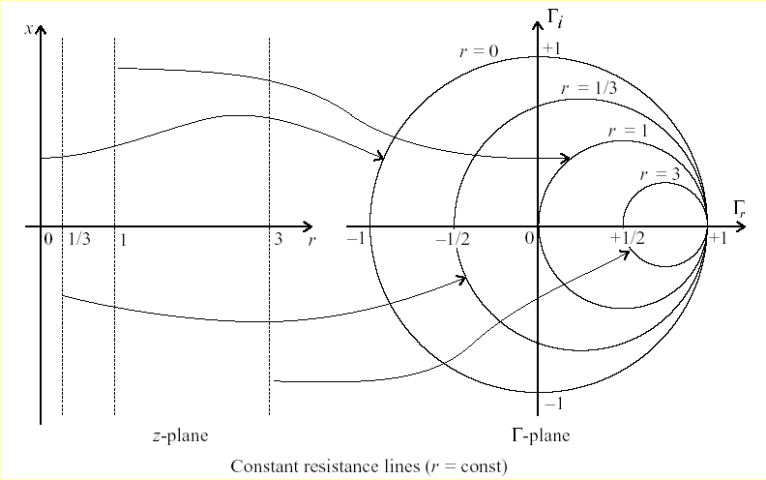
The Smith Chart – Circles of Constant *r*

$$\left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2$$



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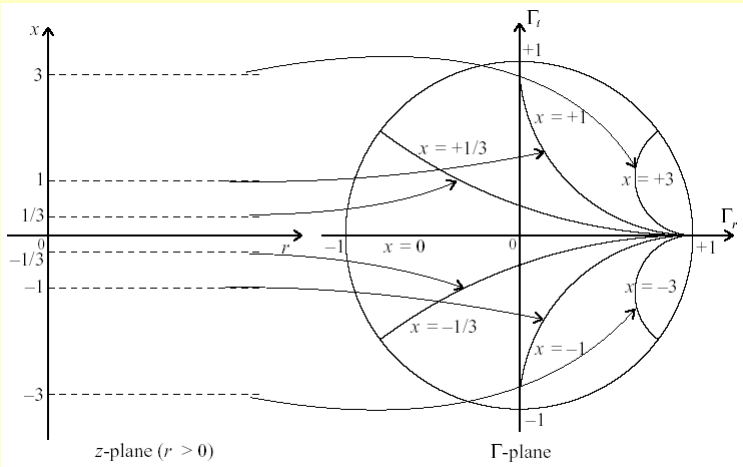
The Smith Chart – Circles of Constant *x*

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$



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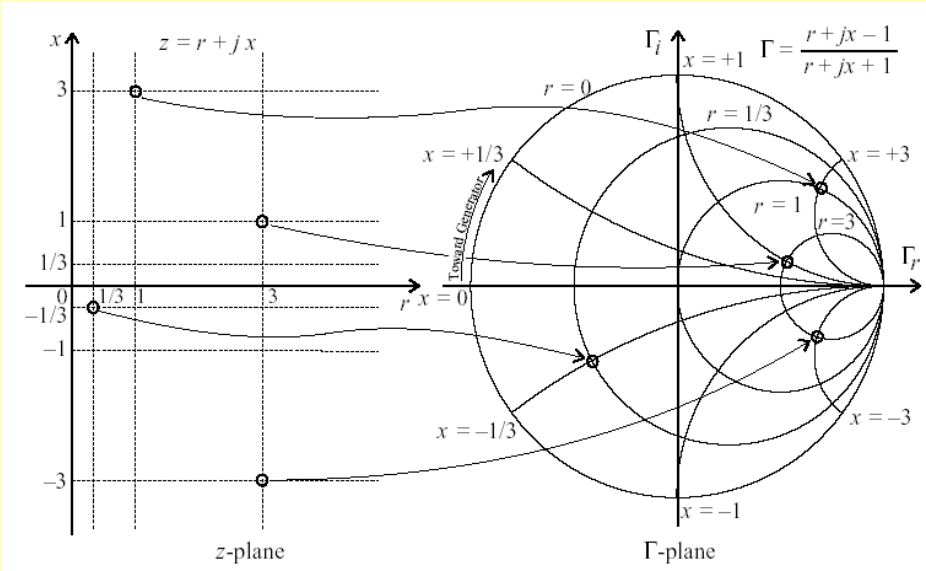
The Smith Chart – Combining both Circles



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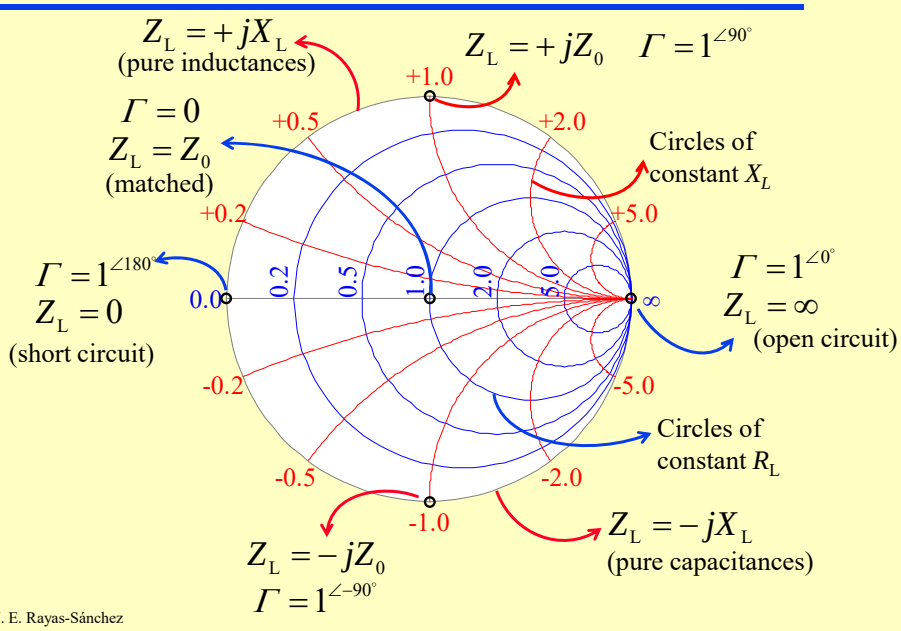
The Smith Chart – Interpretation



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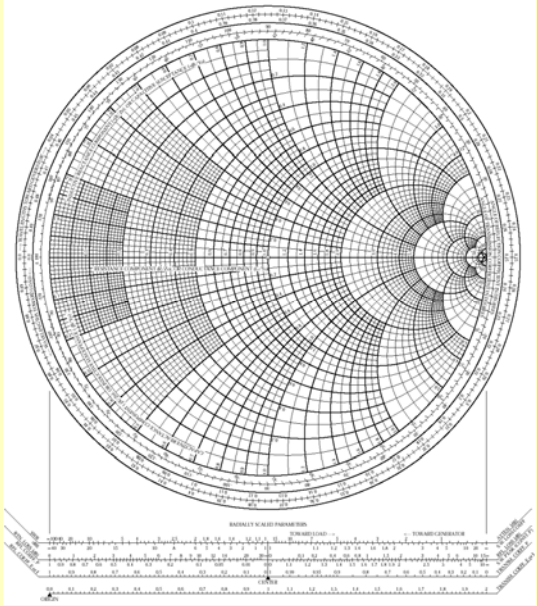
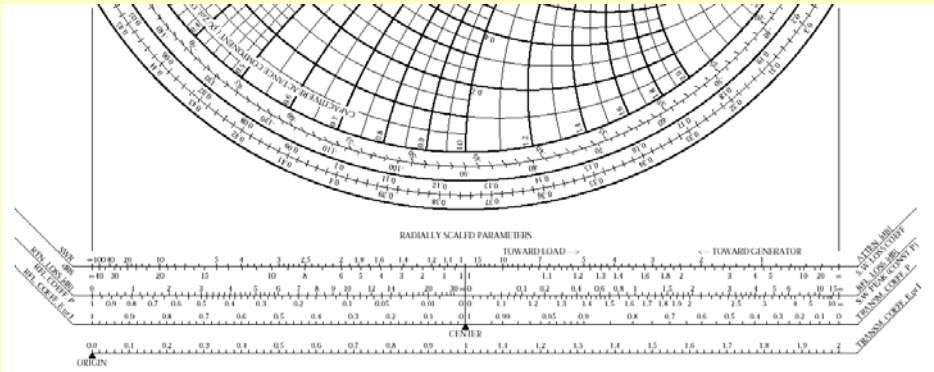
Practical Smith Chart



Practical Smith Chart (cont.)



Smith Chart – Calculating Γ, RL, SWR

Example:

$Z_0 = 50\ \Omega$

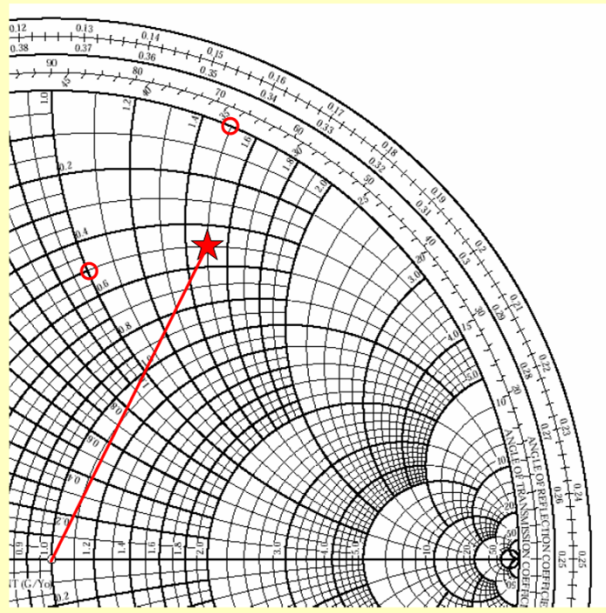
$Z_L = 25 + j75\ \ \Omega$

$\Gamma = ?$

$RL = ?$

$SWR = ?$

$z_L = 0.5 + j1.5$



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Smith Chart – Calculating Γ, RL, SWR (cont.)

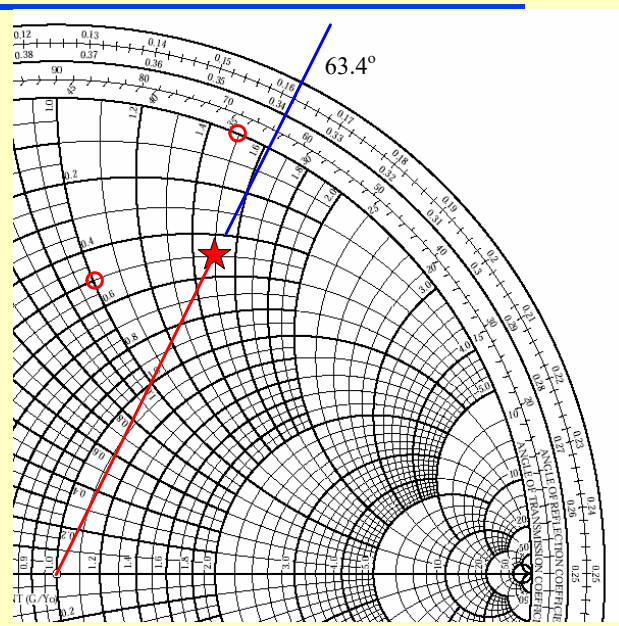
Example:

$Z_0 = 50\ \Omega$

$Z_L = 25 + j75\ \ \Omega$

$z_L = 0.5 + j1.5$

$\Gamma = |\Gamma|^{\angle 63.4^\circ}$



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Smith Chart – Calculating Γ, RL, SWR (cont.)

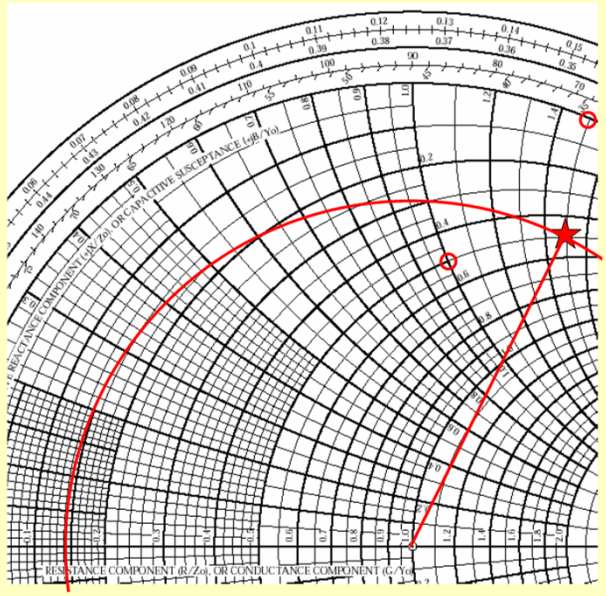
Example:

$Z_0 = 50\ \Omega$

$Z_L = 25 + j75\ \ \Omega$

$z_L = 0.5 + j1.5$

$\Gamma = |\Gamma|^{\angle 63.4^\circ}$



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Smith Chart – Calculating Γ, RL, SWR (cont.)

Example:

$Z_0 = 50\ \Omega$

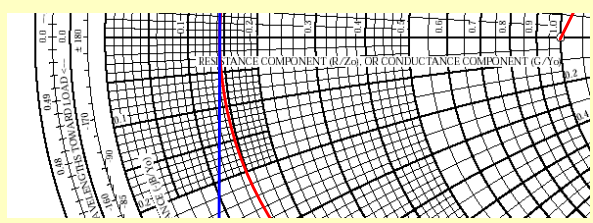
$Z_L = 25 + j75\ \ \Omega$

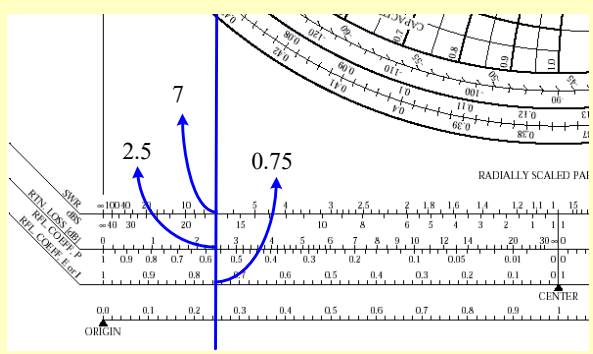
$z_L = 0.5 + j1.5$

$SWR \approx 7$

$RL \approx 2.5\ \text{dB}$

$\Gamma \approx 0.75^{\angle 63.4^\circ}$





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Smith Chart – Circles of *SWR*

Example:

$Z_0 = 50\ \Omega$

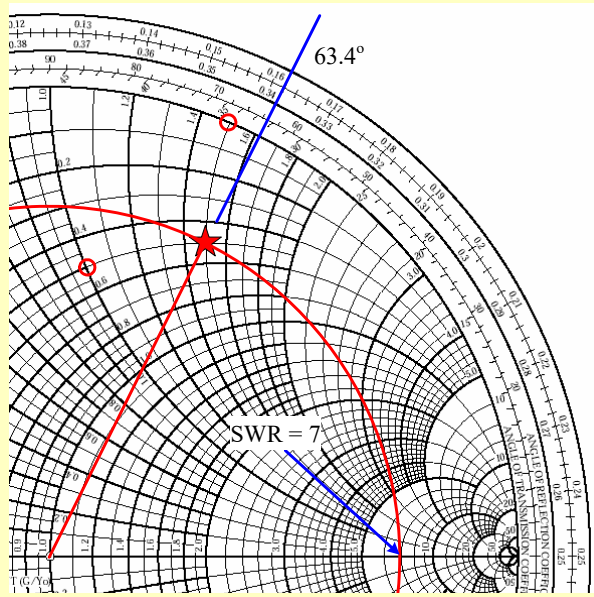
$Z_L = 25 + j75\ \ \Omega$

$z_L = 0.5 + j1.5$

$SWR \approx 7$

$RL \approx 2.5\ \text{dB}$

$\Gamma \approx 0.75\ ^{\angle 63.4^\circ}$



Smith Chart – Circles of *SWR* (cont.)

Examples:

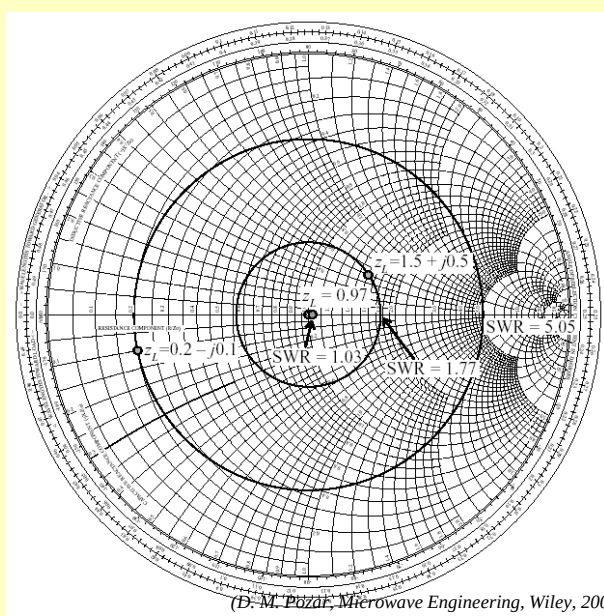
$Z_0 = 50\ \Omega$

a) $Z_L = 50\ \ \Omega$

b) $Z_L = 48.5\ \ \Omega$

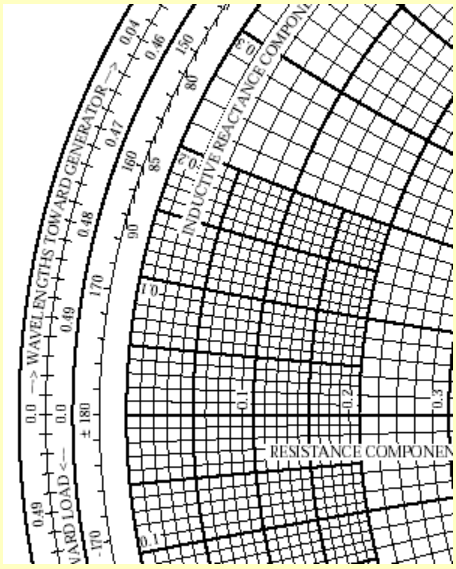
c) $Z_L = 75 + j25\ \ \Omega$

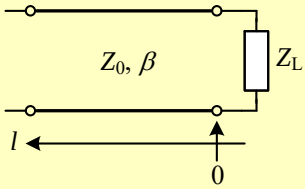
d) $Z_L = 10 - j5\ \ \Omega$



(D. M. Pozar, Microwave Engineering, Wiley, 2005)

Smith Chart – Calculating Γ_l





$$\Gamma_l(l) = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l}} = \frac{V_0^-}{V_0^+} e^{-2j\beta l} = \Gamma e^{-2j\beta l}$$

Since $\beta = \dfrac{2\pi}{\lambda}$

$$\Gamma_l(l) = \Gamma e^{-2j\frac{2\pi}{\lambda}l} \qquad \text{(Period} = \lambda/2)$$

$\dfrac{l}{\lambda}$ is the length in wavelengths

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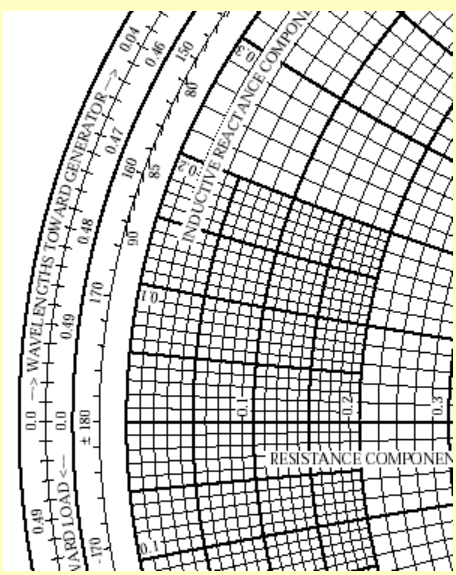
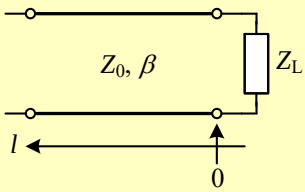
Smith Chart – Calculating Z_{in}





$$\Gamma_l(l) = \Gamma e^{-2j\frac{2\pi}{\lambda}l}$$

$\dfrac{l}{\lambda}$ is the length in wavelengths

$$\Gamma_l = \frac{Z_{in}(l) - Z_0}{Z_{in}(l) + Z_0}$$

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Smith Chart – Calculating Γ_l and Z_{in}

Example:

$Z_0 = 50\ \Omega$

$Z_L = 25 + j75\ \Omega$

$l = 1\ \text{cm}$

$f = 1\ \text{GHz}$

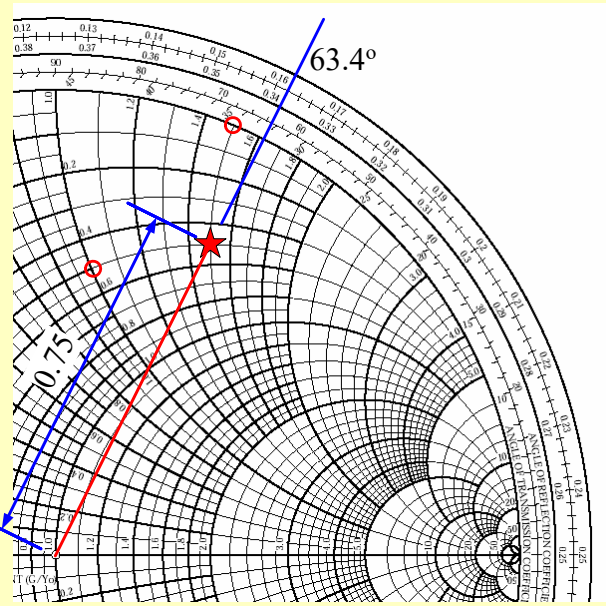
$\varepsilon_e = 4$

$\Gamma_l = ?$

$Z_{in} = ?$

$z_L = 0.5 + j1.5$

$\Gamma \approx 0.75\ \angle 63.4^\circ$



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Smith Chart – Calculating Γ_l and Z_{in} (cont.)

Example:

$Z_0 = 50\ \Omega$

$Z_L = 25 + j75\ \Omega$

$l = 1\ \text{cm}$

$f = 1\ \text{GHz}$

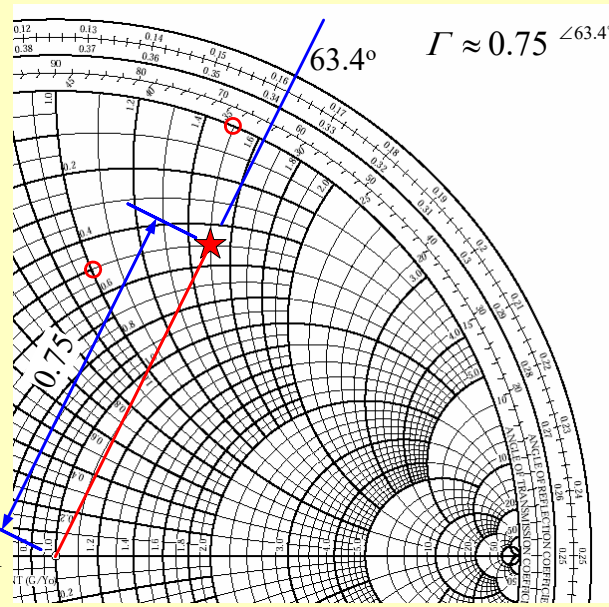
$\varepsilon_e = 4$

$\Gamma_l = ?$

$Z_{in} = ?$

$\lambda = \dfrac{v_p}{f} = \dfrac{c}{f\sqrt{\varepsilon_e}}$

$\lambda = \dfrac{0.3\text{Gm/s}}{1\text{GHz}\sqrt{4}} = 15\text{cm}$



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Smith Chart – Calculating Γ_l and Z_{in} (cont.)

Example:

$Z_0 = 50\ \Omega$

$Z_L = 25 + j75\ \Omega$

$l = 1\ \text{cm}$

$f = 1\ \text{GHz}$

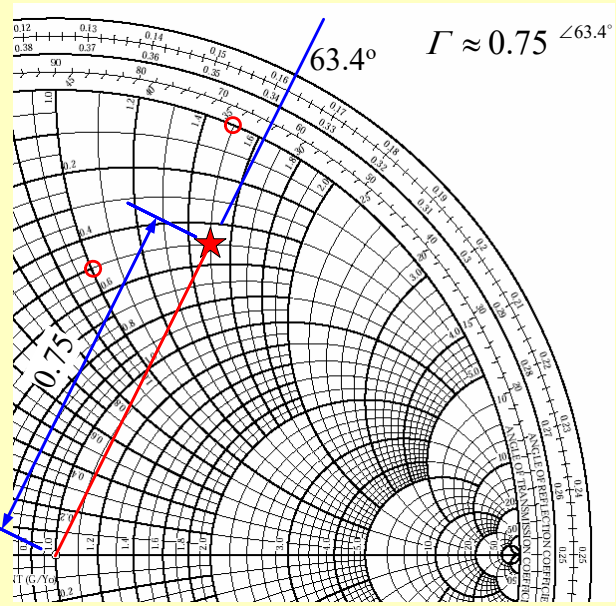
$\varepsilon_e = 4$

$\Gamma_l = ?$

$Z_{in} = ?$

$$\frac{l}{\lambda} = \frac{1\text{cm}}{15\text{cm}} = 0.067$$

$$0.162 + 0.067 = 0.229$$



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Smith Chart – Calculating Γ_l and Z_{in} (cont.)

Example:

$Z_0 = 50\ \Omega$

$Z_L = 25 + j75\ \Omega$

$l = 1\ \text{cm}$

$f = 1\ \text{GHz}$

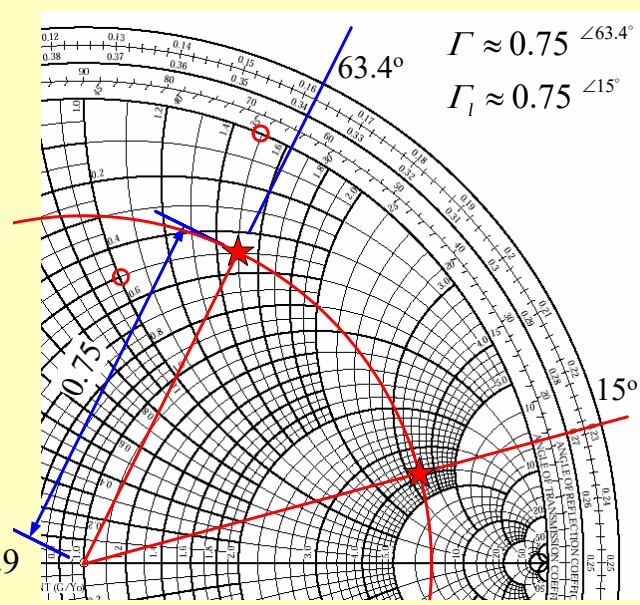
$\varepsilon_e = 4$

$\Gamma_l = ?$

$Z_{in} = ?$

$$\frac{l}{\lambda} = \frac{1\text{cm}}{15\text{cm}} = 0.067$$

$$0.162 + 0.067 = 0.229$$



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Smith Chart – Calculating Γ_l and Z_{in} (cont.)

Example:

$Z_0 = 50\ \Omega$

$Z_L = 25 + j75\ \ \Omega$

$l = 1\ \text{cm}$

$f = 1\ \text{GHz}$

$\varepsilon_e = 4$

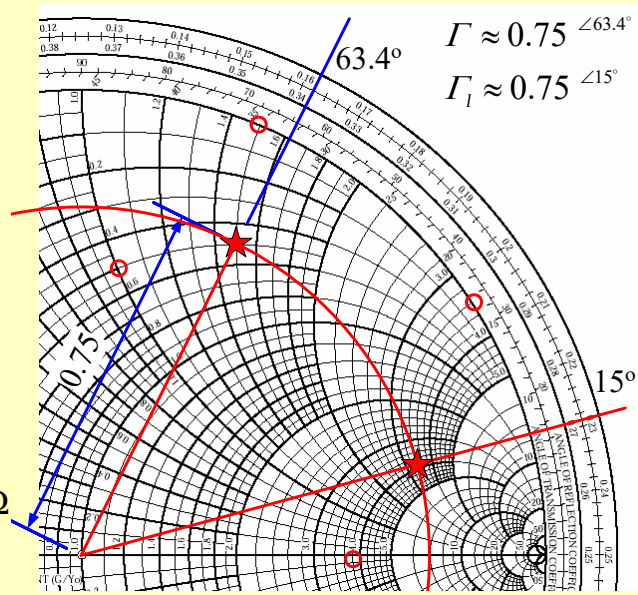
$\Gamma_l = ?$

$Z_{in} = ?$

$z_{in} = 3.8 + j3.4$

$Z_{in} = 50(3.8 + j3.4)\ \Omega$

$Z_{in} \approx 190 + j170\ \ \Omega$



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Smith Chart – Circles of *SWR*, V_{max} and V_{min}

Since $|V(l)| = |V_0^+|\ \ |1 + |\Gamma| e^{+j(\theta - 2\beta l)}|$

where $\Gamma = |\Gamma| e^{j\theta}$

We can locate the positions of V_{max} and V_{min} along the line

Example:

$Z_0 = 50\ \Omega$

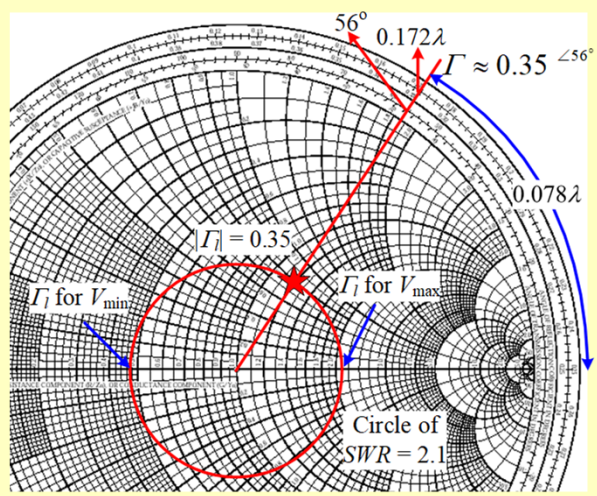
$Z_L = 60 + j40\ \ \Omega$

$z_L = 1.2 + j0.8\ \ \Omega$

$SWR \approx 2.1$

$WL_{max} = 0.25\lambda - 0.172\lambda$

$WL_{max} = 0.078\lambda$



$WL_{min} = 0.078\lambda + 0.25\lambda = 0.328\lambda$

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