## IMPEDANCE MATCHING CIRCUITS

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## Problems

1. Using an L-section, find the two possible solutions to match a $75-\Omega$ transmission line to a load impedance $Z_{L}=(10+j 30) \Omega$.
2. Using an L-section, find the two possible solutions to match a $50-\Omega$ line at 100 MHz to a load impedance $Z_{\mathrm{L}}$ consisting of a $100-\Omega$ resistor in parallel with a $10-\mathrm{pF}$ capacitor.
3. Using an L-section, find the two possible solutions to match a $50-\Omega$ line at 380 MHz to a load impedance $Z_{\mathrm{L}}$ consisting of a $35-\Omega$ resistor in series with a $6-\mathrm{nH}$ inductor. Implement in Keysight ADS both solutions and confirm the correct performance of the matching circuits by plotting $\left|S_{11}\right|$ from 10 MHz to 800 MHz
4. The transmitter illustrated below is operating at 1 GHz on a $75-\Omega$ system and has an output impedance $Z_{\mathrm{S}}=(90+j 30) \Omega$. The transmitter must be directly connected to an antenna whose input impedance is equivalent to the series combination of a resistor $R_{\mathrm{L}}=75 \Omega$ and an inductor $L_{\mathrm{L}}=3 \mathrm{nH}$. Using the L-Section matching network indicated below, find the values of $L_{X}$ and $C_{B}$ such that maximum power is delivered to the antenna.

5. A load impedance $Z_{\mathrm{L}}$ consisting of a $100-\Omega$ resistor in parallel with a $1-\mathrm{pF}$ capacitor is to be match at 5 GHz to a transmission line whose characteristic impedance is $Z_{0}=50 \Omega$. Using a single-stub series tuning network as illustrated below: a) find the first two distances $d_{1}$ and $d_{2}$ from the load where the stub could be connected; b) connecting the stub at the minimum distance from the load, find the length $l_{\mathrm{oc}}$ for an open-circuited stub, and the length $l_{\mathrm{sc}}$ for a short-circuited stub; c) assuming transmission lines on air (or free-space), implement in Keysight ADS the solution for $d$ minimum and $l_{\text {oc }}$.

6. Repeat problem 5 using a single-stub shunt tuning network.
7. A load impedance at 5 GHz is $Z_{\mathrm{L}}=(30+j 90) \Omega$. A single-stub shunt tuning network, as illustrated below, is used to match $Z_{\mathrm{L}}$ to $Z_{0}=50 \Omega$. a) Find the first two distances $d_{1}$ and $d_{2}$ from the load where the stub could be connected; b) Connecting the stub at the minimum distance from the load, find the length $l_{\text {oc }}$ for an open-circuited stub, and the length $l_{\mathrm{sc}}$ for a short-circuited stub.

8. Design a 3-section binomial impedance transformer to match at 1 GHz a load impedance $\mathrm{Z}_{\mathrm{L}}=100 \Omega$ to a transmission line with characteristic impedance $Z_{0}=50 \Omega$. Calculate the theoretical bandwidth $\Delta f$ at $\Gamma_{\mathrm{m}}=-20 \mathrm{~dB}$. Calculate the corresponding lengths of the three sections assuming ideal transmission lines on air. Implement your design in Keysight ADS and confirm the theoretical results.
9. Design a 5-section Butterworth impedance transformer to match a load impedance $Z_{\mathrm{L}}=200 \Omega$ to a transmission line whose characteristic impedance is $Z_{0}=50 \Omega$. Calculate the theoretical fractional bandwidth for a maximum reflection of -60 dB . Calculate the corresponding lengths of the five sections to achieve a perfect match at 15 GHz , assuming ideal transmission lines with effective relative dielectric constant $\varepsilon_{\mathrm{e}}=3.2$.
10. Implement in microstrip technology the impedance transformer of Problem 8. Assume that the substrate height is $H=2 \mathrm{~mm}$, with a relative dielectric constant $\varepsilon_{\mathrm{r}}=4$. For the dielectric losses, use a loss tangent $\tan (\delta)=0.01$. For the metallic losses, assume that all metal traces (including the ground plane) are made of copper with conductivity $\sigma=5.8 \times 10^{7} \mathrm{~S} / \mathrm{m}$ and thickness $t=0.6 \mathrm{mil}=15.24 \mu \mathrm{~m}$ (half-once copper). Simulate the corresponding circuit in Keysight ADS.

## Solutions

1. 



2.


Solution B:
$X=-55.4951 \Omega$
$B=-0.033993 \mathrm{~S}$
Solution B:


Solution B:

3.

Solution A:




Solution B:




4. $L_{X}=3.89 \mathrm{nH} ; C_{B}=1.45 \mathrm{pF}$.
5. $d_{1}=0.279 \lambda, d_{2}=0.392 \lambda$. Using $d=0.279 \lambda$, then $l_{\mathrm{oc}}=0.0645 \lambda$ and $l_{\mathrm{sc}}=0.3145 \lambda$. Since $\varepsilon_{\mathrm{e}}=1$ (air or free-space) and $f=5 \mathrm{GHz}, \lambda=60 \mathrm{~mm}$. Hence $d=16.74 \mathrm{~mm}$ and $l_{\mathrm{oc}}=3.87 \mathrm{~mm}$. ADS simulation:


6. $d_{1}=0.0288 \lambda, d_{2}=0.1416 \lambda$. Using $d=0.0288 \lambda$, then $l_{\mathrm{oc}}=0.3145 \lambda$ and $l_{\mathrm{sc}}=0.0645 \lambda$. Since $\varepsilon_{\mathrm{e}}=1$ and $f=5 \mathrm{GHz}, \lambda=60 \mathrm{~mm}$. Hence $d=1.728 \mathrm{~mm}$ and $l_{\mathrm{oc}}=18.87 \mathrm{~mm}$. ADS simulation:


7. $d_{1}=0.2696 \lambda, d_{2}=0.3808 \lambda$. Using $d=0.2696 \lambda$, then $l_{\mathrm{oc}}=0.3133 \lambda$ and $l_{\mathrm{sc}}=0.0633 \lambda$.
8. $Z_{1}=54.535 \Omega, Z_{2}=70.71 \Omega, Z_{3}=91.685 \Omega, l_{1}=l_{2}=l_{3}=75 \mathrm{~mm}$. The theoretical bandwidth is $\Delta f=$ 919.13 MHz. Implementation in Keysight ADS with ideal transmission lines:



It is confirmed a Butterworth response centered at 1 GHz , with $|\Gamma|=0.333=-9.54 \mathrm{~dB}$ at low frequencies.

Zooming-in to measure the bandwidth at $\Gamma_{\mathrm{m}}=0.1=-20 \mathrm{~dB}$, and simulating up to 6 GHz :



It is seen that $\Delta f=(1,456-542) \mathrm{MHz}=914 \mathrm{MHz}$ at $\Gamma_{m}=0.1$, which is very close to the theoretical prediction $(919.13 \mathrm{MHz})$. It is also confirmed a periodic impedance matching at $1 \mathrm{GHz}, 3 \mathrm{GHz}, 5$ GHz , etc.
9. $Z_{1}=52.26 \Omega, Z_{2}=64.975 \Omega, Z_{3}=100 \Omega, Z_{4}=153.905 \Omega, Z_{5}=191.35 \Omega$. The expected relative bandwidth for maximum reflection $\Gamma_{m}=0.001=-60 \mathrm{~dB}$ is $34.85 \%$. Centering the impedance transformer at $f_{0}=15 \mathrm{GHz}$, the expected theoretical bandwidth is $\Delta f=5.227 \mathrm{GHz}$. The five TL lengths are $l_{1}=\ldots=l_{5}=2.7951 \mathrm{~mm}$. Implementation in Keysight ADS with ideal transmission lines:



It is confirmed a Butterworth response centered at 15 GHz , with $|\Gamma|=0.6=-4.44 \mathrm{~dB}$ at low frequencies.
Zooming-in at $|\Gamma|=-60 \mathrm{~dB}=0.001$ :


It is seen that the actual bandwidth $\Delta f$ is equal to $(17.56-12.42) \mathrm{GHz}=5.14 \mathrm{GHz}$ at $\Gamma \mathrm{m}$, which is close to the theoretical expected bandwidth (5.227 GHz).
10. Using Gupta's formulas:
$W_{1} / H=1.767$ for $Z_{0}=54.535 \Omega$ and $\mathcal{E}_{\mathrm{r}}=4$
$W_{2} / H=1.0829$ for $Z_{0}=70.71 \Omega$ and $\varepsilon_{\mathrm{r}}=4$
$W_{3} / H=0.61071$ for $Z_{0}=91.685 \Omega$ and $\varepsilon_{\mathrm{r}}=4$
Since $H=2 \mathrm{~mm}$, then $W_{1}=3.534 \mathrm{~mm}, W_{2}=2.1659 \mathrm{~mm}, W_{3}=1.2214 \mathrm{~mm}$.
The effective dielectric constant for each segment of microstrip line is:
$\varepsilon_{\mathrm{e} 1}=3.0813$ for $W_{1} / H=1.767$ and $\varepsilon_{\mathrm{r}}=4$
$\varepsilon_{\mathrm{e} 2}=2.9689$ for $W_{2} / H=1.0829$ and $\varepsilon_{\mathrm{r}}=4$
$\varepsilon_{\mathrm{e} 3}=2.8599$ for $W_{3} / H=0.61071$ and $\varepsilon_{\mathrm{r}}=4$
The corresponding physical lengths are ( $f_{0}=1 \mathrm{GHz}$ ):
$l_{1}=\lambda_{1} / 4=42.7264 \mathrm{~mm}, l_{2}=\lambda_{2} / 4=43.5276 \mathrm{~mm}, l_{3}=\lambda_{3} / 4=44.3495 \mathrm{~mm}$
Implementing the circuit in Keysight ADS:


It is seen that the impedance matching is centered at 1 GHz , with $|\Gamma|=0.333=-9.54 \mathrm{~dB}$ at low frequencies, as expected. However, the response is strictly no longer a Butterworth response (maximally flat).

Zooming-in to measure the bandwidth at $\Gamma_{\mathrm{m}}=0.1=-20 \mathrm{~dB}$, and simulating up to 6 GHz :



It is seen that $\Delta f=(1,470-535.1) \mathrm{MHz}=934.9 \mathrm{MHz}$ at $\Gamma_{m}=0.1$, which is still close to the theoretical prediction ( 919.13 MHz ). It is also seen that the periodic impedance matching at $1 \mathrm{GHz}, 3 \mathrm{GHz}, 5$ GHz , etc. is affected by the microstrip losses.

