FUNDAMENTAL TRANSMISSION LINE THEORY

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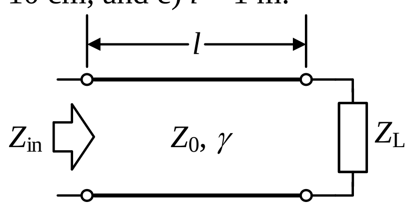
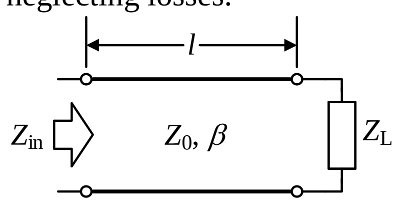
PROBLEMS

1. A lossy transmission line has the following per-unit length parameters: L = 289.05 nH/m, C = 115.62 pF/m, R = 6.3 Ω/m, and G = 5 mS/m. For simplicity, it is assumed that these four parameters are frequency-independent. a) Calculate the characteristic impedance of the line, Z_0, at 10 MHz and 100 MHz; b) Make 3 plots versus frequency, from 10 MHz to 10 GHz, of $|Z_0|$, α (attenuation constant) and β (propagation constant).

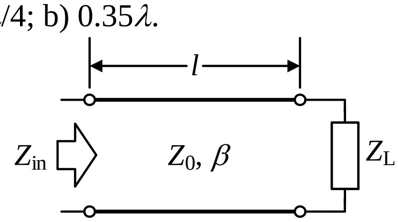
2. The transmission line illustrated below uses the same per-unit length parameters as in the previous problem. The load impedance Z_L consists of an 80-Ω resistor in series with a 10-nH inductor. Calculate the input impedance of the transmission line Z_{in} at 100 MHz, at the following distances from the load: a) l = 1 cm, b) l = 10 cm, and c) l = 1 m.



3. Solve previous problem but now neglecting losses.



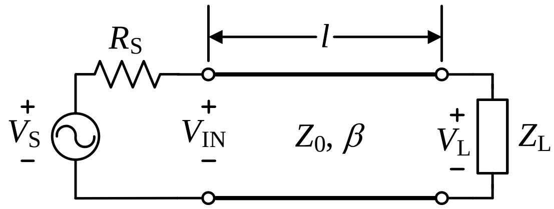
4. For the following lossless transmission line circuit, calculate the reflection coefficient at the load, Γ, the SWR on the line, and the input impedance, Z_{in}, at a at a distance l from the load. Assume Z_0 = 50 Ω, Z_L = 40 + j20 Ω, and l =: a) $\lambda/4$; b) 0.35λ.



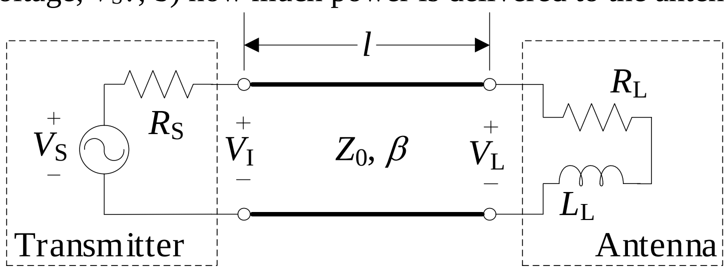
5. A lossless transmission line is connected to a R_L = 75 Ω. a) If the measured SWR at the load is 2.2, find the two possible values for Z_0. b) What is the value of the SWR when measured at a distance l = 0.3λ away from the load?

6. A coaxial line has a characteristic impedance Z_0 = 75 Ω, a physical length l = 2.5 m, and is filled with a dielectric whose ε_r = 2.56. If it is operating at 5 GHz and is connected to a load impedance Z_L = 37.5 + j75 Ω, calculate the reflection coefficient at the load, Γ, the reflection coefficient at the input, $\Gamma_{l=2.5\text{m}}$, the SWR on the line, and the impedance at the input of the coaxial line, Z_{in}. Use c = 0.3 Gm/s.

7. The following transmission line circuit has V_S = 15 Vrms, R_S = 75 Ω, Z_0 = 75Ω, Z_L = 55 – j30 Ω at the operating frequency. Calculate the average power delivered to the load if: a) $l = \lambda/4$; and b) $l = 0.3\lambda$.



8. A 50-Ω radio transmitter is connected to an antenna through a 50-Ω coaxial line. The antenna can be represented by a resistor R_L = 70 Ω in series with an inductor L_L = 5 nH when operating at 3 GHz. If the transmitter can delivered 25 W when connected to a matched load, a) What is the value of the equivalent source voltage, V_S?, b) how much power is delivered to the antenna at 3 GHz?

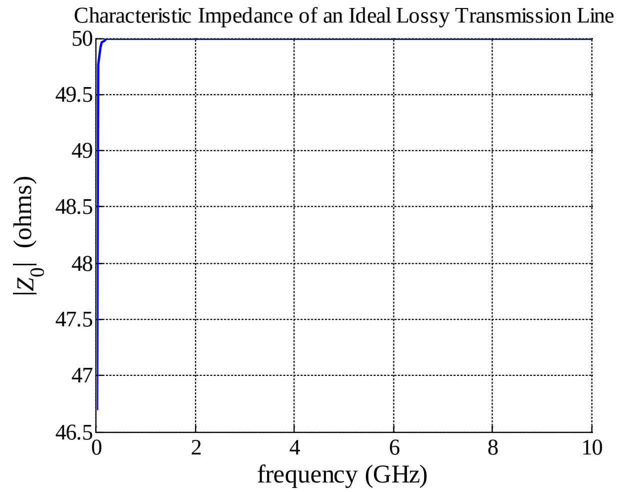


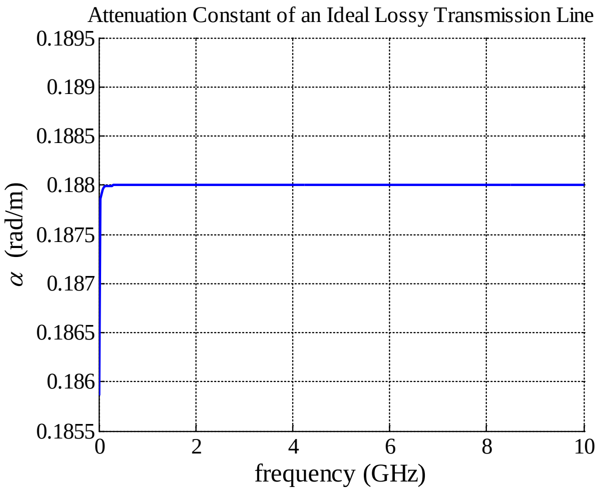
9. A lossless transmission line (TL) has a characteristic impedance Z_0 = 50 Ω and uses an effective permittivity ε_e = 3.8. If a load impedance Z_L = 500 Ω is connected to the transmission line, calculate the magnitude of the reflection coefficient at the load, $|\Gamma|$, at the following frequencies: a) 3 GHz, b) 3 GHz + 150 MHz, and c) 3 GHz – 150 MHz. If a quarter-wave transformer (QWT) is inserted between the TL and Z_L to achieve a perfect match at 3 GHz, calculate: d) its characteristic impedance Z_T, and e) its physical length l_T. Finally, calculate the magnitude of the reflection coefficient at the end of the TL (or the QWT input), $|\Gamma|$, at the following frequencies: f) 3 GHz, g) 3 GHz + 150 MHz, and h) 3 GHz – 150 MHz.

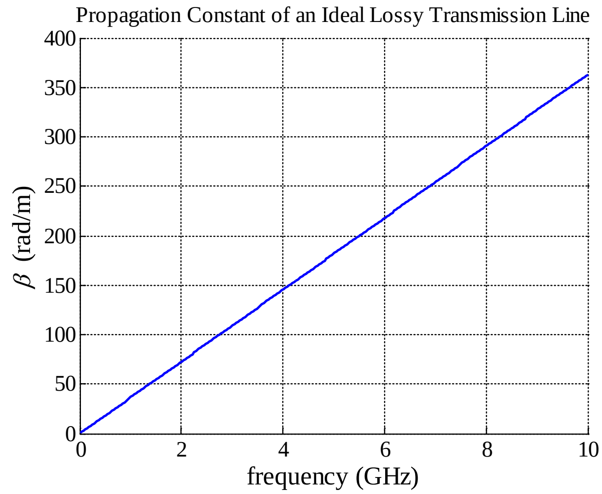
SOLUTIONS

1. a) Z_0 = 46.69 Ω ∠7.7° at 10 MHz;
   Z_0 = 49.96 Ω ∠0.97° at 100 MHz;

   b)







2. a) Z_{in} = 80.38 Ω ∠2.43°; b) Z_{in} = 72.35 Ω ∠–14.13°; c) Z_{in} = 61.52 Ω ∠–13.08°.
   Notice that Z_L = 80.25 Ω ∠4.49° (when the transmission line is electrically very short, $Z_{in} \approx Z_L$).
3. a) Z_{in} = 80.64 Ω ∠2.44°; b) Z_{in} = 74.26 Ω ∠–15.02°; c) Z_{in} = 68.39 Ω ∠–20.09°.
4. Γ = 0.2425 ∠104.04° and SWR = 1.64 in both cases. a) Z_{in} = 55.90 Ω ∠–26.56°; b) Z_{in} = 33.19 Ω ∠–15.29°.
5. a) Z_{01} = 165 Ω, Z_{02} = 34.09 Ω. b) Same value (SWR = 2.2) because the transmission line is lossless.
6. Γ = 0.62 ∠82.87°, Γ_l = 0.62 ∠–37.12°, SWR = 4.2656, and Z_{in} = 183.70 Ω ∠–50.58°.
7. a) P_{avg} = 695.22 mW; b) P_{avg} = 695.22 mW (same value because the transmission line is lossless and $R_S = Z_0$).
8. a) V_S = 100 V; b) P_{avg} = 15.03 W.
9. a), b), c) $|\Gamma|$ = 0.8182 at all frequencies; d) Z_T = 158.114 Ω; e) l_T = 12.825 mm; f) $|\Gamma|$ = 0 at f = 3 GHz; g) $|\Gamma|$ = 0.11096 at f = 3 GHz + 150 MHz; and h) $|\Gamma|$ = 0.11096 at f = 3 GHz – 150 MHz.