

# **Fundamental Transmission Line Theory**

(Part 1)

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## Outline

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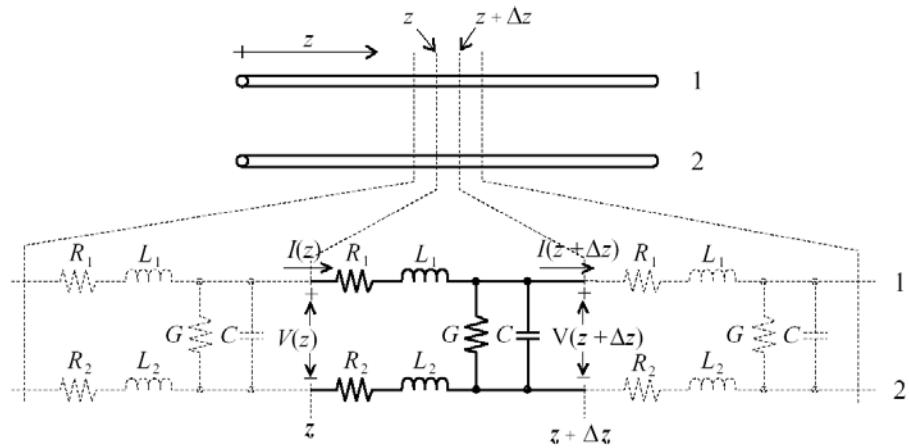
- Distributed equivalent circuits
- Telegrapher equations and wave equation
- Traveling waves
- Characteristic impedance
- Reflection coefficient along the line
- Transmission line symbol
- Input impedance along the line

## Fundamental Transmission Line Theory (Part 1)

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### Transmission Line Model

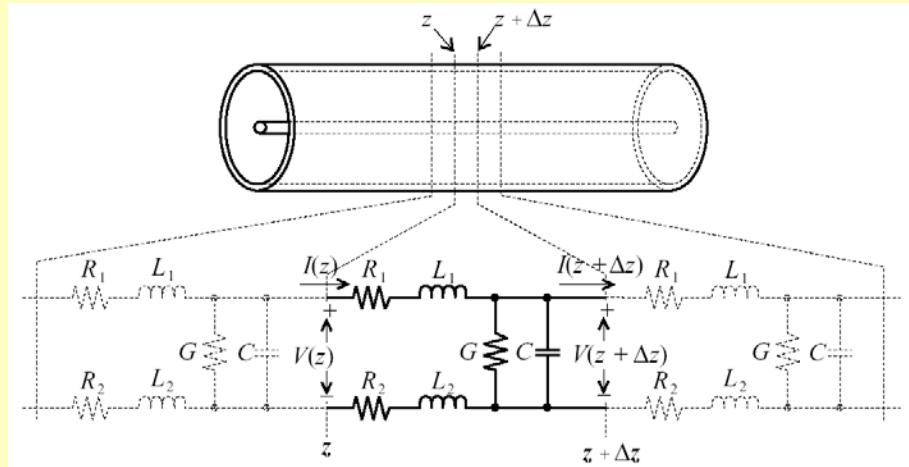


The interconnect is modeled using an infinite number of RLCG sections

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(R. Ludwig and P. Bretschko, RF Circuit Design, Prentice Hall, 2000) 3

### Transmission Line Model

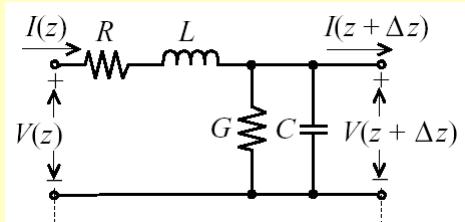


The interconnect is modeled using an infinite number of RLCG sections

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## Transmission Line Model



- Generic equivalent circuit for each section ( $R, L, C$  and  $G$  are per unit length)
- The interconnect is modeled using an infinite number of these sections, making  $\Delta z \rightarrow 0$

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## Transmission Line Equations

- Telegrapher equations in time-domain

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L \frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C \frac{\partial v(z,t)}{\partial t}$$

- Telegrapher equations in frequency-domain

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z)$$

$$\frac{dI(z)}{dz} = -(G + j\omega C)V(z)$$

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## Transmission Line Equations (cont.)

- Wave equation (frequency-domain)

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \quad \frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0$$

where  $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \equiv \alpha + j\beta$

$\gamma$  is the complex propagation constant,  $\alpha$  is the attenuation constant, and  $\beta$  is the propagation constant

- Solutions to the wave equation are

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

## Transmission Line Equations (cont.)

- Solutions in the frequency domain

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \equiv \alpha + j\beta$$

- Solutions in the time domain

$$v(z, t) = |V_0^+| \cos(\omega t - \beta z + \phi^+) e^{-\alpha z} + |V_0^-| \cos(\omega t + \beta z + \phi^-) e^{+\alpha z}$$

- Wavelength

$$\lambda = \frac{v_p}{f} = \frac{2\pi}{\beta}$$

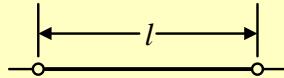
- Phase velocity, wave velocity or propagation speed

$$v_p = \frac{dz}{dt} = \frac{\omega}{\beta}$$

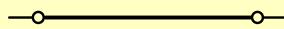
(speed at which a constant phase point travels down the line)

## Transmission Line Symbol

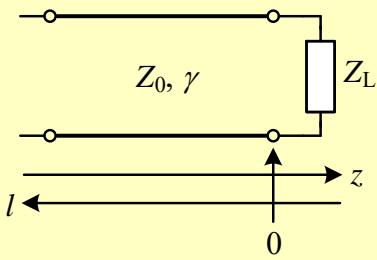
- (Lossy) Transmission line



$Z_0, \gamma$



- Length along the line



## Characteristic Impedance

- Characteristic impedance of the TL

$$Z_0 \equiv \frac{V_0^+}{I_0^+} = \frac{V_0^-}{-I_0^-}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

## Reflection Coefficient

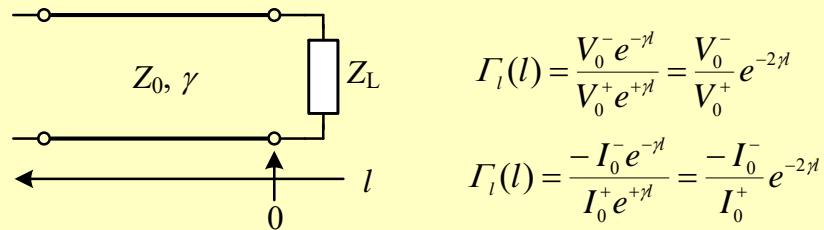
- Reflection coefficient along the line,  $\Gamma_z$

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$\Gamma_z(z) = \frac{V_0^- e^{+\gamma z}}{V_0^+ e^{-\gamma z}} = \frac{V_0^-}{V_0^+} e^{+2\gamma z}$$

## Reflection Coefficient (cont.)

- Reflection coefficient along the line,  $\Gamma_l$

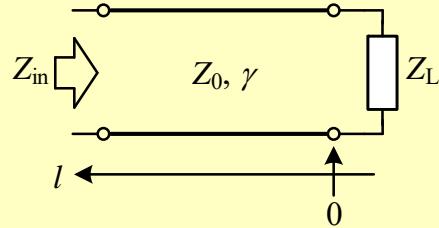


- Reflection coefficient at the load,  $\Gamma$

$$\Gamma = \Gamma_l(l=0) = \frac{V_0^-}{V_0^+} = \frac{-I_0^-}{I_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

## Input Impedance

- Input impedance along the line



$$Z_{in}(l) = \frac{V(l)}{I(l)} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}$$