# High-Frequency Filters

(Part 3)

**Dr. José Ernesto Rayas-Sánchez**

1

## Outline

- Richard's transformation
- Example of a low-pass filter using transmission lines
- Kuroda's identities
- Low-pass filter example implemented with microstrips
- Stepped impedance low-pass filters

Dr. J. E. Rayas-Sánchez

2

## Richard's Transformation

- It maps $\omega$ to $\Omega$, such that we can synthesize inductors and capacitors by using transmission lines

$$\Omega = \tan \beta l = \tan\left(\frac{2\pi}{\lambda} l\right) = \tan\left(\frac{2\pi}{v_p / f} l\right) = \tan\left(\frac{\omega l}{v_p}\right)$$

- The transformation has a period of $\omega l / v_p = \pi$
- Using this new frequency, the reactance of an inductor is

$$jX = j\Omega L = jL \tan(\beta l)$$

and the susceptance of a capacitor is

$$jB = j\Omega C = jC \tan(\beta l)$$



## Richard's Transformation (cont.)

- In the $\Omega$ domain, an inductor can be replaced with a short-circuited transmission line with characteristic impedance $L$ and electrical length $\beta l$:

$$jX = j\Omega L = jL \tan(\beta l) \longrightarrow Z_{\text{in}} = jZ_0 \tan(\beta l)$$

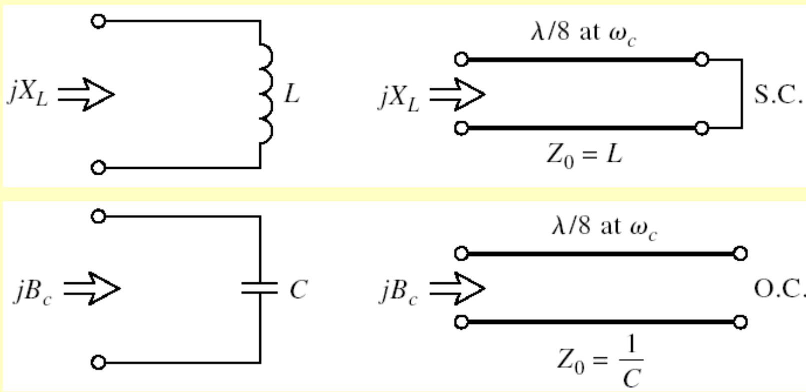
- In the $\Omega$ domain, a capacitor can be replaced with an open-circuited transmission line with characteristic admittance $C$ and electrical length $\beta l$:

$$jB = j\Omega C = jC \tan(\beta l) \longrightarrow Y_{\text{in}} = j(1/Z_0) \tan(\beta l)$$

## Richard's Transformation (cont.)

- Taking $l = \lambda/8$ at $f_c$,

$$\Omega = \tan \beta l = \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{8}\right) = 1 \text{ (normalized frequency)}$$




*(D. M. Pozar, Microwave Engineering, Wiley, 2005)*



## Example: Low-Pass Filter Design

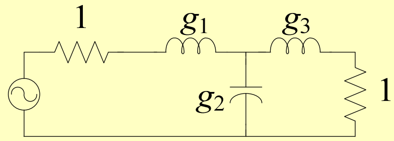
Design a low-pass filter with the following specifications:

- Third-order Chebyshev response with a 3dB ripple level
- Cutoff frequency at 4 GHz
- For a 50-Ω system
- Implemented with transmission lines

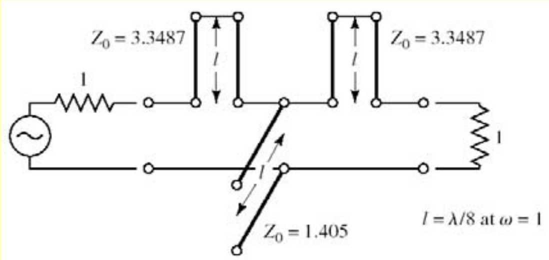
# Example: Low-Pass Filter Design (cont.)

From Tables:

$g_0 = 1$
$g_1 = 3.3487$
$g_2 = 0.7117$
$g_3 = 3.3487$
$g_4 = 1$




Using Richard's transformations:




Dr. J. E. Rayas-Sánchez

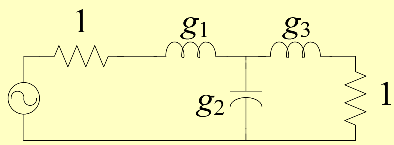
(*D. M. Pozar, Microwave Engineering, Wiley, 2005*)



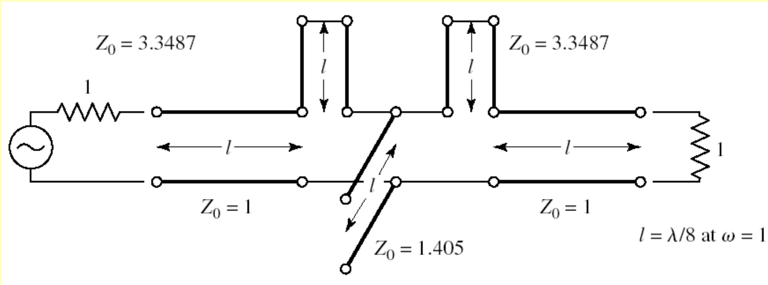
# Example: Low-Pass Filter Design (cont)

From Tables:

$g_0 = 1$
$g_1 = 3.3487$
$g_2 = 0.7117$
$g_3 = 3.3487$
$g_4 = 1$




Adding input-output lines:




Dr. J. E. Rayas-Sánchez

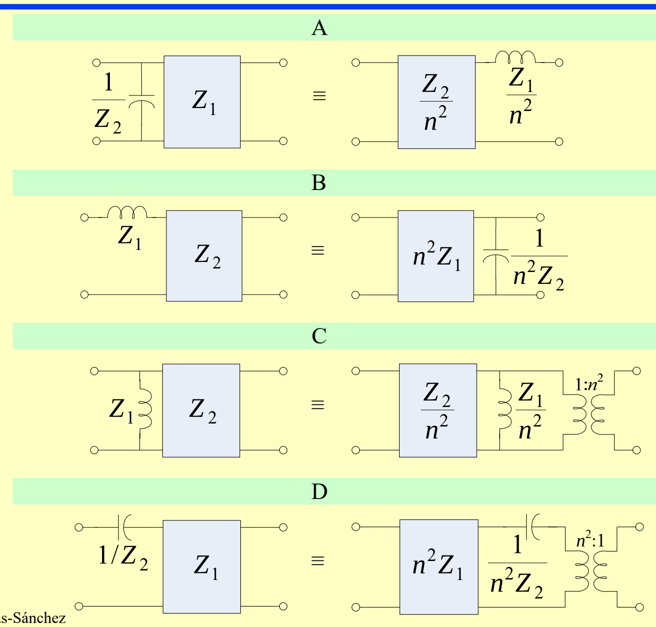
(*D. M. Pozar, Microwave Engineering, Wiley, 2005*)

## Kuroda's Identities

- They are used to generate more practical filter implementations
- They provide 4 possible transformations
- In the circuits of next slide:
  - Each box represents a transmission line with the indicated characteristic impedance and length $\lambda/8$ at $\omega_c$
  - Each capacitor represents a open-circuited shunt stub of length $\lambda/8$ at $\omega_c$
  - Each inductor represents a short-circuited series stub of length $\lambda/8$ at $\omega_c$



## Kuroda's Identities (cont.)

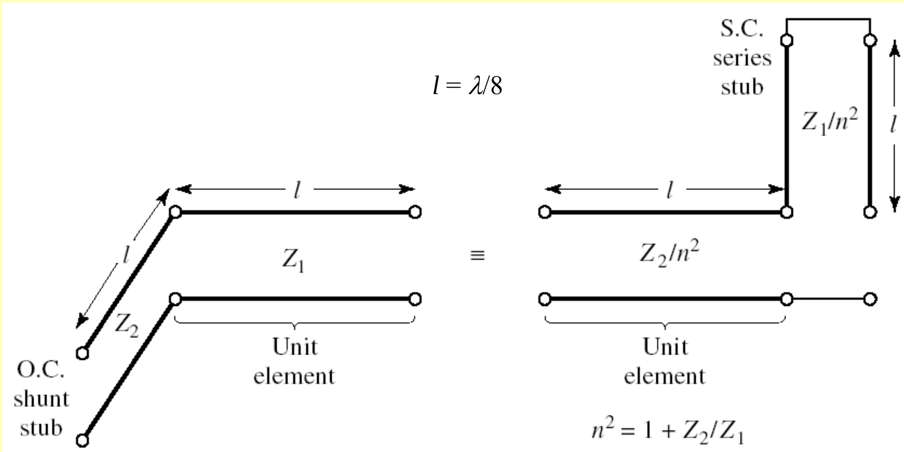



where

$$n^2 = 1 + \frac{Z_2}{Z_1}$$

# Kuroda's Identities (cont.)

Kuroda Identity A

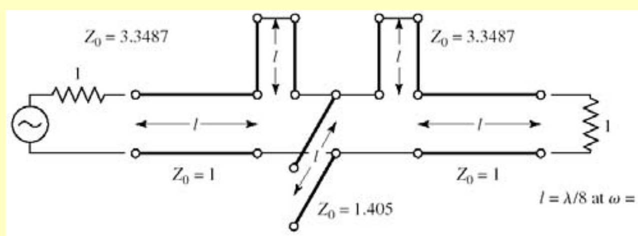




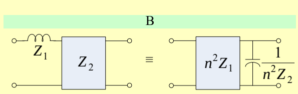
*(D. M. Pozar, Microwave Engineering, Wiley, 2005)*


# Example: Low-Pass Filter Design (cont.)

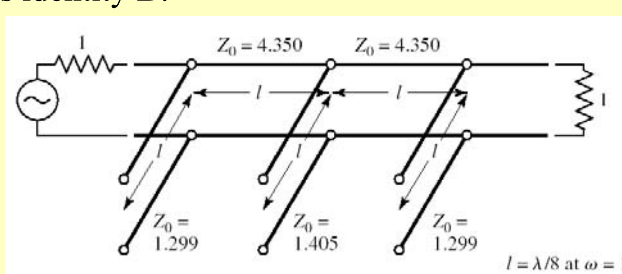
Previous solution:




Applying Kuroda's identity B:



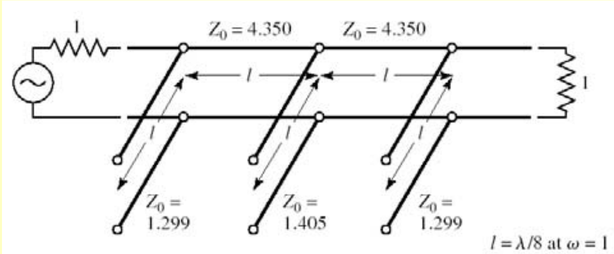

$Z_1 = 3.3487$ ; $Z_2 = 1$
$n^2 = 1+Z_2/Z_1 = 1.299$
$n^2Z_1 = 4.35$
$n^2Z_2 = 1.299$



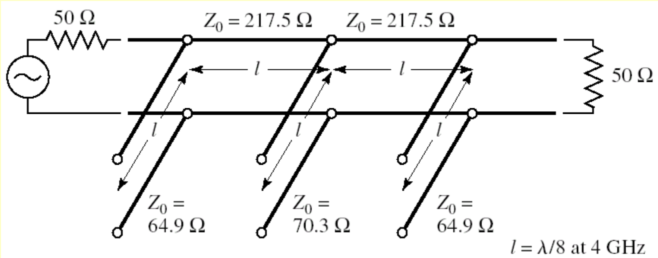


*(D. M. Pozar, Microwave Engineering, Wiley, 2005)*

## Example: Low-Pass Filter Design (cont.)



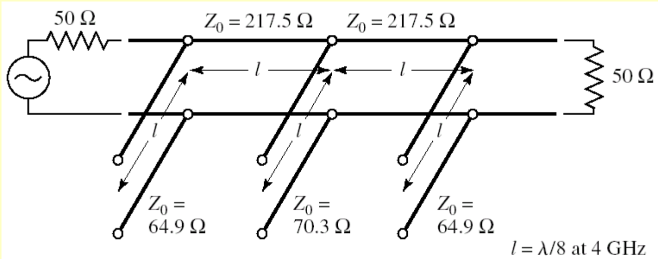

De-scaling the characteristic impedances:



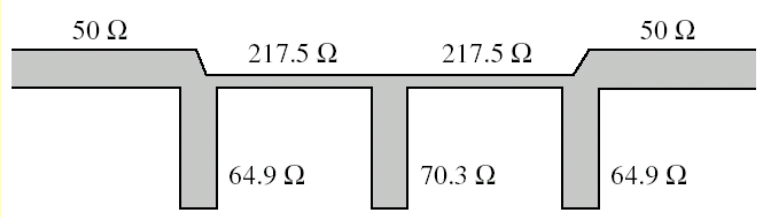


*(D. M. Pozar, Microwave Engineering, Wiley, 2005)*


## Example: Low-Pass Filter Design (cont.)



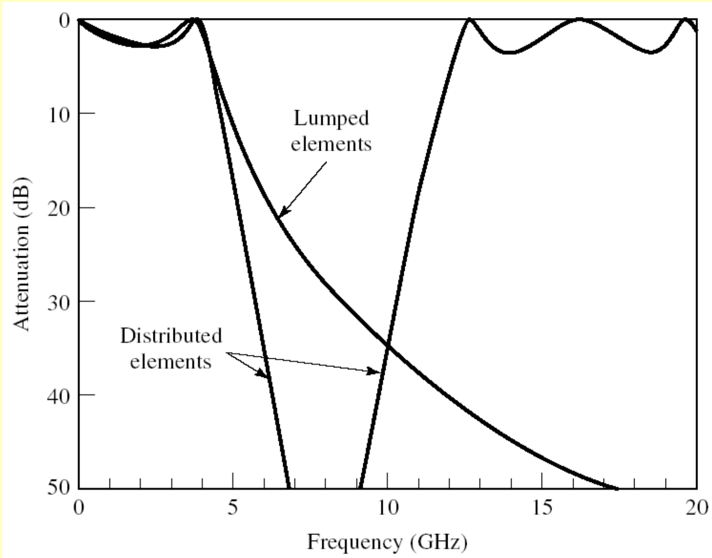

Implementing each transmission line with a microstrip line:





*(D. M. Pozar, Microwave Engineering, Wiley, 2005)*

## Example: Low-Pass Filter Design (cont.)




Dr. J. E. Rayas-Sánchez

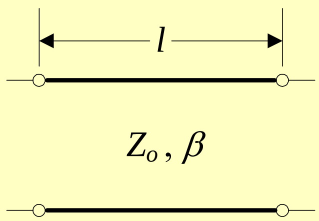
*(D. M. Pozar, Microwave Engineering, Wiley, 2005)* 15

## Stepped-Impedance Low-Pass Filters

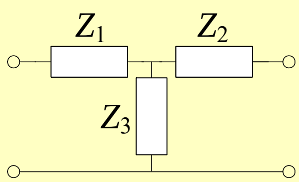
- They alternate short transmission lines with very high and very low characteristic impedances (HZ-LZ filters)
- They are easier to design and require less area than microstrip implementations using Richard's transformation and Kuroda's identities, but have poorer electrical performance
- They are a very good alternative when a sharp cutoff is not required

Dr. J. E. Rayas-Sánchez 16

## From ABCD Parameters…



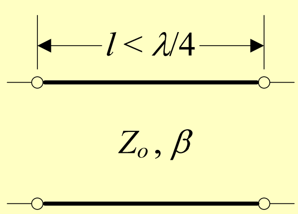

$$A = \cos \beta l \qquad B = jZ_o \sin \beta l$$
$$C = jY_o \sin \beta l \qquad D = \cos \beta l$$

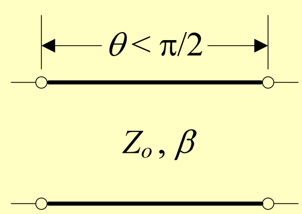



$$A = 1 + \frac{Z_1}{Z_3} \qquad B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$
$$C = \frac{1}{Z_3} \qquad D = 1 + \frac{Z_2}{Z_3}$$

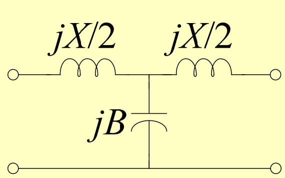


## Approximating Short Transmission Lines



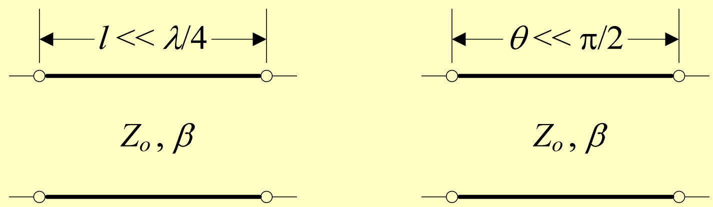

Can be approximated by




where

$$X/2 = Z_o \tan(\beta l / 2)$$
$$B = Y_o \sin(\beta l)$$

## Approximating Short Transmission Lines (cont.)




Can be approximated by

if $Z_o$ is very high, if $Z_o$ is very low,




## Stepped-Impedance LP Filter – Implementation

- Series inductors are replaced by high-impedance transmission lines ($Z_{\text{high}}$)
- Parallel capacitors are replaced by low-impedance transmission lines ($Z_{\text{low}}$)
- Lengths of the transmission line sections are calculated using:

$$\beta l_k = \frac{g_k R_0}{Z_{\text{high}}} \quad (g_k \text{ corresponds to an inductor})$$

$$\beta l_k = \frac{g_k Z_{\text{low}}}{R_0} \quad (g_k \text{ corresponds to a capacitor})$$

## Stepped-Impedance LP Filter – Example

Design a low-pass stepped impedance microstrip filter with the following specifications:

- Maximally flat response
- $IL_{\text{min}}$ = 20 dB at 3.8 GHz
- Cutoff frequency at 2.5 GHz
- For a 50-Ω system
- Highest practical impedance is 150 Ω, and lowest is 10 Ω



*(D. M. Pozar, Microwave Engineering, Wiley, 2005)*

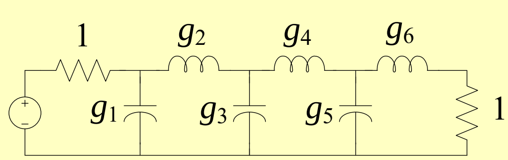


## Stepped-Impedance LP Filter – Example (cont.)

$IL_{\text{min}}$ = 20 dB at 3.8 GHz requires $N = 6$

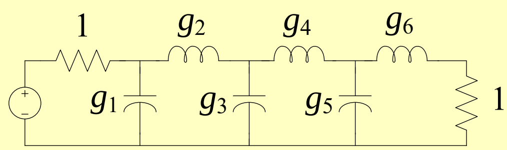
From Tables:

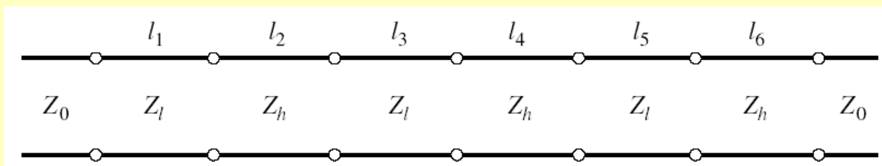
$g_0 = 1$
$g_1 = 0.5176$
$g_2 = 1.4142$
$g_3 = 1.9318$
$g_4 = 1.9318$
$g_5 = 1.4142$
$g_6 = 0.5176$
$g_7 = 1$

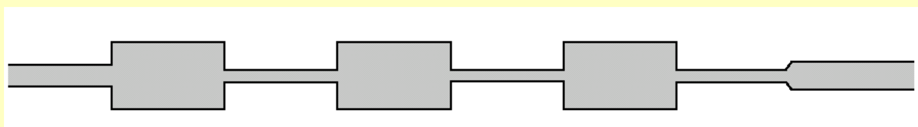


## Stepped-Impedance LP Filter – Example (cont.)

Implementation:



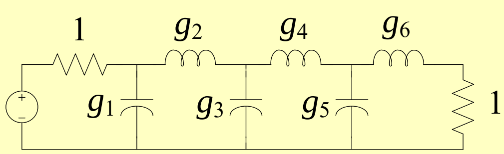


*(D. M. Pozar, Microwave Engineering, Wiley, 2005)*



## Stepped-Impedance LP Filter – Example (cont.)

Calculating transmission line lengths:




$Z_{\text{low}} = 10\ \Omega$
$Z_{\text{high}} = 150\ \Omega$

$g_1 = 0.5176$
$g_2 = 1.4142$
$g_3 = 1.9318$
$g_4 = 1.9318$
$g_5 = 1.4142$
$g_6 = 0.5176$

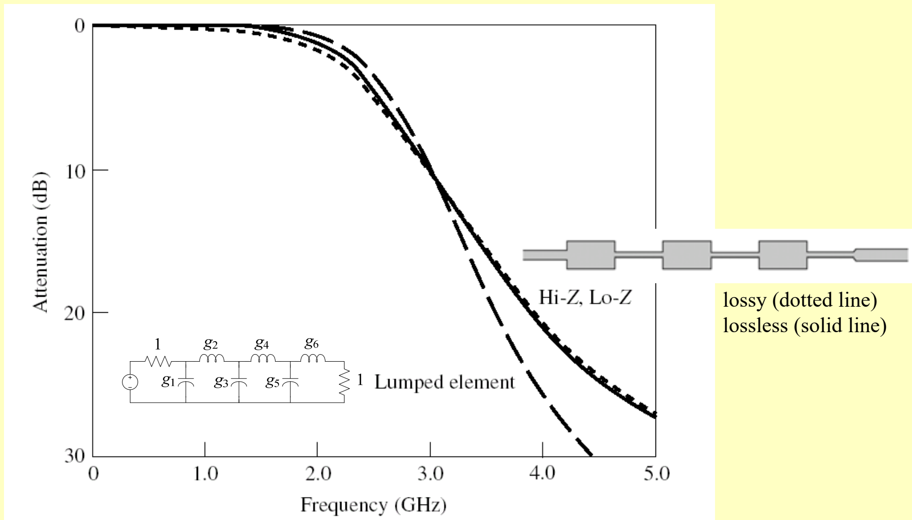
→

$(\beta l)_1 = 5.93°$
$(\beta l)_2 = 27.00°$
$(\beta l)_3 = 22.14°$
$(\beta l)_4 = 36.89°$
$(\beta l)_5 = 16.20°$
$(\beta l)_6 = 9.88°$

$$\beta_k = \frac{2\pi}{\lambda_k}$$

$$\lambda_k = \frac{v_{pk}}{f_c}$$

# Stepped-Impedance LP Filter – Example (cont.)




lossy (dotted line)
lossless (solid line)

Dr. J. E. Rayas-Sánchez

*(D. M. Pozar, Microwave Engineering, Wiley, 2005)* 25