Impedance Matching Circuits
(Part 4)

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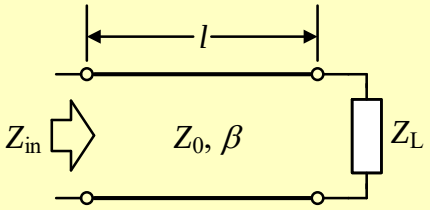
Outline

- Quarter-wave transformer
- Bandwidth of a quarter-wave transformer
- Multi-sectional transformers

Quarter-Wave Transformer

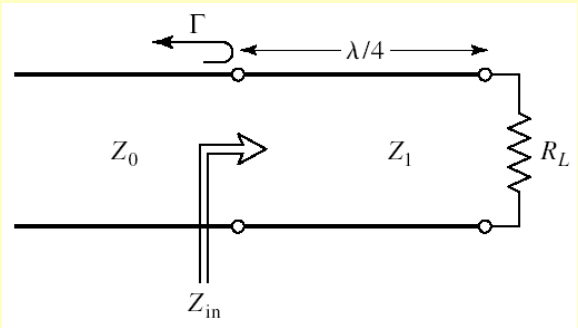
- Simple technique for impedance matching
- It can achieve perfect match at a single frequency



$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

$$\text{If } l = \frac{\lambda}{4} \quad \Rightarrow \quad Z_{in} = \frac{Z_0^2}{Z_L}$$

Quarter-Wave Transformer (cont.)



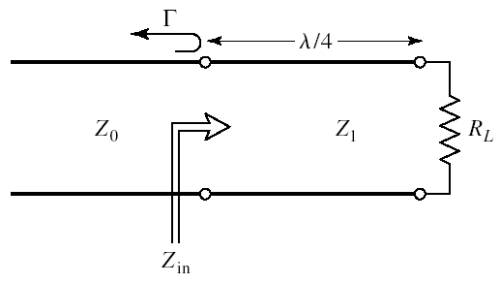
$$Z_{in} = \frac{Z_1^2}{R_L}$$

To make $\Gamma = 0$,

$$Z_1 = \sqrt{R_L Z_0}$$

Z_1 must be the geometric mean of Z_0 and R_L

Quarter-Wave Transformer - Example



$Z_0 = 50\Omega$
$R_L = 100\Omega$
$Z_1 = ?$
$\Gamma(f) = ?$

$$Z_1 = \sqrt{(50)(100)} = 70.71\Omega$$

$\Gamma = 0$ only at the frequencies at which the electrical length of the matching section is 90°, 270°, …

Since $\Gamma_l = \dfrac{Z_{in}(l) - Z_0}{Z_{in}(l) + Z_0}$ and $Z_{in}(l) = Z_0 \dfrac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$

$$\beta l = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda_0}{4}\right) = \left(\frac{2\pi f}{v_p}\right)\left(\frac{v_p}{4 f_0}\right) = \frac{\pi}{2}\frac{f}{f_0}$$

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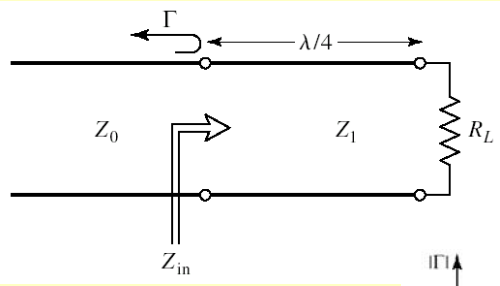
Quarter-Wave Transformer - Example



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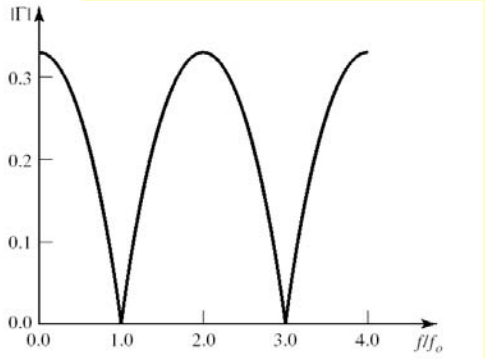

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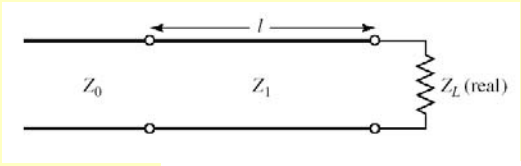
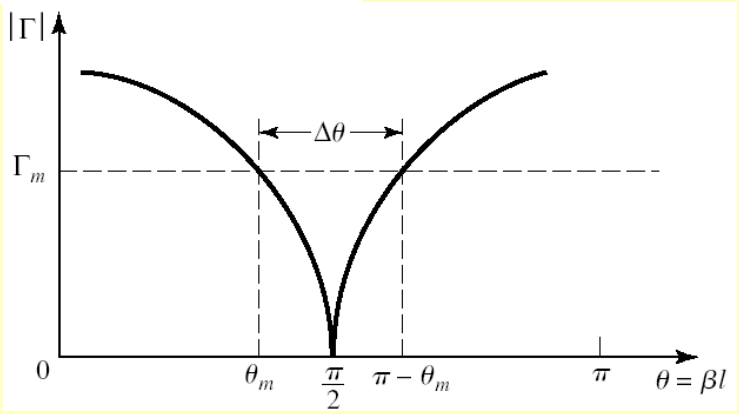
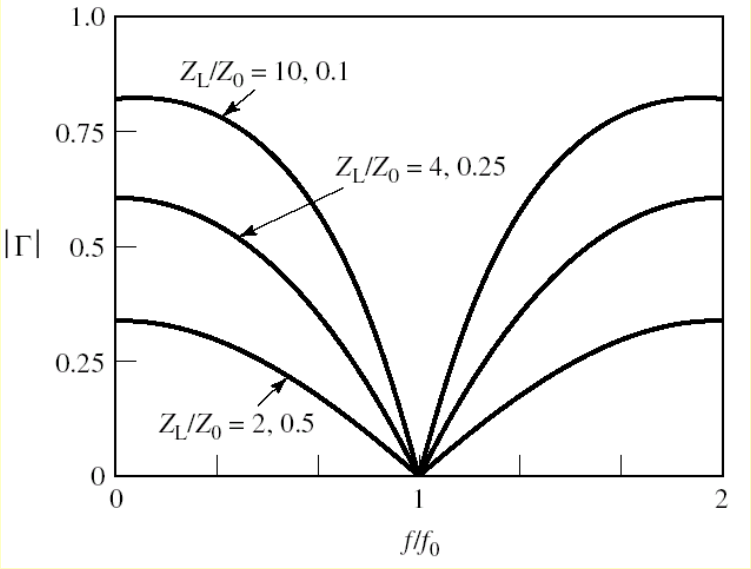
Quarter-Wave Transformer - Bandwidth





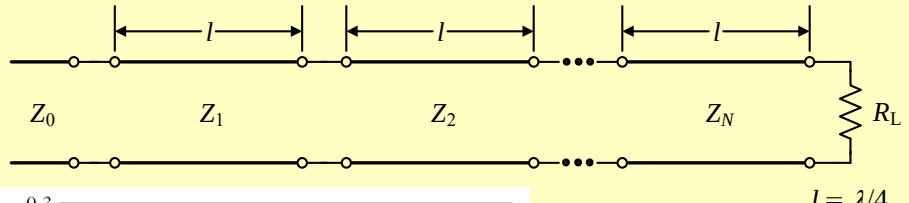
(D. M. Pozar, Microwave Engineering, Wiley, 2005)

Quarter-Wave Transformer - Bandwidth (cont.)

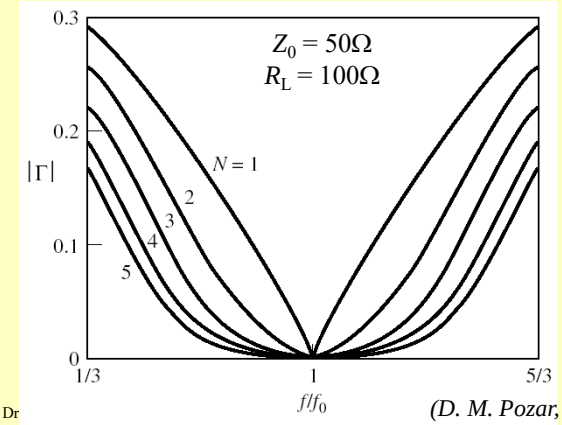


(D. M. Pozar, Microwave Engineering, Wiley, 2005)

Muti-Section Binomial Transformer Matching



$l = \lambda/4$



(D. M. Pozar, Microwave Engineering, Wiley, 2005)

Muti-Section Transformer - Binomial Coeff.

	$N=2$		$N=3$			$N=4$			
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110

	$N=5$					$N=6$					
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_6/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

(D. M. Pozar, Microwave Engineering, Wiley, 2005)

Muti-Section Transformer - Binomial (cont.)

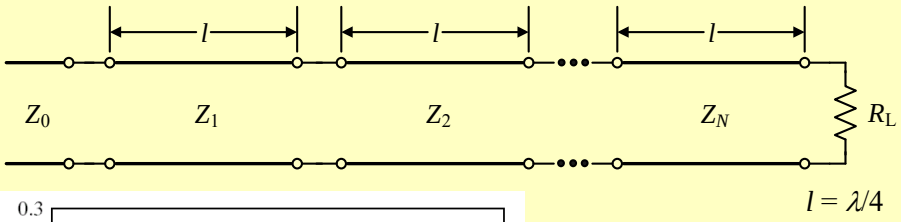
- Previous tables provide line impedances to achieve a binomial (Butterworth) response for the following cases:

  $N = 2, 3, \ldots, 6$

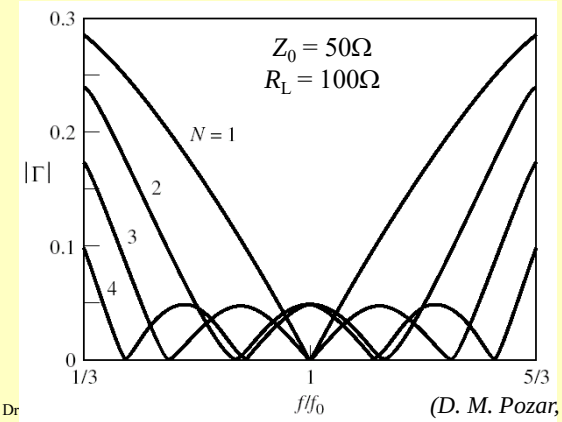
  $T = Z_L/Z_0 = 1, 1.5, 2, 3, 4, 6, 8,$ and 10
- If $Z_L < Z_0$, the same tables can be used by using the row that corresponds to $T = Z_0/Z_L$, and making the coefficient of section k equal to Z_0/Z_k
- The fractional bandwidth at Γ_m is given by

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_m}{A}\right)^{1/N}\right] \qquad A = 2^{-(N+1)}\left|\ln(T)\right|$$

Muti-Section Chebyshev Transformer Matching



$l = \lambda/4$



(D. M. Pozar, Microwave Engineering, Wiley, 2005)

Muti-Section Transformer - Chebyshev Coeff.

	$N=2$				$N=3$					
	$\Gamma_m=0.05$		$\Gamma_m=0.20$		$\Gamma_m=0.05$			$\Gamma_m=0.20$		
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920

	$N=4$							
	$\Gamma_m=0.05$				$\Gamma_m=0.20$			
Z_L/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0892	1.1742	1.2775	1.3772	1.2247	1.2247	1.2247	1.2247
2.0	1.1201	1.2979	1.5409	1.7855	1.2727	1.3634	1.4669	1.5715
3.0	1.1586	1.4876	2.0167	2.5893	1.4879	1.5819	1.8965	2.0163
4.0	1.1906	1.6414	2.4369	3.3597	1.3692	1.7490	2.2870	2.9214
6.0	1.2290	1.8773	3.1961	4.8820	1.4415	2.0231	2.9657	4.1623
8.0	1.2583	2.0657	3.8728	6.3578	1.4914	2.2428	3.5670	5.3641
10.0	1.2832	2.2268	4.4907	7.7930	1.5163	2.4210	4.1305	6.5950

(D. M. Pozar, Microwave Engineering, Wiley, 2005)

Muti-Section Transformer - Binomial Example

- Design a four-section Butterworth impedance transformer to match at 5 GHz a load impedance $Z_L = 150\ \Omega$ to a transmission line with characteristic impedance $Z_0 = 50\ \Omega$
- Calculate the theoretical fractional bandwidth for a maximum reflection of –40 dB
- Calculate the corresponding lengths of the four sections of the impedance transformer, assuming an effective dielectric constant $\varepsilon_e = 3.38$