FREQUENCY-DOMAIN ANALYSIS OF T. L. CIRCUITS

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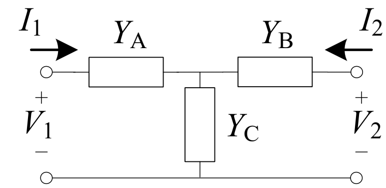
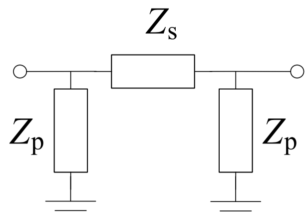
March 2, 2020

PROBLEMS

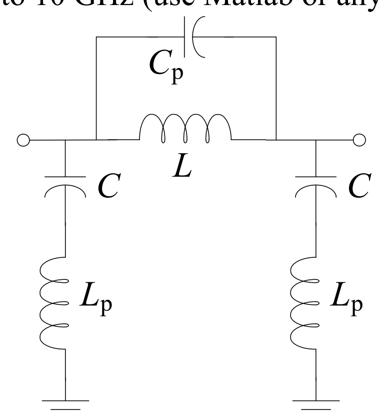
1. For the following lumped circuit, derive formulas to calculate the corresponding Y-parameters.



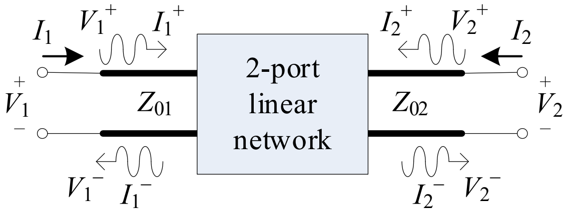
2. For the following lumped circuit, derive formulas to calculate the corresponding S-parameters. Assume that the S-parameters are measured with respect to a reference impedance Z_0.



3. The following π-section low-pass filter uses L = 1.62 nH, C = 0.9124 pF, C_p = 0.05 pF and L_p = 0.07 nH. Using the formulas derived in the previous problem (taking Z_0 = 50Ω), plot the magnitude and phase of S_{11} and S_{21} from 10 Hz to 10 GHz (use Matlab or any other similar software).



4. The two-port network shown below is driven at both ports such that the port voltages and currents have the following values:



$$V_1 = 1.5^{\angle 0^\circ} \text{ V}$$

$$V_2 = 0.5^{\angle -90^\circ} \text{ V}$$

$$I_1 = 3^{\angle 90^\circ} \text{ mA}$$

$$I_2 = 2^{\angle 0^\circ} \text{ mA}$$

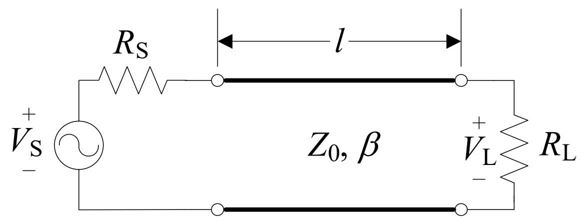
Calculate the input impedance at each port (Z_{in1} and Z_{in2}), and the incident and reflected voltage waves at each port (V_1^+, V_1^-, V_2^+, and V_2^-), measured with respect to a reference impedance of Z_{01} = Z_{02} = 50 Ω.

5. A four-port network has the S parameters shown below (measured at a given operating frequency).

$$\boldsymbol{S} = \begin{bmatrix} 0.15^{\angle 80^\circ} & 0.9^{\angle 45^\circ} & 0.2^{\angle -45^\circ} & 0 \\ 0.9^{\angle 45^\circ} & 0 & 0 & 0.35^{\angle 60^\circ} \\ 0.2^{\angle -45^\circ} & 0 & 0 & 0.75^{\angle -35^\circ} \\ 0 & 0.35^{\angle 60^\circ} & 0.75^{\angle -35^\circ} & 0 \end{bmatrix}$$

a) Is the network reciprocal?; b) Is the network lossless?; c) What is the return loss at port 1 when all other ports are terminated with matched loads?; d) What is the insertion loss and phase delay (in degrees) between port 2 and 4, when all other ports are terminated with matched loads?; e) What is the return loss at port 1 if port 3 is terminated with a short circuit and all other ports are terminated with matched loads?

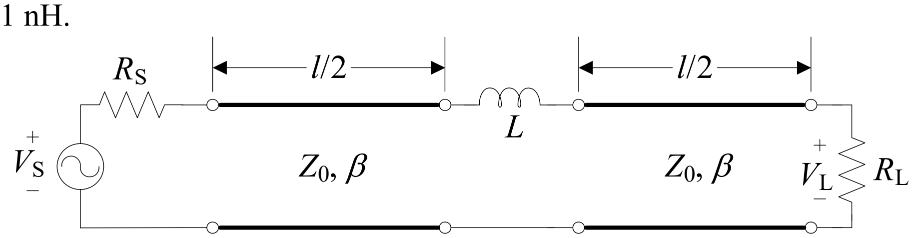
6. Using ABCD parameters, calculate the voltage at the load of the following circuit when the frequency of the input signal is 1 GHz. Assume $V_S = 1.5\text{ V}^{\angle 0^\circ}$, $R_S = 25\ \Omega$, $R_L = 75\ \Omega$, $Z_0 = 50\ \Omega$, $\varepsilon_e = 4$, and $l = 2.5$ cm.



7. Using again ABCD parameters in problem 5, and taking $V_S = 1.5\text{ V}^{\angle 0^\circ}$, $Z_0 = 50\ \Omega$, $\varepsilon_e = 4$, and $l = 2.5$ cm, plot the magnitude and phase of the voltage at the load from 100 MHz to 10 GHz when: a) $R_S = 25\ \Omega$ and $R_L = 75\ \Omega$; b) $R_S = 50\ \Omega$ and $R_L = 50\ \Omega$. Use Matlab or any other similar software.

8. Using again ABCD parameters in problem 5, and taking $V_S = 1.5\text{ V}^{\angle 0^\circ}$, $Z_0 = 50\ \Omega$, $\varepsilon_e = 4$, and $f = 1$ GHz, plot the magnitude and phase of the voltage at the load from $l = 0.1$ cm to $l = 15$ cm when: a) $R_S = 25\ \Omega$ and $R_L = 75\ \Omega$; b) $R_S = 50\ \Omega$ and $R_L = 50\ \Omega$. Use Matlab or any other similar software.

9. Using ABCD parameters, calculate the voltage at the load of the following circuit when the frequency of the input signal is 1 GHz. Assume $V_S = 1.5\text{ V}^{\angle 0^\circ}$, $R_S = 25\ \Omega$, $R_L = 75\ \Omega$, $Z_0 = 50\ \Omega$, $\varepsilon_e = 4$, $l = 2.5$ cm, and $L = 1$ nH.

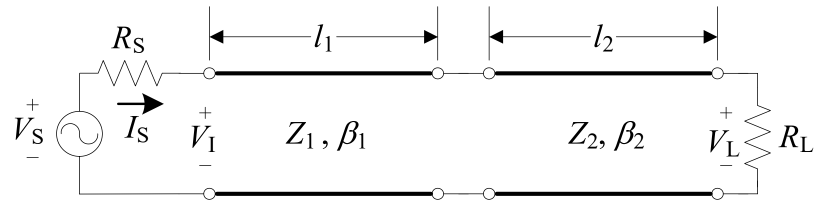


10. Using again ABCD parameters in problem 8, and taking $V_S = 1.5\text{ V}^{\angle 0^\circ}$, $Z_0 = 50\ \Omega$, $\varepsilon_e = 4$, $l = 2.5$ cm, and $L = 1$ nH, plot the magnitude and phase of the voltage at the load from 100 MHz to 10 GHz when: a) $R_S = 25\ \Omega$, $R_L = 75\ \Omega$; b) $R_S = 50\ \Omega$, $R_L = 50\ \Omega$. Use Matlab or any other similar software.

11. Using again ABCD parameters in problem 8, and taking $V_S = 1.5\text{ V}^{\angle 0^\circ}$, $Z_0 = 50\ \Omega$, $\varepsilon_e = 4$, $L = 1$ nH, and $f = 1$ GHz, plot the magnitude and phase of the voltage at the load from $l = 0.1$ cm to $l = 30$ cm when: a) $R_S = 25\Omega$ and $R_L = 75\ \Omega$; b) $R_S = 50\ \Omega$ and $R_L = 50\ \Omega$. Use Matlab or any other similar software.

12. A load resistance $R_L = 80\ \Omega$ must be connected to a transmission line whose characteristic impedance is $Z_1 = 50\ \Omega$, physical length $l_1 = 15$cm, and effective dielectric constant $\varepsilon_e = 3.6$. To avoid reflections at the operating frequency $f = 3$GHz, a quarter-wave impedance transformer with the same ε_e is

inserted between the transmission line and the load (see circuit below). Assuming $V_S = 1\ \text{V}\angle 0°$ and $R_S = 50\ \Omega$: a) find the required characteristic impedance Z_2 and physical length l_2 of the quarter-wave transformer to achieve a perfect match at 3 GHz, b) Using ABCD parameters and Matlab, plot the magnitude and phase of the source current I_S before and after inserting the quarter-wave transformer, from 60 MHz to 6 GHz.

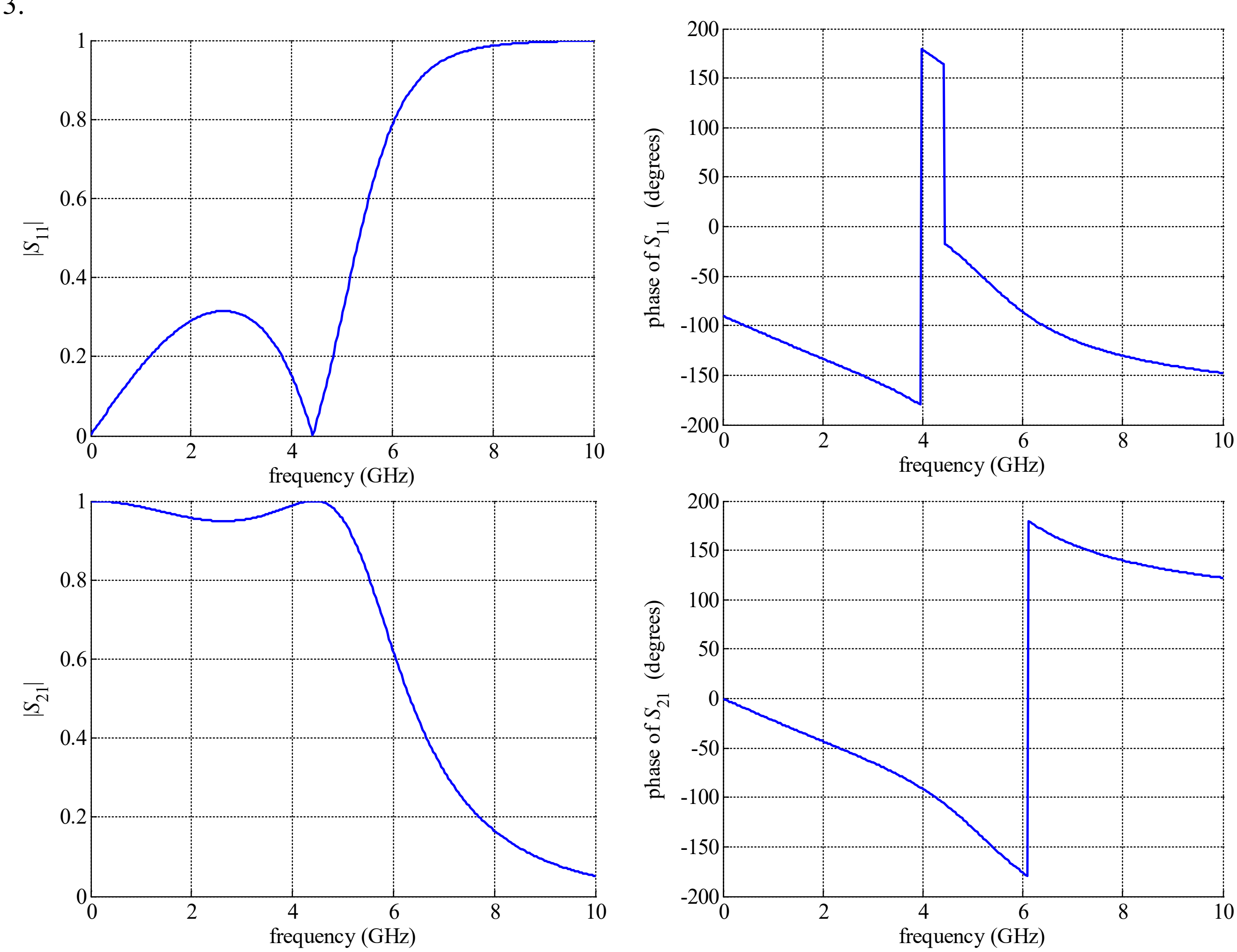


SOLUTIONS

1. $Y_{11} = \dfrac{Y_A(Y_B + Y_C)}{Y_A + Y_B + Y_C}$, $Y_{22} = \dfrac{Y_B(Y_A + Y_C)}{Y_A + Y_B + Y_C}$, $Y_{21} = Y_{12} = \dfrac{-Y_A Y_B}{Y_A + Y_B + Y_C}$.

2. $S_{11} = S_{22} = \dfrac{Z_{in} - Z_0}{Z_{in} + Z_0}$, where $Z_{in} = Z_p \,\|\, [Z_s + (Z_p \,\|\, Z_0)]$, and $S_{21} = S_{12} = (1 + S_{11})\dfrac{(Z_p \,\|\, Z_0)}{Z_s + (Z_p \,\|\, Z_0)}$.

3.

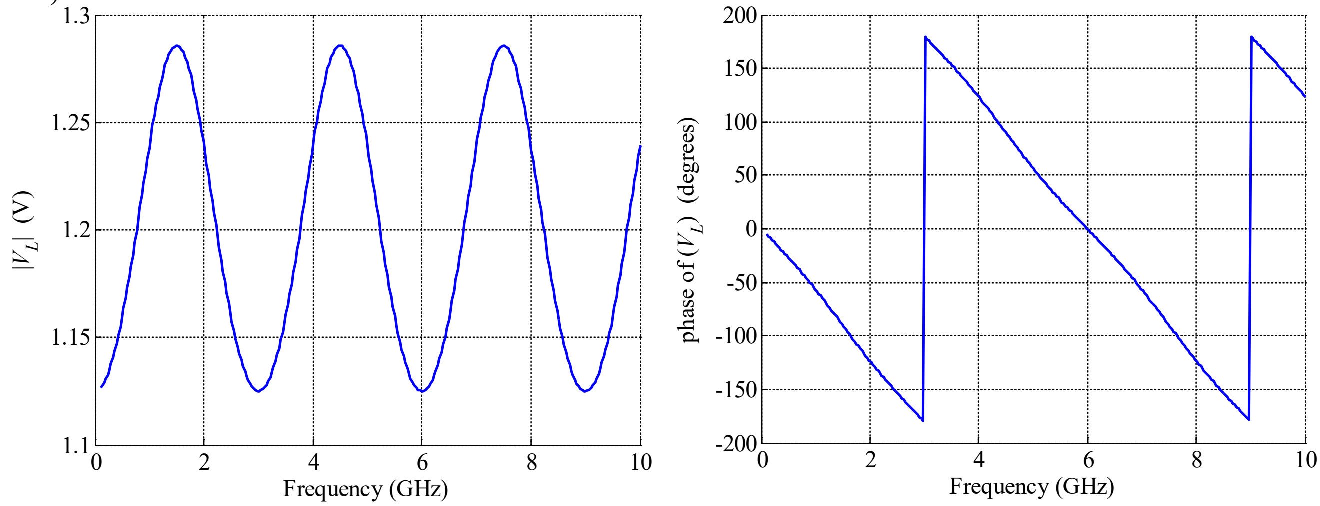


4. $Z_{in1} = 500\ \Omega\ ^{\angle -90°}$; $Z_{in2} = 250\ \Omega\ ^{\angle -90°}$; $V_1^+ = 0.7537\ \text{V}\ ^{\angle 5.71°}$; $V_1^- = 0.7537\ \text{V}\ ^{\angle -5.71°}$; $V_2^+ = 0.255\ \text{V}\ ^{\angle -78.7°}$; $V_2^- = 0.255\ \text{V}\ ^{\angle -101.31°}$;

5. a) Since matrix $\boldsymbol{S}$ is symmetrical, then the network is reciprocal; b) Since $\boldsymbol{S}_1^T \boldsymbol{S}_1^* = 0.8725 \neq 1$, then the network is lossy; c) $RL = 16.48$ dB; d) $IL = 9.12$ dB, $\theta_{24} = 60°$; e) $RL = 14.45$dB.

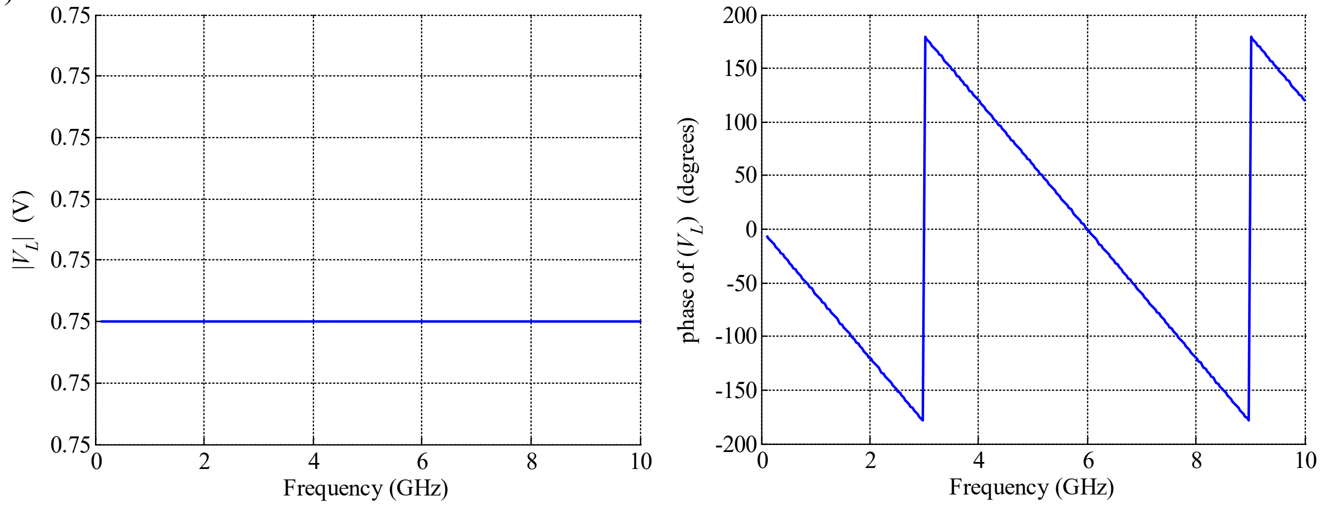
6. $V_L = 1.2392\ \text{V}\ ^{\angle -56.58°}$.

7. a)



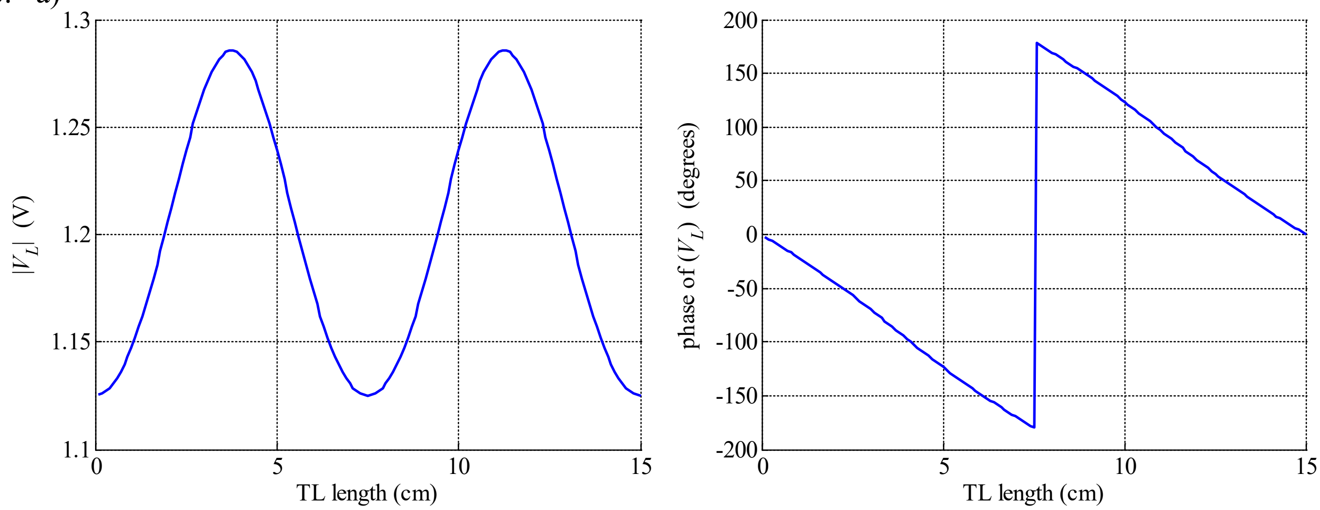
|V_L| (V)
Frequency (GHz)
phase of (V_L) (degrees)
Frequency (GHz)

b)



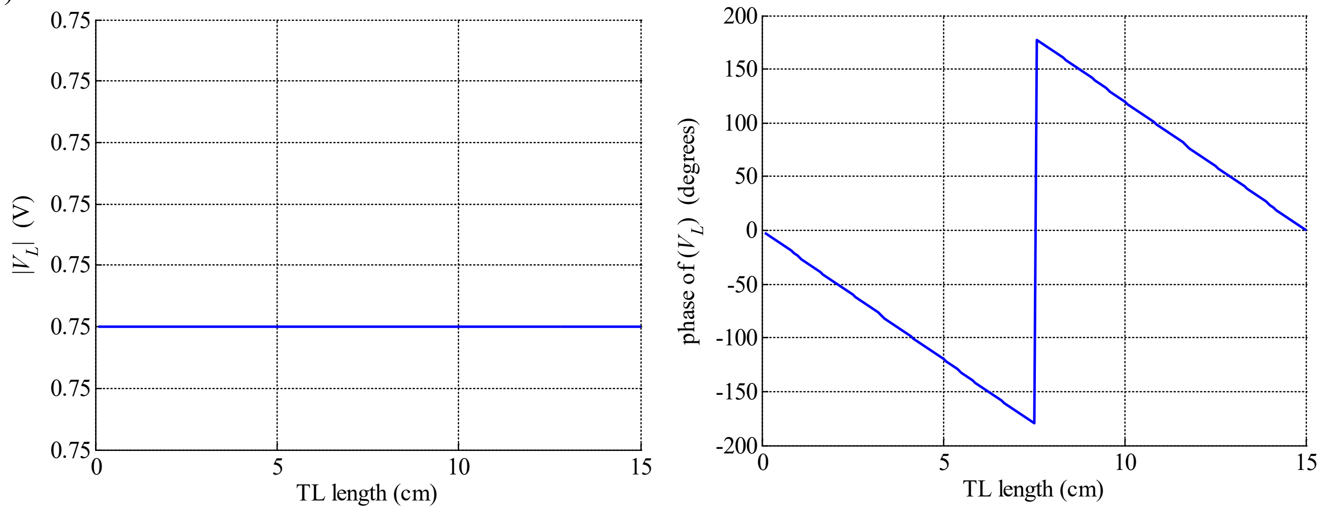
|V_L| (V)
Frequency (GHz)
phase of (V_L) (degrees)
Frequency (GHz)

8. a)



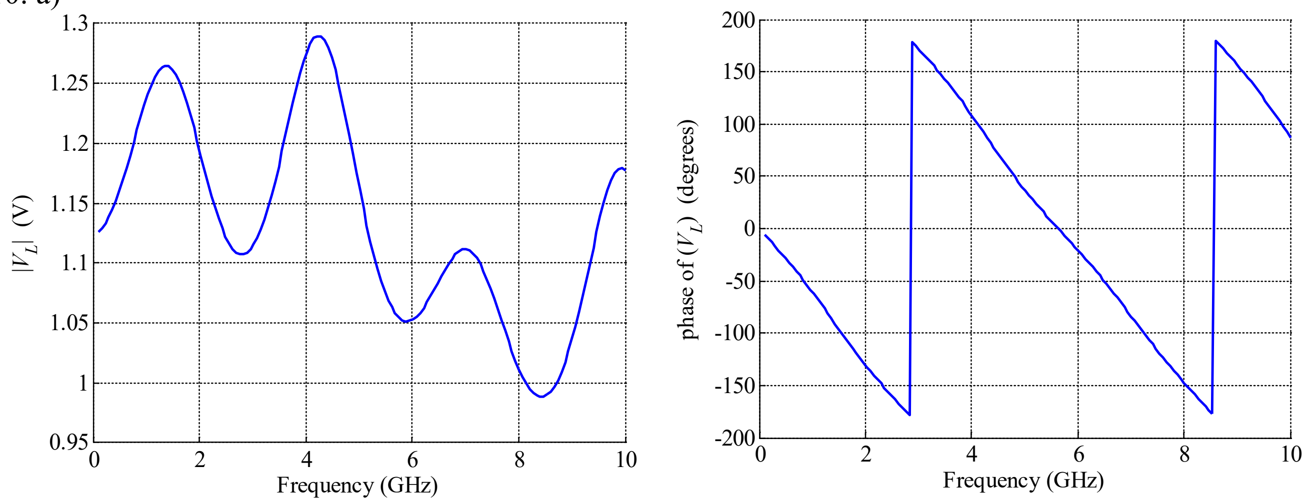
|V_L| (V)
TL length (cm)
phase of (V_L) (degrees)
TL length (cm)

b)

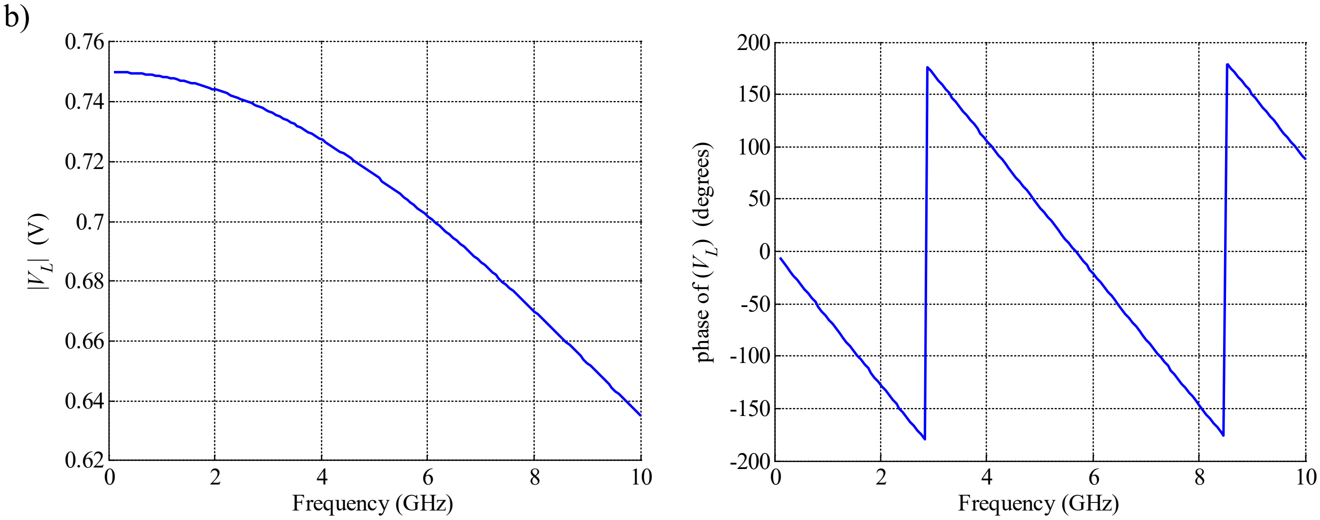


9. $V_L = 1.2367\ \mathrm{V}^{\angle -60.67°}$.

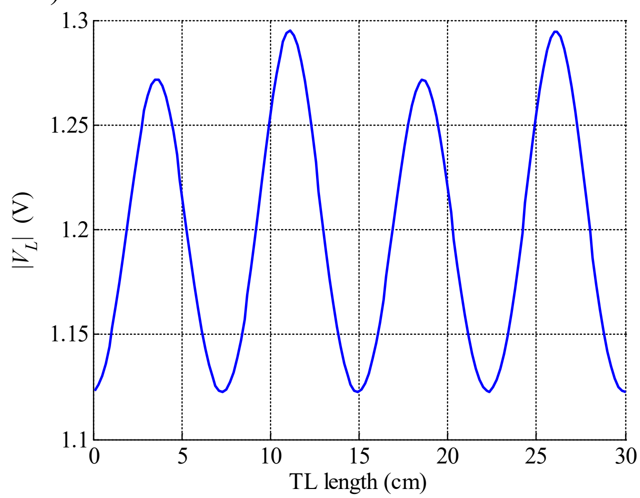
10. a)

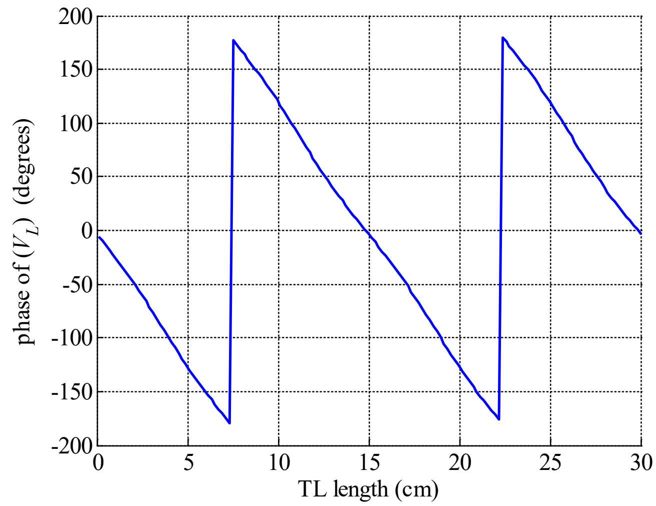


b)

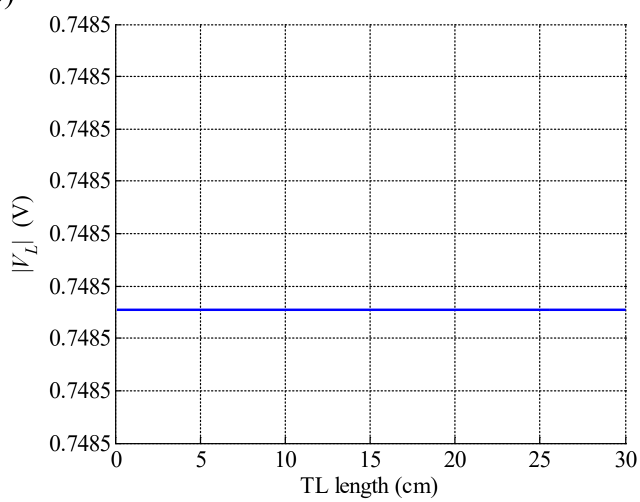


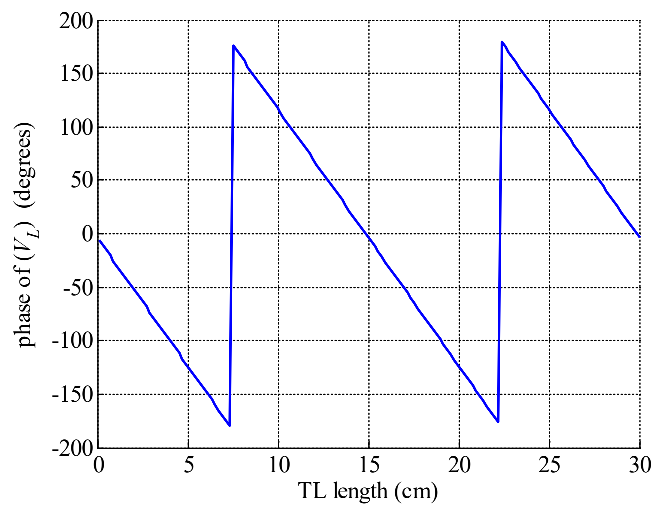
11. a)





b)

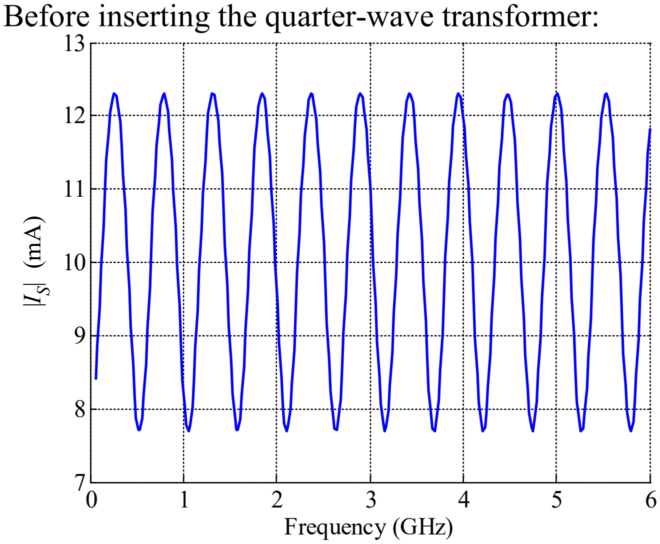


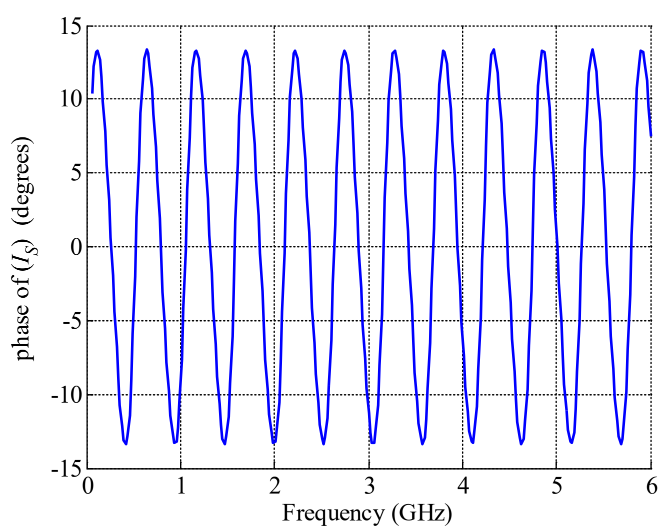


12. a) $Z_2 = 63.2456\ \Omega$; $l_2 = 1.3176$ cm.

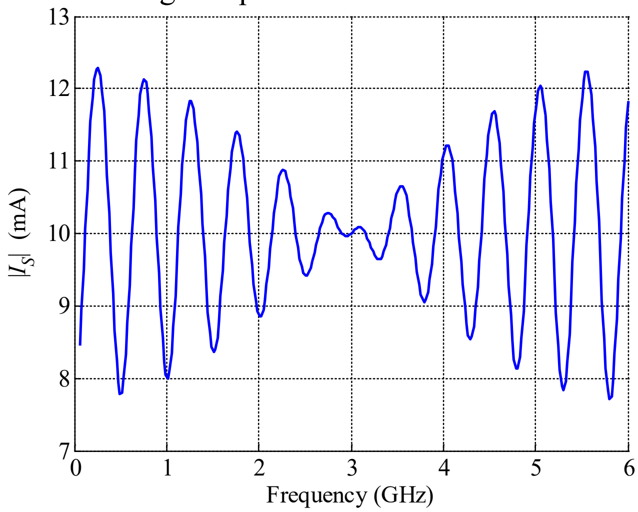
b)

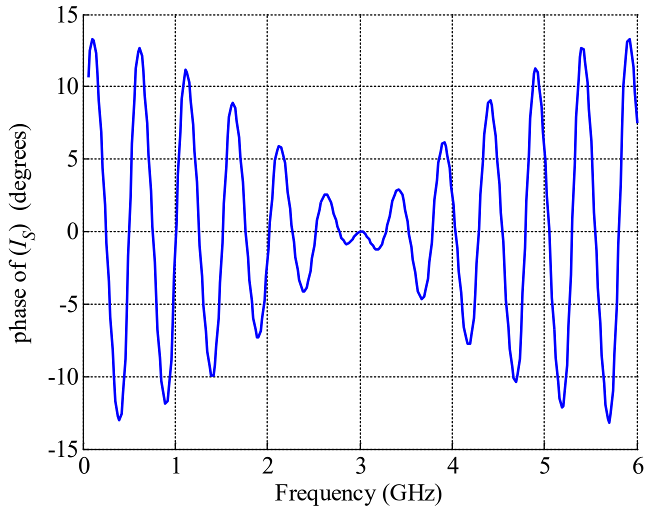
Before inserting the quarter-wave transformer:





After inserting the quarter-wave transformer:





Notice that, after inserting the quarter-wave transformer, the magnitude of I_S becomes less sensitive to the frequency around 3 GHz ($|I_S| = |V_S|/(R_S + Z_1) = 10$ mA at 3 GHz), and the phase of I_S is almost 0° around 3 GHz (because Z_{in} is resistive around 3 GHz, since $Z_{in} = Z_1 = 50\ \Omega$ at 3 GHz).