Frequency Response

(Part 1)

Dr. José Ernesto Rayas Sánchez

Most of the figures of this presentation were taken from the web site of the authors of the book:

A.S. Sedra and K.C. Smith, *Microelectronic Circuits*. New York, NY: Oxford University Press, 1998.

Frequency Response

- Introduction
- s-Domain Analysis
- The Amplifier Transfer Function
- Low Frequency Responses
- The FET and BJT Hybrid-π models
- High Frequency Responses
- Frequency Response by Computer Simulation
- Cascode Configuration

Introduction

- Frequency response analysis is crucial to amplifier´s performance
- Exact frequency response analysis can be very complicated (CAD)
- The Time Constant Method will be used

s-Domain Analysis

Voltage gain as a transfer function of the complex frequency $s = j\omega$

$$T(s) \equiv \frac{v_o(s)}{v_i(s)}$$

$$T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \cdots + a_1 s + a_0}{s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}$$

n is the order of the circuit ($m < n$)

For a stable circuit, all the roots of the denominator polynomial must have negative real parts

s-Domain Analysis (cont.)

Poles and Zeros

$$T(s) = a_m \frac{(s - Z_1)(s - Z_2)\cdots(s - Z_m)}{(s - P_1)(s - P_2)\cdots(s - P_n)}$$

$Z_1, \ldots, Z_m$ are the transfer function zeros, or transmission zeros

$P_1, \ldots, P_m$ are the transfer function poles, transmission poles or natural modes

s-Domain Analysis (cont.)

First-Order Functions

$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

$$T(s) = \frac{a_0}{s + \omega_0}$$ low-pass first-order network

$$T(s) = \frac{a_1 s}{s + \omega_0}$$ high-pass first-order network

Assignment

Read appendix F in the textbook, on Single Time Constant (STC) circuits

Bode Plots

$s + a = a + j\omega$ (a pole or zero term)

$$|s + a| = \sqrt{a^2 + \omega^2}$$

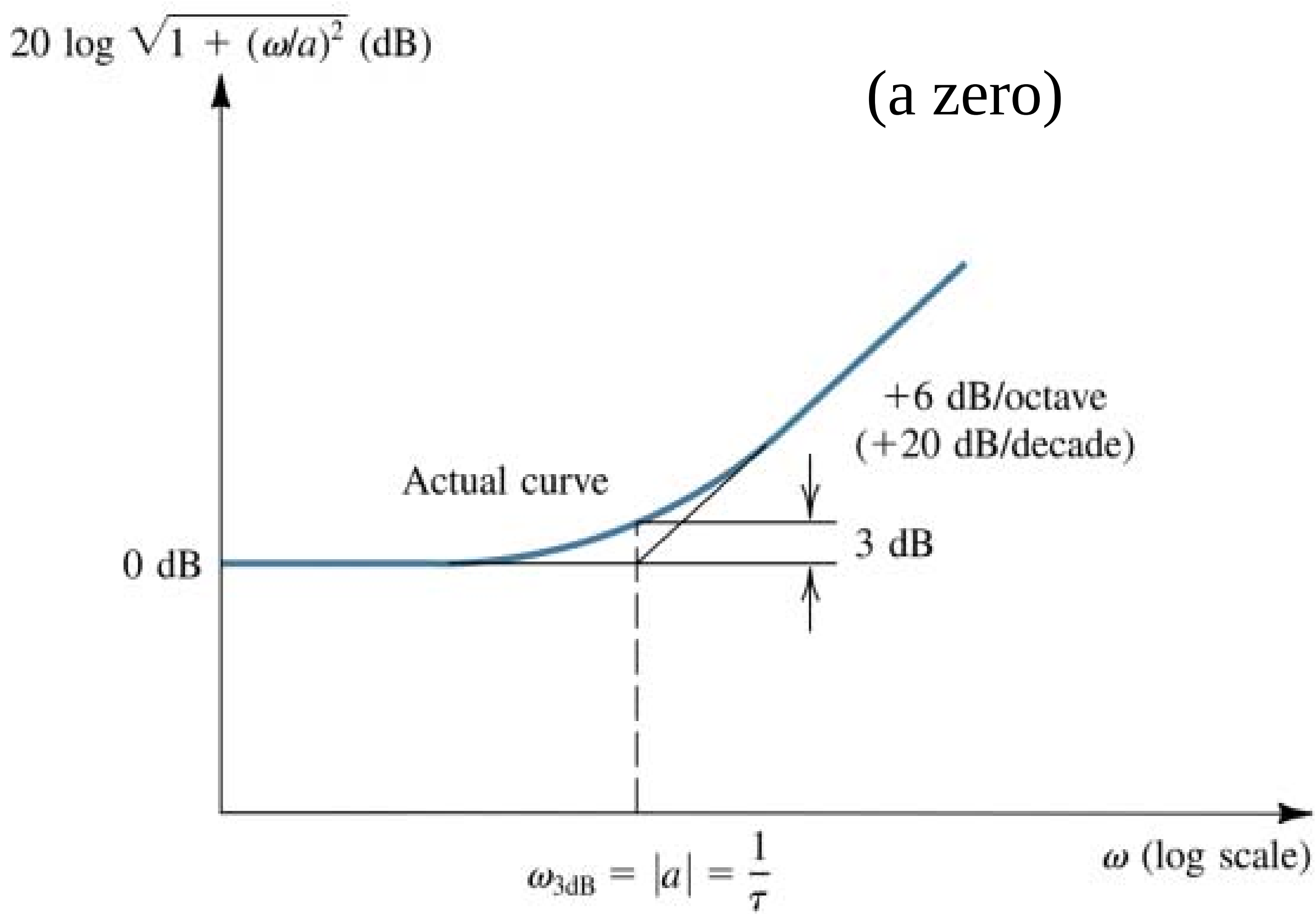
$$|s + a| = a\sqrt{1 + (\omega / a)^2}$$

$$\left.\frac{|s + a|}{a}\right|_{\text{dB}} = 20 \log \sqrt{1 + (\omega / a)^2} \text{ dB}$$

$$\angle(s + a) = \tan^{-1}(\omega / a)$$

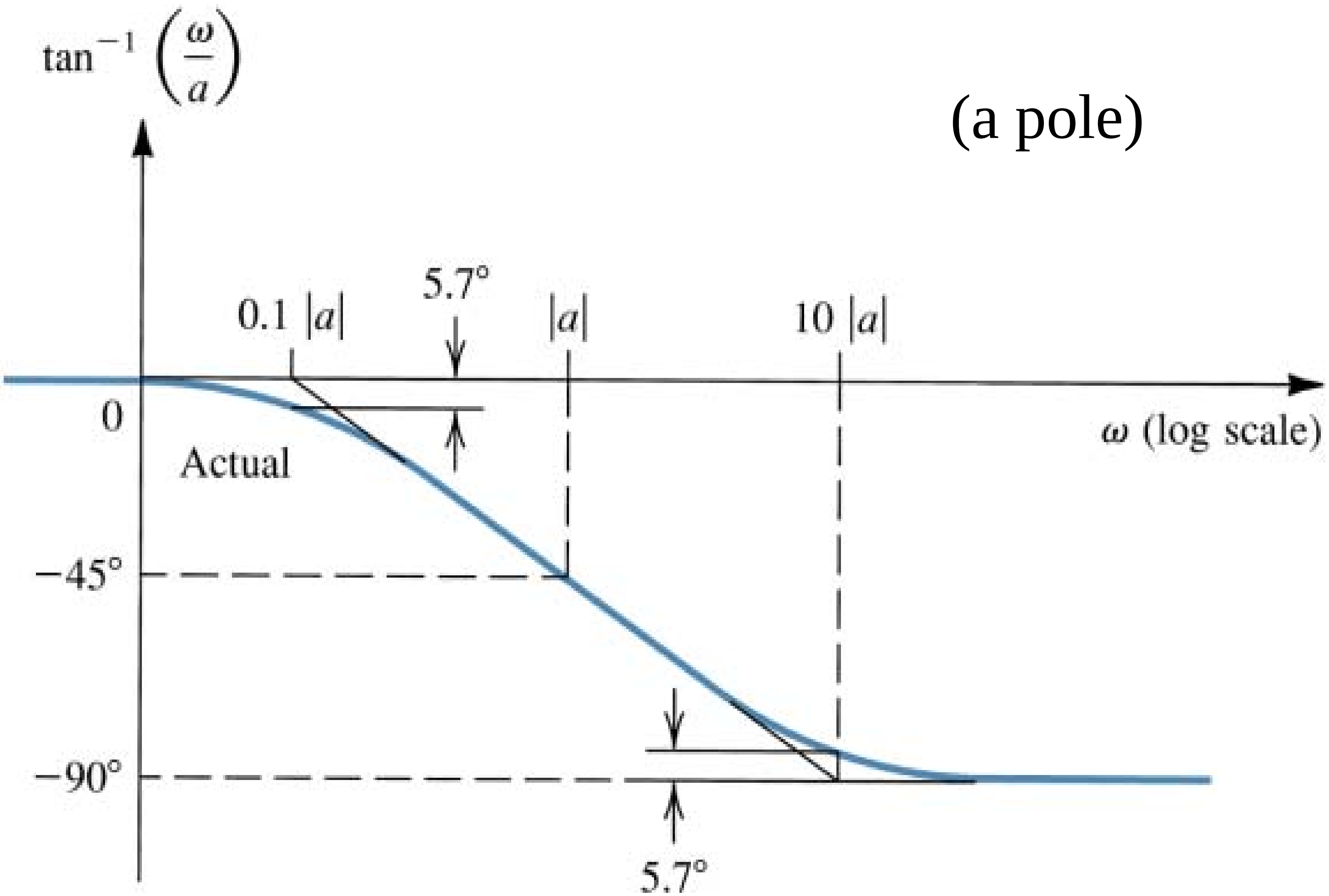
Bode Plots (continue)

$$\left.\frac{|s+a|}{a}\right|_{\text{dB}} = 20\log\sqrt{1+(\omega/a)^2}\ \text{dB}$$



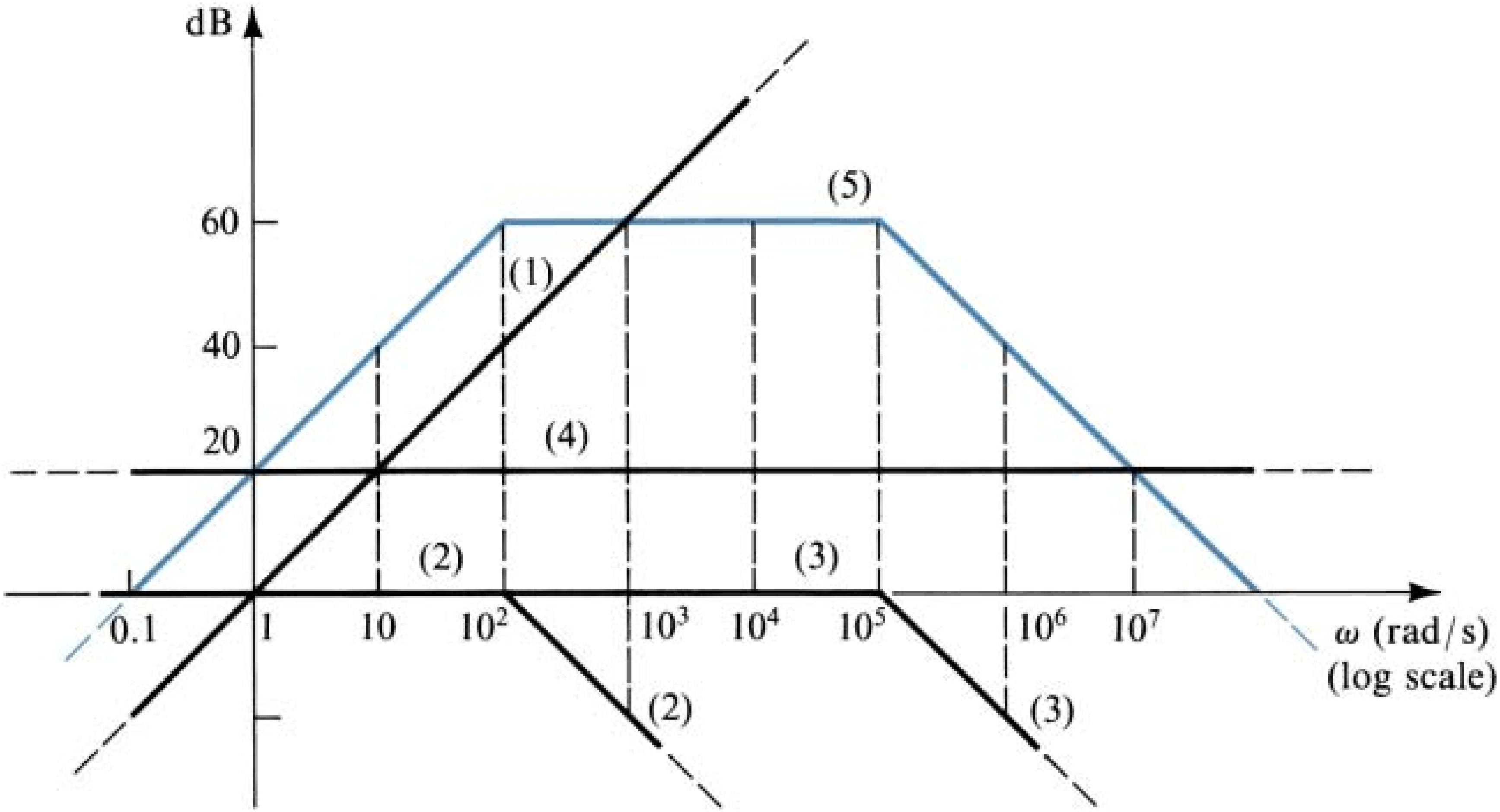
Bode Plots (continue)

$$\angle(s+a) = \tan^{-1}(\omega / a)$$



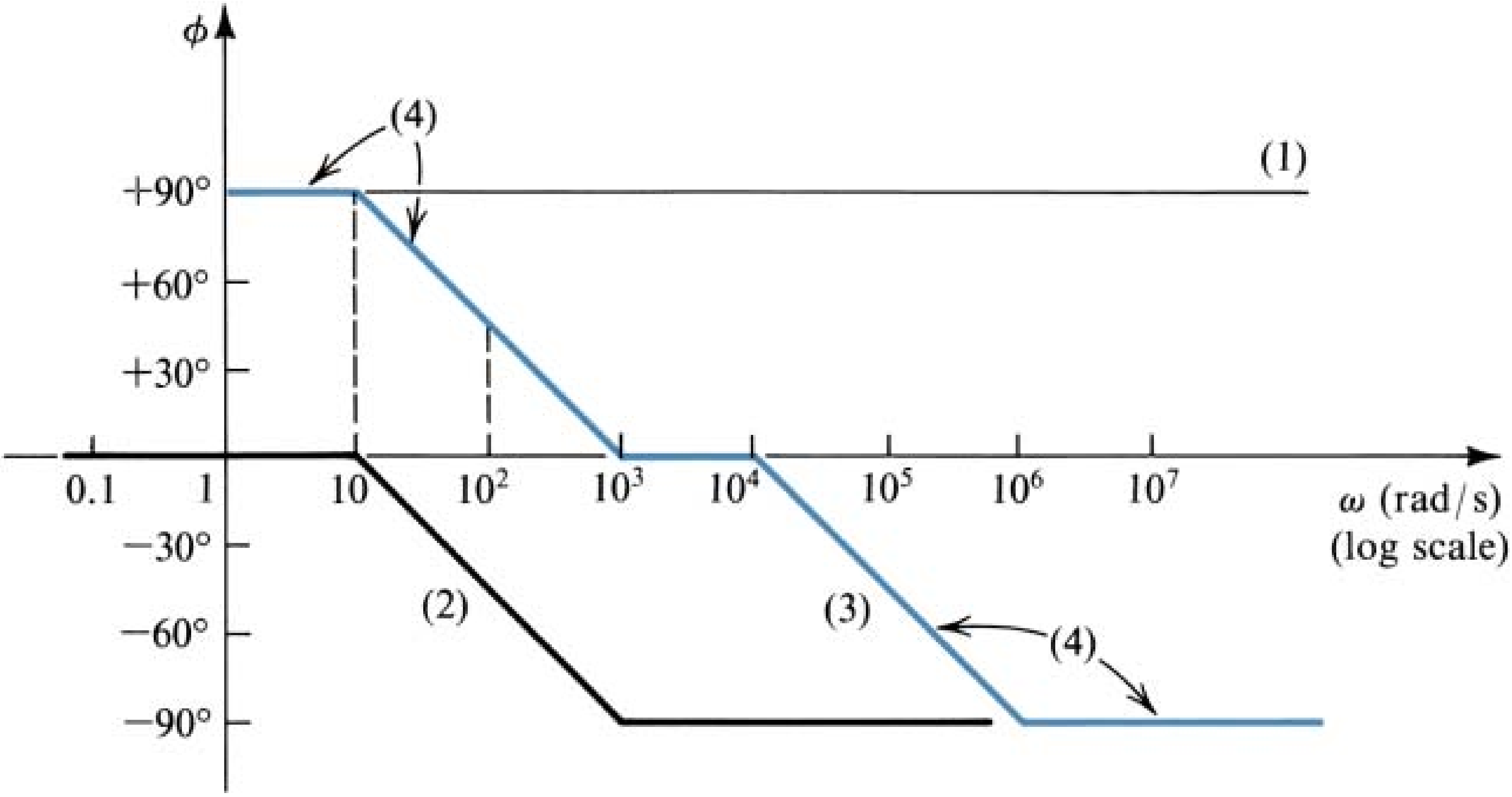
Bode Plots, an Example

$$T(s) = \frac{10s}{(1+s/10^2)(1+s/10^5)}$$



Bode Plots, an Example (continue)

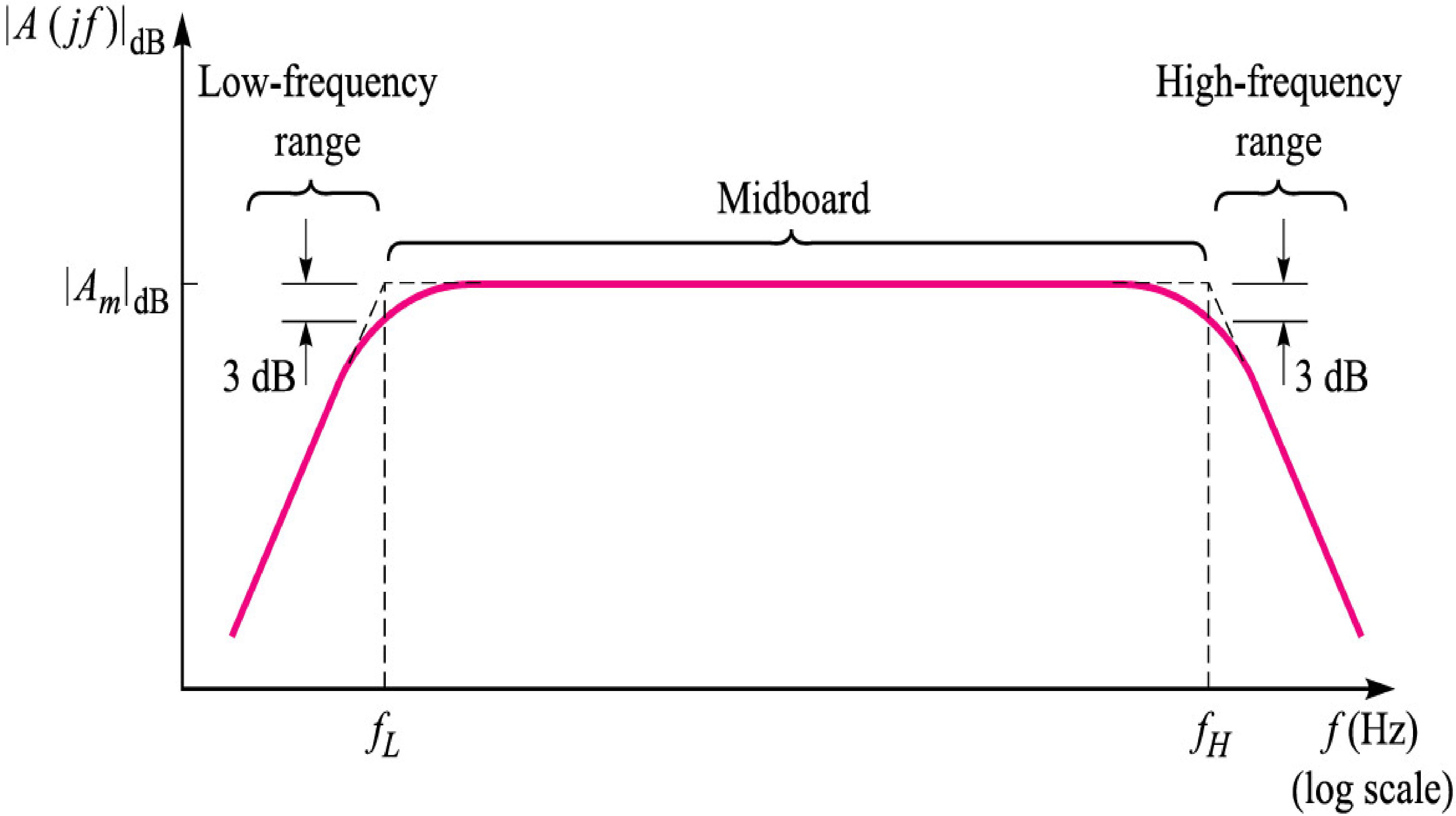
$$T(s)=\frac{10s}{(1+s/10^2)(1+s/10^5)}$$



Assignment

Solve problems 7.1, 7.5 and 7.10 from the textbook

The Amplifier Transfer Function



The Amplifier Transfer Function (continue)

A_M midband gain

ω_L cutoff low frequency, 3-dB low frequency

ω_H cutoff high frequency, 3-dB high frequency

Bandwidth (*BW*)

$BW = f_H - f_L$ (Hz) $BW = \omega_H - \omega_L$ (rad/sec)

usually $\omega_H >> \omega_L$, $BW \approx \omega_H$

Gain-Bandwith Product (*GB*)

$GB = A_M\, \omega_H$

The Gain Function

$$A(s) = \frac{v_o(s)}{v_i(s)} = A_M F_L(s) F_H(s)$$

A_M midband gain

$F_L(s)$ low frequency response

$F_H(s)$ high frequency response

When $\omega_H >> \omega >> \omega_L$, $A(s) \approx A_M$

When $\omega >> \omega_L$, $F_L(s) \approx 1$

When $\omega << \omega_H$, $F_H(s) \approx 1$

Low Frequency Response

$$F_L(s) = \frac{(s+\omega_{Z1})(s+\omega_{Z2})\cdots(s+\omega_{Zn_L})}{(s+\omega_{P1})(s+\omega_{P2})\cdots(s+\omega_{Pn_L})}$$

n_L number of poles (or zeros) of $F_L(s)$

if $\omega_{P1} >> \omega_{P2},\ldots,\omega_{Pn_L},\omega_{Z1},\ldots,\omega_{Zn_L}$ then

$F_L(s) \approx \frac{s}{(s+\omega_{P1})}$ and $\omega_L \approx \omega_{P1}$ (ω_{P1} is a dominant pole)

If there is no dominant pole

$$\omega_L \approx \sqrt{\omega_{P1}^2 + \omega_{P2}^2 + \ldots + \omega_{Pn_L}^2 - 2(\omega_{Z1}^2 + \omega_{Z2}^2 + \ldots + \omega_{Zn_L}^2)}$$

Low Frequency Response, Example

Calculate ω_L for $F_L(s) = \dfrac{s(s+10)}{(s+100)(s+25)}$

1) $\omega_{P1} = 100$ is a dominant pole, then $\omega_L \approx 100$ rad/sec

2) $\omega_L \approx \sqrt{100^2 + 25^2 - 2(10^2)} = 102$ rad/sec

3) Using Bode plots... $\omega_L \approx 105$ rad/sec

High Frequency Response

$$F_H(s) = \frac{(1+s/\omega_{Z1})(1+s/\omega_{Z2})\cdots(1+s/\omega_{Zn_H})}{(1+s/\omega_{P1})(1+s/\omega_{P2})\cdots(1+s/\omega_{Pn_H})}$$

n_H number of poles (or zeros) of $F_H(s)$

if $\omega_{P1} << \omega_{P2},\ldots,\omega_{Pn_L},\omega_{Z1},\ldots,\omega_{Zn_L}$ then

$F_H(s) \approx \frac{1}{(1+s/\omega_{P1})}$ and $\omega_H \approx \omega_{P1}$ (ω_{P1} is a dominant pole)

If there is no dominant pole

$$\omega_H \approx \frac{1}{\sqrt{1/\omega_{P1}^2 + 1/\omega_{P2}^2 + \ldots + 1/\omega_{Pn_H}^2 - 2(1/\omega_{Z1}^2 + 1/\omega_{Z2}^2 + \ldots + 1/\omega_{Zn_H}^2)}}$$

High Frequency Response, Example

Calculate ω_H for $F_H(s) = \dfrac{1 - s/10^5}{(1 + s/10^4)(1 + s/4\times10^4)}$

1) $\omega_{P1} = 10^4$ is a dominant pole, then $\omega_H \approx 10^4$ rad/sec

2) $\omega_H \approx 1/\sqrt{1/10^8 + 1/(16\times10^8) - 2/10^{10}} = 9{,}800$ rad/sec

3) Using Bode plots... $\omega_H \approx 9{,}537$ rad/sec

Assignment

Solve problems 7.14, 7.21 and 7.27 from the textbook