

Fundamental Transmission Line Theory

(Part 2)

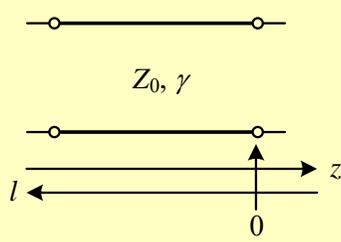
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1

Outline

- Summary on lossy transmission lines
- Lossless transmission lines
- Reflection coefficient
- Input impedance
- Input impedance *vs* reflection coefficient
- Input impedance periodicity
- Special cases of lossless terminated TL

Summary on Lossy Transmission Lines



$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \equiv \alpha + j\beta$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$V = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} = V_0^+ e^{+\gamma l} + V_0^- e^{-\gamma l}$$

$$I = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z} = I_0^+ e^{+\gamma l} + I_0^- e^{-\gamma l}$$

Since $Z_0 \equiv \frac{V_0^+}{I_0^+} = \frac{V_0^-}{-I_0^-}$ then $I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$

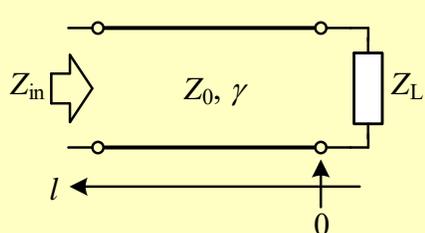
Since $\Gamma = \frac{V_0^-}{V_0^+}$ then $V(z) = V_0^+ [e^{-\gamma z} + \Gamma e^{+\gamma z}]$

$$I(z) = \frac{V_0^+}{Z_0} [e^{-\gamma z} - \Gamma e^{+\gamma z}]$$

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3

Summary on Lossy Transmission Lines (cont.)



$$\Gamma_l(l) = \frac{V_0^-}{V_0^+} e^{-2\gamma l}$$

$$\Gamma = \Gamma_l(l=0) = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_{in}(l) = \frac{V(l)}{I(l)} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}$$

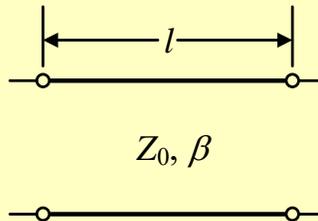
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4

Lossless Transmission Line

- Conductive and dielectric losses are zero
- It is a non-dispersive transmission line
- Ideally, practical interconnects should behave as lossless transmission lines
- For analytical purposes (first order approximations), many practical interconnects can be treated as lossless

Symbol:



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5

Lossless Transmission Line Basic Equations

The diagram shows a horizontal line with terminals at both ends. A coordinate system z is defined with the origin 0 at the right terminal and l at the left terminal. An arrow points from 0 to l .

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = j\omega\sqrt{LC}$$

$$\alpha = 0 \quad \beta = \omega\sqrt{LC}$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \quad \text{non-dispersive TL: all waves propagate at the same speed}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}} \quad Z_0 \text{ is frequency independent} \quad \lambda = \frac{v_p}{f} = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}}$$

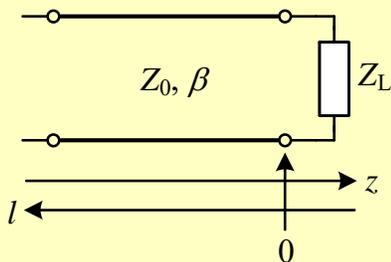
$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} \quad V(l) = V_0^+ e^{+j\beta l} + V_0^- e^{-j\beta l}$$

$$I(z) = I_0^+ e^{-j\beta z} + I_0^- e^{+j\beta z} \quad I(l) = I_0^+ e^{+j\beta l} + I_0^- e^{-j\beta l}$$

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6

Reflection Coefficient, Γ



$$\Gamma_l(l) = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{+j\beta l}} = \frac{V_0^-}{V_0^+} e^{-2j\beta l}$$

$|\Gamma_l|$ is constant along the line

$$\Gamma = \Gamma_l(l=0) = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

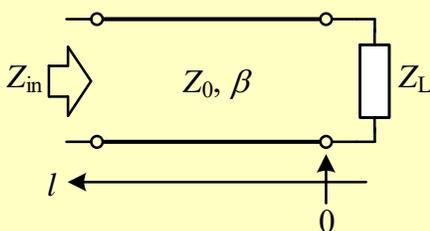
$$V(z) = V_0^+ [e^{-j\beta z} + \Gamma e^{+j\beta z}]$$

$$V(l) = V_0^+ [e^{+j\beta l} + \Gamma e^{-j\beta l}]$$

$$I(z) = \frac{V_0^+}{Z_0} [e^{-j\beta z} - \Gamma e^{+j\beta z}]$$

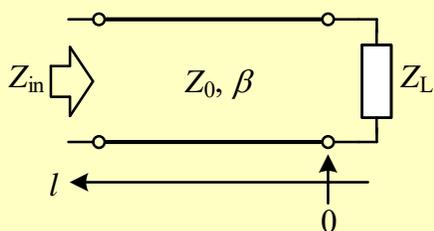
$$I(l) = \frac{V_0^+}{Z_0} [e^{+j\beta l} - \Gamma e^{-j\beta l}]$$

Input Impedance, Z_{in}



$$Z_{in}(l) = \frac{V(l)}{I(l)} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

Input Impedance vs Reflection Coefficient



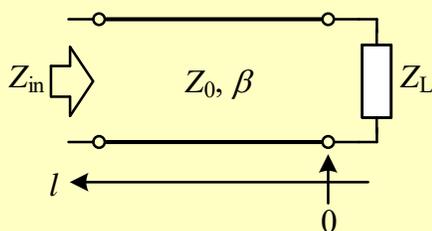
$$V(l) = V_0^+ [e^{+j\beta l} + \Gamma e^{-j\beta l}]$$

$$I(l) = \frac{V_0^+}{Z_0} [e^{+j\beta l} - \Gamma e^{-j\beta l}]$$

$$Z_{in}(l) = \frac{V(l)}{I(l)} = Z_0 \frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} = Z_0 \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}}$$

$$Z_{in}(l) = Z_0 \frac{1 + \Gamma_l}{1 - \Gamma_l} \quad \Gamma_l = \frac{Z_{in}(l) - Z_0}{Z_{in}(l) + Z_0}$$

Input Impedance – Periodicity



$$Z_{in}(l) = \frac{V(l)}{I(l)} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

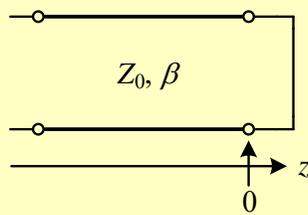
Since $\beta = \frac{2\pi}{\lambda}$

βl is the electrical length

$$Z_{in}(l = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots) = Z_L \quad Z_{in}(l = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots) = \frac{Z_0^2}{Z_L}$$

(Period = $\lambda/2$)

Short-Circuited Lossless TL



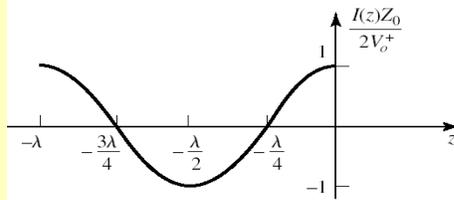
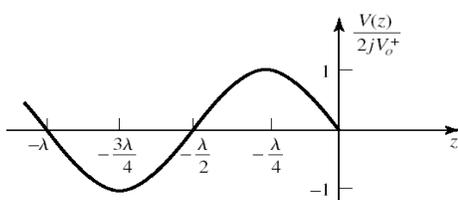
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

$$V(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{+j\beta z}) = V_0^+ (e^{-j\beta z} - e^{+j\beta z})$$

$$I(z) = \frac{V_0^+}{Z_0} (e^{-j\beta z} - \Gamma e^{+j\beta z}) = \frac{V_0^+}{Z_0} (e^{-j\beta z} + e^{+j\beta z})$$

$$V(z) = -2jV_0^+ \sin \beta z$$

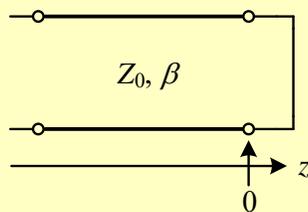
$$I(z) = \frac{2V_0^+}{Z_0} \cos \beta z$$



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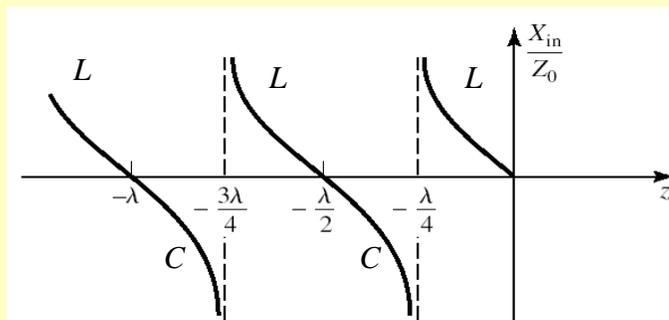
Short-Circuited Lossless TL (cont.)



$$Z_{in}(l) = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

$$Z_{in}(l) = jZ_0 \tan(\beta l)$$

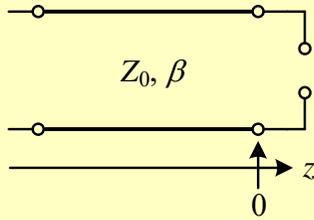
Z_{in} is purely reactive



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Open-Circuited Lossless TL



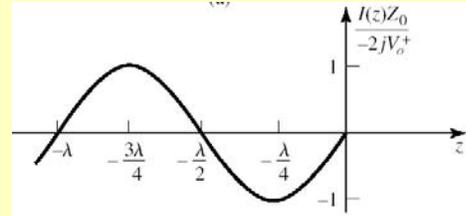
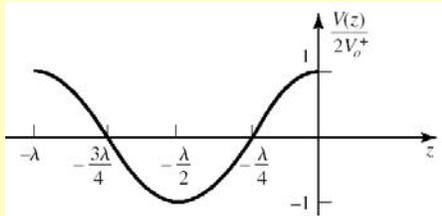
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = +1$$

$$V(z) = V_0^+ (e^{-j\beta z} + e^{+j\beta z})$$

$$I(z) = \frac{V_0^+}{Z_0} (e^{-j\beta z} - e^{+j\beta z})$$

$$V(z) = 2V_0^+ \cos \beta z$$

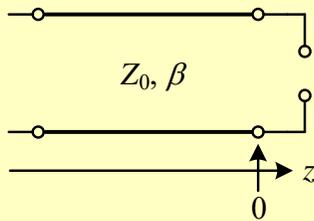
$$I(z) = \frac{j2V_0^+}{Z_0} \sin \beta z$$



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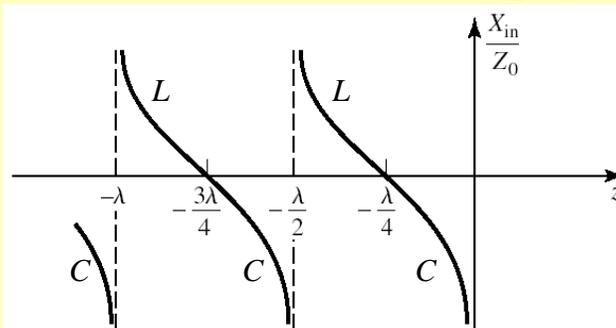
Open-Circuited Lossless TL (cont.)



$$Z_{in}(l) = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

$$Z_{in}(l) = \frac{Z_0}{j \tan(\beta l)} = -jZ_0 \cot(\beta l)$$

Z_{in} is purely reactive



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