

Fundamental Transmission Line Theory (Part 4)

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Outline

- Lossy transmission lines
- The low-loss TL
- The lossy distortionless TL
- Lossy vs lossless transmission lines

Lossy Transmission Lines

- In practice, all transmission lines have losses due to some finite conductivity and/or lossy dielectric
- These losses are usually small
- For analysis purposes (first-order approximations), these losses may be neglected
- There are two special cases of interest:
 - The low-loss line
 - The lossy distortionless line

The Low-Loss Transmission Line

- It is a line where $R \ll \omega L$ and $G \ll \omega C$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{(j\omega L)(j\omega C) \left(1 + \frac{R}{j\omega L} \right) \left(1 + \frac{G}{j\omega C} \right)}$$

$$\gamma = j\omega \sqrt{LC} \sqrt{1 - j \left(\frac{R}{\omega L} + \frac{G}{\omega C} \right) - \frac{RG}{\omega^2 LC}}$$

$$\gamma \approx j\omega \sqrt{LC} \sqrt{1 - j \left(\frac{R}{\omega L} + \frac{G}{\omega C} \right)}$$

$$\text{Using } \sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{2 \cdot 4}x^2 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6}x^3 - \dots$$

The Low-Loss Transmission Line (cont.)

- It is a line where $R \ll \omega L$ and $G \ll \omega C$

$$\gamma \approx j\omega\sqrt{LC} \left[1 - \frac{j}{2} \left(\frac{R}{\omega L} + \frac{G}{\omega C} \right) \right]$$

$$\gamma = \alpha + j\beta$$

$\beta \approx \omega\sqrt{LC}$ β is almost a linear function of ω
(no dispersion)

$$\alpha \approx \frac{1}{2} \left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}} \right) \quad Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \approx \sqrt{\frac{L}{C}}$$

$$\alpha \approx \frac{1}{2} \left(\frac{R}{Z_0} + GZ_0 \right) \quad \alpha_c \approx \frac{R}{2Z_0} \quad \alpha_d \approx \frac{GZ_0}{2}$$

The Lossy Distortionless Transmission Line

- It is a line where $R/L = G/C$ (Heaviside condition)

Since $\gamma = j\omega\sqrt{LC} \sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}}$

$$\gamma = j\omega\sqrt{LC} \sqrt{1 - 2j\frac{R}{\omega L} - \frac{R^2}{\omega^2 L^2}}$$

$$\gamma \equiv \alpha + j\beta$$

$$\gamma = j\omega\sqrt{LC} \left(1 - j\frac{R}{\omega L} \right)$$

$$\beta = \omega\sqrt{LC} \quad \alpha = R\sqrt{\frac{C}{L}}$$



Oliver Heaviside
(1850-1925)

β is a linear function of ω (non dispersive TL)

α is independent of ω

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Lossy vs Lossless Transmission Lines

$$V(z) = V_0^+ [e^{-\gamma z} + \Gamma e^{+\gamma z}]$$

$$I(z) = \frac{V_0^+}{Z_0} [e^{-\gamma z} - \Gamma e^{+\gamma z}]$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma \equiv \alpha + j\beta$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\Gamma_z(z) = \Gamma e^{+2j\beta z}$$

$$V(z) = V_0^+ [e^{-j\beta z} + \Gamma e^{+j\beta z}]$$

$$I(z) = \frac{V_0^+}{Z_0} [e^{-j\beta z} - \Gamma e^{+j\beta z}]$$

$$\beta = \omega \sqrt{LC} \quad Z_0 = \sqrt{\frac{L}{C}}$$

$$\Gamma_z(z) = \Gamma e^{+2j\beta z}$$

$$Z_0 \equiv \frac{V_0^+}{I_0^+} = \frac{V_0^-}{-I_0^-} \quad \Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \lambda = \frac{v_p}{f} = \frac{2\pi}{\beta} \quad v_p = \frac{\omega}{\beta}$$