On Optimizing the Feedback Components in a Voltage-Feedback Amplifier

Philip Hoff

Abstract—For a voltage-feedback amplifier in which the open-loop gain will be much greater than the closed-loop gain, it is possible to calculate values of the feedback resistors that will maximize the open-loop gain. This paper presents the derivation and compares the results to a series of PSPICE simulations. Agreement is found to be excellent.

Index Terms—Feedback amplifiers, gain maximization simulation-feedback amplifiers, voltage feedback.

The two-stage transistor voltage feedback amplifier such as the one shown in Fig. 1 is taught in all electronics programs. Hopefully, students also have the opportunity to design such an amplifier. If they have already learned how to design a single-stage amplifier, the largest challenge involved in such a design assignment is to choose the values of \( R_f \) and \( R_u \) (see Fig. 1). If the open-loop gain of the amplifier is much larger than the desired closed-loop gain, the correct ratio of these two resistors is determined to a good accuracy by the desired closed-loop gain at midband in accordance with the equation [1]

\[ R_F/R_u = A_f - 1 \]  

where \( A_f \) is the desired closed-loop gain at midband.

We generally tell our students that one of the first considerations in designing negative feedback amplifiers is to maximize the open-loop gain. Not only is this necessary to validate the usage of (1), but because, at least at midband, all of the “nice” features of negative feedback will accrue to us to an increasing degree as we “give away” open-loop gain. So in the design process, the values of these two resistors must be chosen judiciously, because if we make them too small, \( R_u \) will load the output of stage two excessively and cut the available open-loop gain. On the other hand, if we make these resistors too large, \( R_u \) will give excessive degeneration in stage one, which will also degrade the open-loop gain.

The above-mentioned limiting factors suggest that there will therefore be some value of these two resistors that will maximize the open-loop gain (everything else remaining constant). This paper derives and reports expressions for the optimum values of these two feedback resistors.

We begin with the minimal low-frequency model for the bipolar shown in Fig. 2 [2]. When the equivalent circuit of Fig. 2 is inserted into the amplifier circuit shown in Fig. 1, the loop is opened by removing \( R_f \), and the resulting circuit is simplified for ac midband (shorting all capacitors plus the dc source voltage), the circuit of Fig. 3 results. All of the circuit parameters used in analyzing the circuit are essentially defined by this schematic except

1) \( \beta_1 \) is the short-circuit current transfer ratio of the first transistor;
2) \( A_o \) is the midband open-loop gain;
3) \( R_{TH1} \) is the parallel combination of \( R_{i1} \) and \( R_{i2} \);
4) \( R_{TH2} \) is the parallel combination of \( R_{i1} \) and \( R_{i2} \);
5) \( R_L \) is the parallel combination of \( R_C \) and \( R_{TH2} \) and \( r_{\pi1} \);
6) \( A_f \) is the desired closed-loop gain at midband

Analysis of the circuit of Fig. 3 yields the transfer function of (2).

\[ A_o = \frac{g_2 R_C \beta_1 R_L}{(\beta_1 + 1) R_u + r_{\pi1}} \left( 1 + \frac{R_S}{R_{TH1}} + R_S \right) \]  

This expression, unfortunately, does not take account of the fact that the output of the amplifier is loaded by the feedback elements, which are themselves loaded by the input of the amplifier. Even though we conceptually open the loop, the open loop gain will only be calculated correctly if these loading factors are taken into account. This can be accomplished by replacing \( R_C \) in the numerator by \( R_C (R_F + R_u) \) and replacing \( R_u \) in the denominator by \( R_u (R_F) \) [3].

\[ A_o = \frac{g_2 [R_C] (R_F + R_u) \beta_1 R_L}{(\beta_1 + 1) R_u [R_F + r_{\pi1}]} \left( 1 + \frac{R_S}{R_{TH1}} + R_S \right) \]  

Next we use (1) to eliminate \( R_u \) in (3), yielding (4) at the bottom of the following page. This equation for \( A_o \) can be differentiated with respect to \( R_F \) and the derivative set equal to zero to solve for the value of \( R_F \) that will maximize \( A_o \). Although the algebra is laborious, the final expression for \( R_F \) so obtained is surprisingly compact.

\[ R_F = \left[ \frac{R_C \beta_1 (A_f^2 + 1)}{A_f (\beta_1 + 1)} (r_{\pi1} + R_S) [R_{TH1}] \right]^{1/2} \]  

For the vast majority of transistors, \( \beta \gg 1 \). Then using \( \beta_1 = g_1 r_{\pi1} [2] \) allows this expression to be further simplified.
It may be noted here that since the simplest equivalent circuit of a field-effect transistor (FET) is topologically identical to that of a bipolar transistor with \( r_{\pi} \rightarrow \infty \), (6) also applies to a FET amplifier if \( R_{C2} \) is replaced by \( R_{D2} \) and the term involving \( r_{\pi 1} \) is set equal to zero.

If \( A_f^2 \gg 1 \), (6) further simplifies to

\[
R_f = \left[ \frac{R_{C2}(A_f^2 - 1)}{A_f g_h} \left( 1 + \frac{R_S |R_{TH1}|}{r_{\pi 1}} \right) \right]^{1/2}. \tag{6}
\]

Since the value of \( R_u \) is determined by the values of \( R_f \) and \( A_f \), once we have found \( R_f \), we can rearrange (1) and write

\[
R_u = R_f / (A_f - 1). \tag{8}
\]

It will be seen that in Fig. 1, \( R_S = 0 \). The optimum values of \( R_f \) and \( R_u \) calculated for this circuit according to (7) and (8) were: \( R_f = 1500 \, \Omega \) and \( R_u = 82.1 \, \Omega \).

To test these results, the circuit was simulated in PSPICE®.

In the simulation, \( R_u \) was varied through standard 5% resistance values, while keeping \( R_f \) equal to 19\( \times \)\( R_u \) to maintain a constant first-order open-loop gain of 20. The loading effects introduced in (3) were incorporated into the circuit being simulated. The results are shown in Fig. 4. According to the PSPICE® results, the maximum open-loop gain occurs at

\[
A_o = g_2 \beta_1 R_L \cdot \frac{R_{C2}(R_f + R_f / (A_f - 1))}{(\beta_1 + 1) \cdot R_f (R_f / (A_f - 1) + r_{\pi 1} + R_S / R_{TH1}) + R_S}. \tag{4}
\]
slightly lower values of $R_u$ and $R_F$ than are predicted by the theoretical result of (7). This difference can be attributed to the much more sophisticated transistor model used in PSpice®. Nevertheless, the peak is so broad that the open-loop gain varies by less than 0.1% between the optimum values predicted by (7) and by PSpice®. This certainly establishes the value of this method as a design tool.

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REFERENCES


Philip Hoff received the B.S. and M.S. degrees from Carnegie-Mellon University, Pittsburg, PA, in 1963 and 1964, respectively, and the Ph.D. degree from the University of California at Berkeley in 1970. He is a Professor of Electrical/Electronic Engineering at California State University at Chico. His interests include consumer electronics and solid-state devices.
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Comment on “On the Geometry of Parallel Impedances”
Vittal P. Pyati

In the above paper\(^1\) the time-honored custom of forbidding the use of angle-measuring devices in geometrical proofs appears to have been violated by its author Karni. If it were otherwise, the problem of trisecting an arbitrary angle would be trivial. Furthermore, the method proposed by Karni suffers from the disadvantage of employing one procedure for resistors and another for impedances. This is bound to prove awkward when there is a combination of the two. By far the most elegant geometrical method of handling impedances and resistors connected in parallel is that of inversion in a circle followed by reflection in the real axis. This requires that the impedances be given in rectangular rather than polar form. I learned the inversion method (invented in the West at least a 100 years ago) in the 1950’s as an undergraduate in my native India. In the modern age of computers, graphical methods can perhaps play a supplementary role.

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Correction to “On Optimizing the Feedback Components in a Voltage-Feedback Amplifier”
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In the above paper,\(^1\) there is an error in Fig. 4. Only one of the loading factors was incorporated into the PSPICE® simulations. The other one effectively lowers the unbypassed emitter resistance in the first stage and so increases the open-loop gain slightly. The revised Fig. 4 is included with this correction. The simulation predicts the maximum open-loop gain for \(R_u = 78 \ \Omega\). To the nearest whole number, this is the same value predicted by the original Fig. 4, so none of the conclusions is modified.

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