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## FOUR-SECTION 3:1 BUTTERWORTH IMPEDANCE TRANSFORMER

### Problem

Design a four-section Butterworth impedance transformer to match at 5 GHz a load impedance $Z_L = 150$ Ω to a transmission line with characteristic impedance $Z_0 = 50$ Ω. Calculate the theoretical fractional bandwidth for a maximum reflection of –40 dB. Calculate the corresponding lengths of the four sections of the impedance transformer, assuming an effective dielectric constant $\varepsilon_e = 3.38$.

### Solution

We have $N = 4$, $Z_L = 150$ Ω, $Z_0 = 50$ Ω, $\Gamma_m = 0.01 = -40$ dB, and $f_0 = 5$ GHz.

Using Table 5.1 (Binomial Transformer Design, [1]) for $N = 4$ and $T = Z_L/Z_0 = 3$:

$$Z_1/Z_0 = 1.0718$$
$$Z_2/Z_0 = 1.4105$$
$$Z_3/Z_0 = 2.1269$$
$$Z_4/Z_0 = 2.7990$$

Since $Z_0 = 50$ Ω, then,

$$Z_1 = 53.5900\ \Omega$$
$$Z_2 = 70.5250\ \Omega$$
$$Z_3 = 106.3450\ \Omega$$
$$Z_4 = 139.9500\ \Omega$$

Since $N = 4$, $Z_L = 150$ Ω, $Z_0 = 50$ Ω,

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = +0.5 = -6.02\text{ dB},\ A = 2^{-N-1}\left|\ln(T)\right| = 0.0343.$$

The expected relative bandwidth for a maximum reflection $\Gamma_m = 0.01 = -40$ dB is

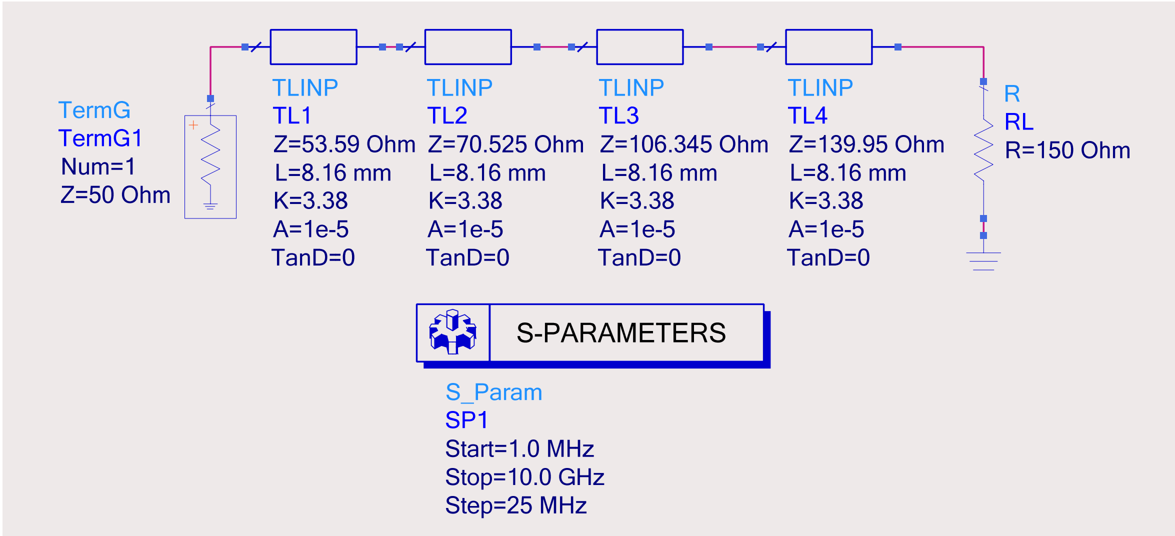
$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_m}{A}\right)^{1/N}\right] = 47.89\%$$

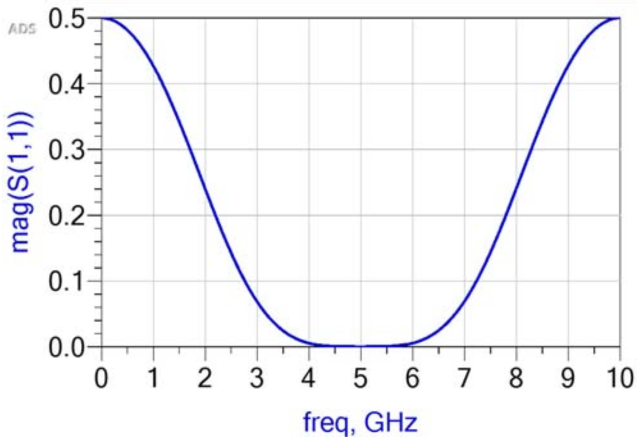
Centering the impedance transformer at $f_0 = 5$ GHz, then the expected bandwidth is $\Delta f = 2.39$ GHz.

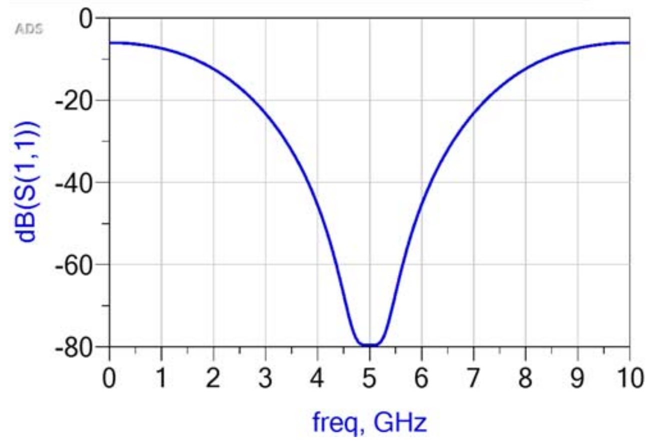
Since $\varepsilon_e = 3.38$,

$$\lambda_0 = \frac{v_p}{f_0} = \frac{c}{f_0\sqrt{\varepsilon_e}} = 3.26\text{ cm};\ \ l = \frac{\lambda_0}{4} = 8.1589\text{ mm}$$

**Implementation in ADS with Ideal Transmission Lines**

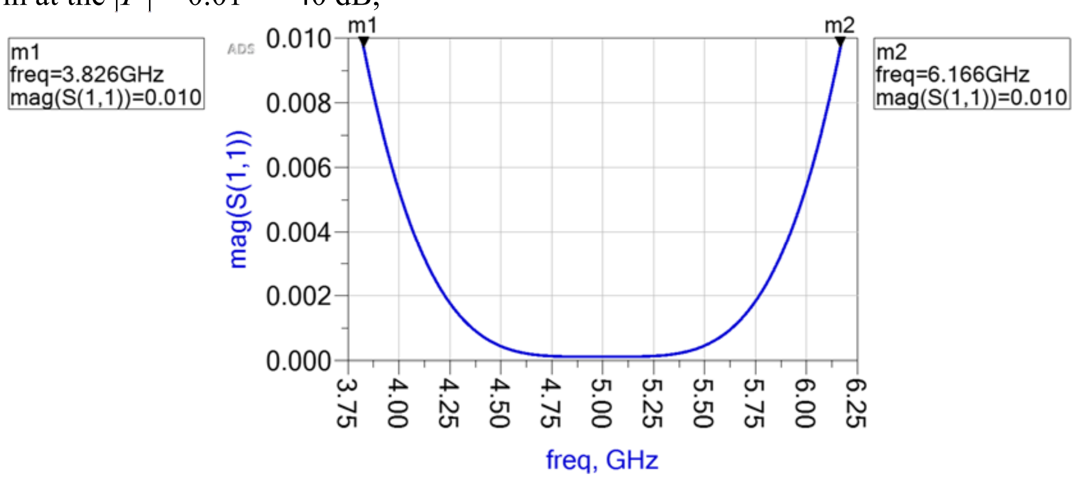



It is confirmed a Butterworth response centered a 5 GHz, with $|\Gamma| = 0.5$ at low frequencies.

Zooming-in at the $|\Gamma| = 0.01 = -40$ dB,




It is seen that the actual bandwidth $\Delta f = (6.166 - 3.826)$ GHz = 2.34 GHz at $\Gamma_m = 0.01$ is very close to the expected bandwidth (2.39 GHz).

[1] D. M. Pozar, *Microwave Engineering*. Amherst, MA: Wiley, 1998.