BJT Small-Signal Models

Dr. José Ernesto Rayas Sánchez

Some figures of this presentation were taken from the instructional resources of the following textbook:

A. S. Sedra and K. C. Smith, *Microelectronic Circuits*. New York, NY: Oxford University Press, 2003.

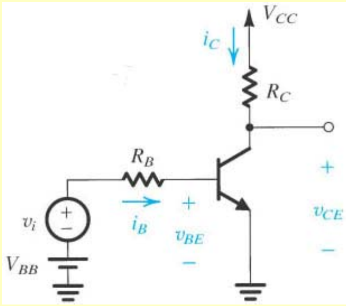
1

Outline

- DC Bias + small-signal excitation
- Load lines
- Amplification process in BJTs
- Small-signal models
- DC and small-signal analysis: example
- N-port networks
- Z, Y and H-parameters
- Manufacturing data sheets

Dr. J. E. Rayas Sánchez 2

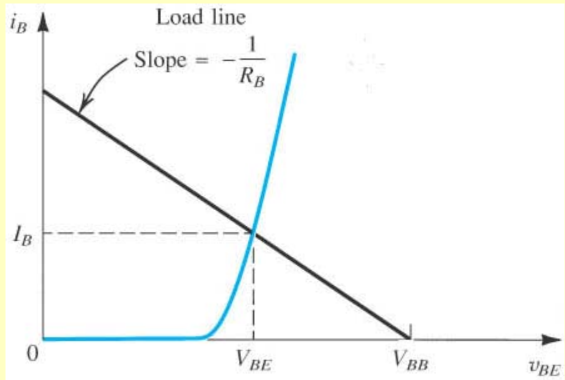
DC Bias + Small-Signal Excitation



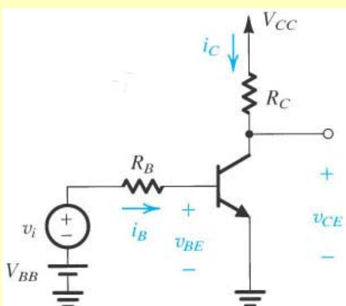
If $v_i = 0$ (bias only):

$$V_{BB} - i_B R_B - v_{BE} = 0$$

$$i_B = \frac{V_{BB} - v_{BE}}{R_B}$$ → Input Load Line



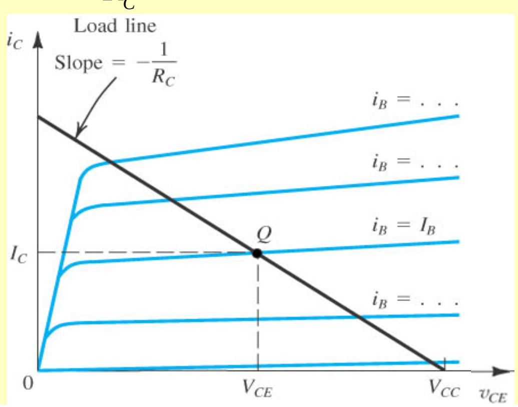
DC Bias + Small-Signal Excitation (cont)



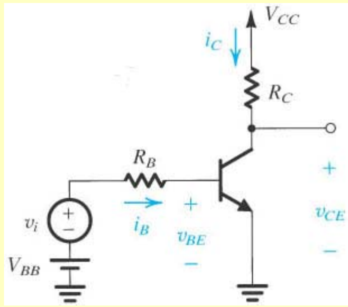
If $v_i = 0$ (bias only):

$$V_{CC} - i_C R_C - v_{CE} = 0$$

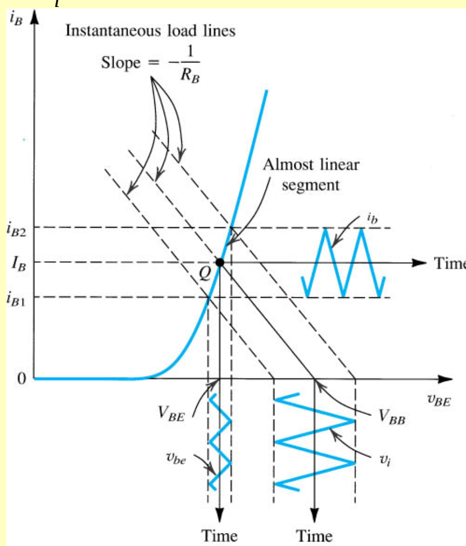
$$i_C = \frac{V_{CC} - v_{CE}}{R_C}$$ → Load Line



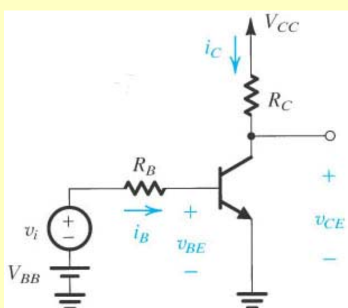
DC Bias + Small-Signal Excitation (cont)



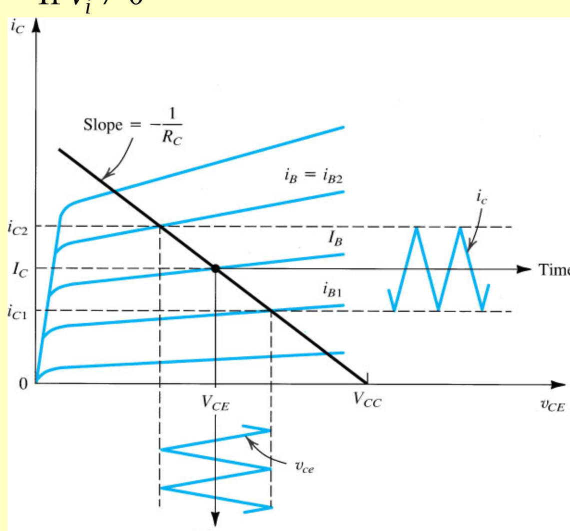
If $v_i \neq 0$



DC Bias + Small-Signal Excitation (cont)



If $v_i \neq 0$

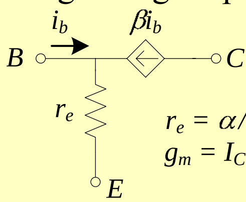


A small variation of v_{BE} can produce a large variation in v_{CE}

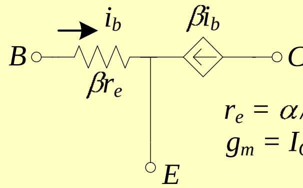
→ Amplification

BJT Small-Signal Models: T Model using β

- Neglecting output resistance, r_o

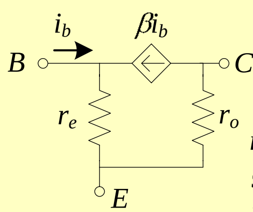


$r_e = \alpha / g_m \approx 1/g_m$
$g_m = I_{CQ} / \eta V_T$

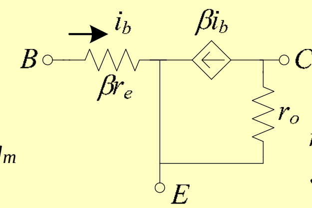


$r_e = \alpha / g_m \approx 1/g_m$
$g_m = I_{CQ} / \eta V_T$

- Considering r_o



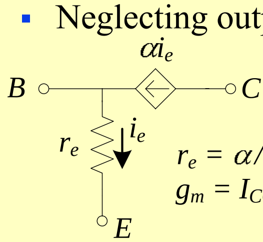
$r_e = \alpha / g_m \approx 1/g_m$
$g_m = I_{CQ} / \eta V_T$
$r_o = V_A / I_{CQ}$



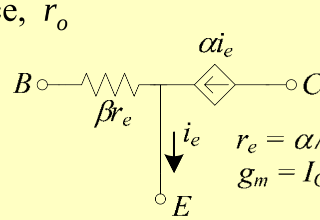
$r_e = \alpha / g_m \approx 1/g_m$
$g_m = I_{CQ} / \eta V_T$
$r_o = V_A / I_{CQ}$

BJT Small-Signal Models: T Model using α

- Neglecting output resistance, r_o

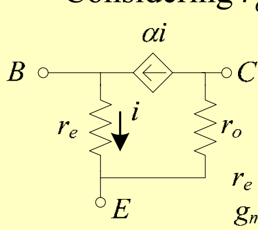


$r_e = \alpha / g_m \approx 1/g_m$
$g_m = I_{CQ} / \eta V_T$

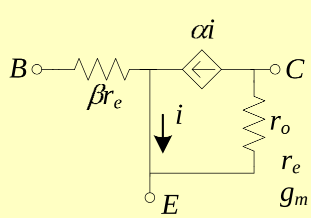


$r_e = \alpha / g_m \approx 1/g_m$
$g_m = I_{CQ} / \eta V_T$

- Considering r_o



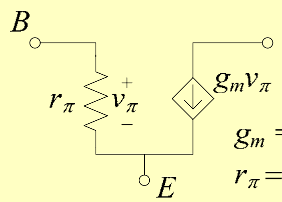
$r_e = \alpha / g_m \approx 1/g_m$
$g_m = I_{CQ} / \eta V_T$
$r_o = V_A / I_{CQ}$



$r_e = \alpha / g_m \approx 1/g_m$
$g_m = I_{CQ} / \eta V_T$
$r_o = V_A / I_{CQ}$

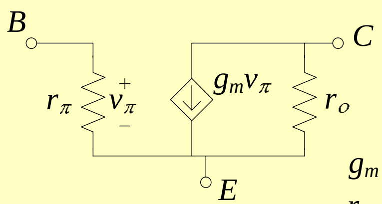
BJT Small-Signal Models: Hybrid π Model

- Neglecting output resistance, r_o



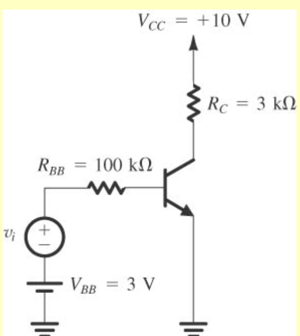
$g_m = I_{CQ}/\eta V_T$
$r_\pi = \beta / g_m$

- Considering r_o



$g_m = I_{CQ}/\eta V_T$
$r_\pi = \beta / g_m$
$r_o = V_A / I_{CQ}$

DC and Small-Signal Analysis – Example

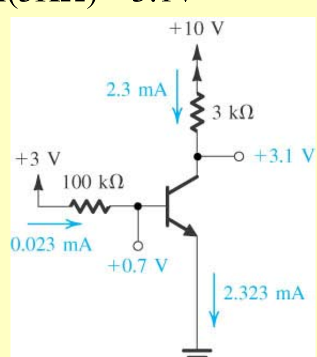


Bias point calculation (DC analysis)

$$I_B = \frac{3\text{V} - 0.7\text{V}}{100\text{K}\Omega} = 0.023\text{mA}$$

$$I_C = 100 I_B = 2.3\text{mA}$$

$$V_{CE} = 10\text{V} - 2.3\text{mA}(3\text{K}\Omega) = 3.1\text{V}$$

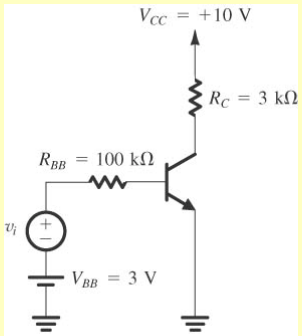


$\beta = 100$
$\eta = 1$

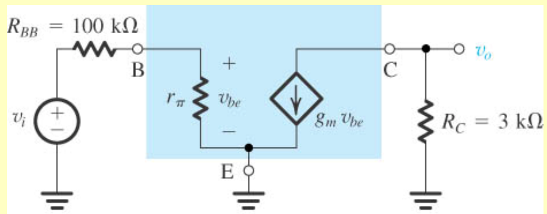
$$g_m = \frac{I_{CQ}}{\eta V_T} = \frac{2.3\text{mA}}{25\text{mV}} = 92\text{m}\Omega^{-1}$$

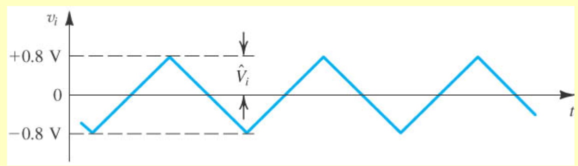
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{92\text{m}\Omega^{-1}} = 1.087\text{K}\Omega$$

DC and Small-Signal Analysis – Example (cont)



Small-signal equivalent circuit (neglecting r_o):





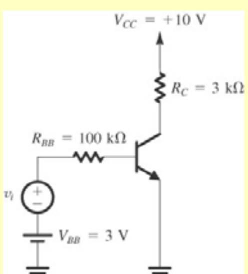
$\beta = 100$

$\eta = 1$

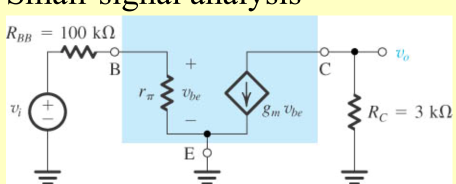
$g_m = 92\text{m}\Omega^{-1}$

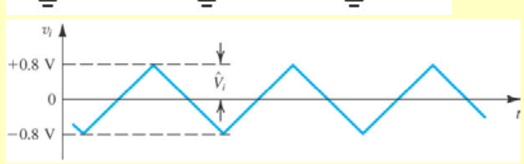
$r_\pi = 1.087\text{K}\Omega$

DC and Small-Signal Analysis – Example (cont)

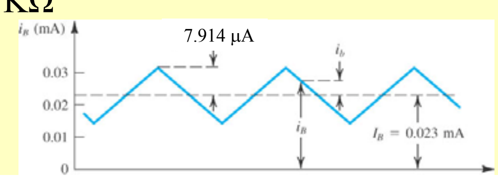


Small-signal analysis

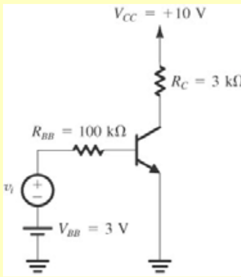




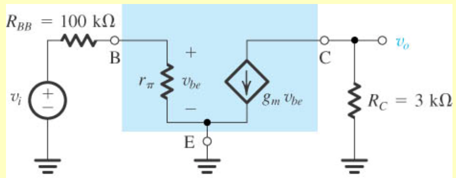
$$i_b = \frac{v_i}{R_{BB} + r_\pi} = \frac{0.8\text{V}}{100\text{K}\Omega + 1.087\text{K}\Omega} = 7.914\mu\text{A}$$

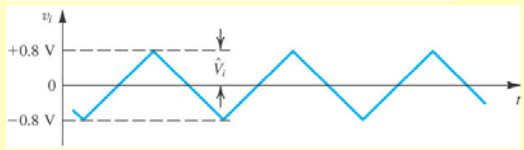


DC and Small-Signal Analysis – Example (cont)



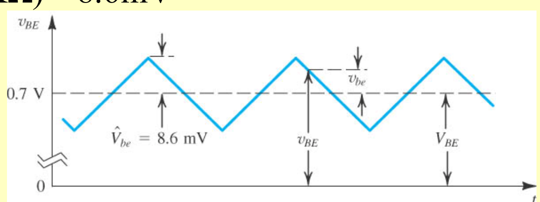
Small-signal analysis



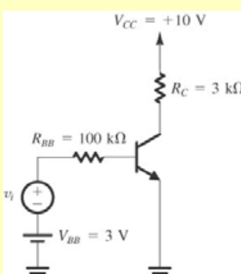


$i_b = 7.914\mu A$

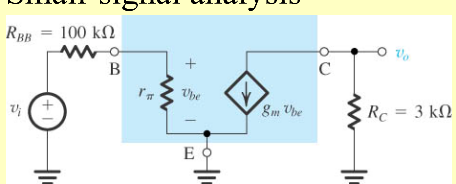
$v_{be} = i_b r_\pi = 7.914\mu A(1.087K\Omega) = 8.6mV$

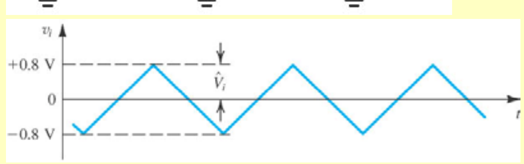


DC and Small-Signal Analysis – Example (cont)



Small-signal analysis



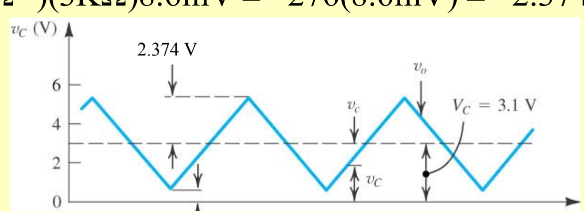


$v_{be} = 8.6mV$

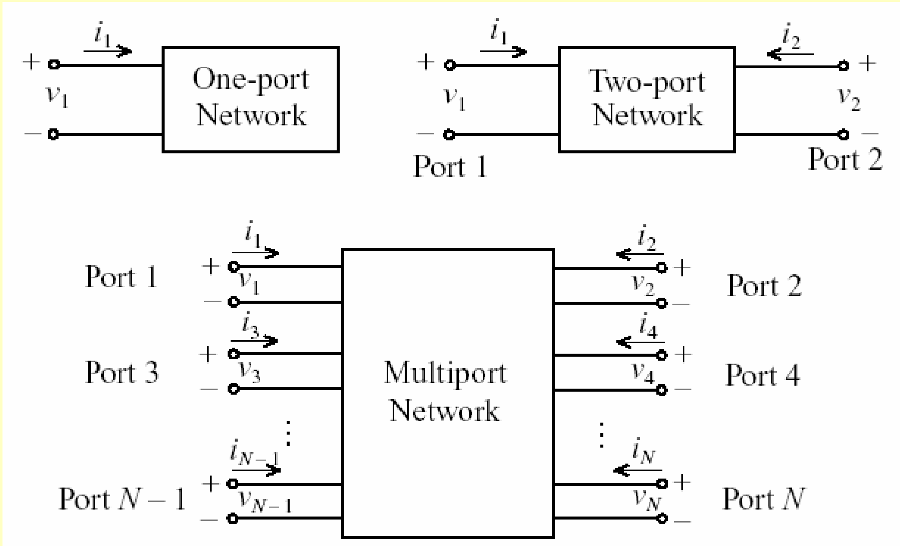
$v_o = -g_m R_C v_{be} = -(92m\Omega^{-1})(3K\Omega)8.6mV = -276(8.6mV) = -2.374V$

$$A_v = \frac{v_o}{v_i} = \frac{-2.374V}{0.8V}$$

$$A_v = -2.96V/V$$



N-Ports Networks (Linear Circuits)



(R. Ludwig and P. Bretchko, RF Circuit Design, Prentice Hall, 2000)

Impedance Matrix Representation (***Z***)

$$\boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \qquad \boldsymbol{I} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

$$\boldsymbol{V} = \boldsymbol{ZI}$$

$$\boldsymbol{Z} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1N} \\ Z_{21} & Z_{22} & \dots & Z_{2N} \\ \vdots & & & \\ Z_{N1} & Z_{N2} & \dots & Z_{NN} \end{bmatrix}$$

Each element of matrix ***Z*** is given by

$$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{I_k = 0 \text{ for } k \neq j}$$

Admittance Matrix Representation (*Y*)

$$\boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \qquad \boldsymbol{I} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

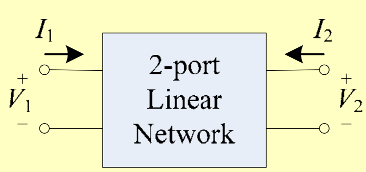
$$\boldsymbol{I} = \boldsymbol{YV}$$

$$\boldsymbol{Y} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1N} \\ Y_{21} & Y_{22} & \dots & Y_{2N} \\ \vdots & & & \\ Y_{N1} & Y_{N2} & \dots & Y_{NN} \end{bmatrix}$$

Each element of matrix ***Y*** is given by

$$Y_{ij} = \left. \frac{I_i}{V_j} \right|_{V_k = 0 \text{ for } k \neq j}$$

Z-Parameters for 2-Port Networks

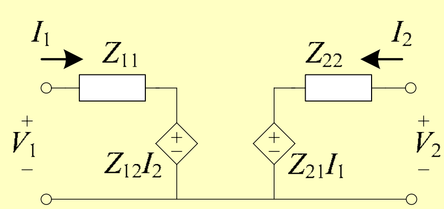


$$\boldsymbol{V} = \boldsymbol{ZI}$$

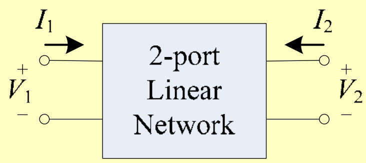
$$\boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \boldsymbol{I} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\boldsymbol{Z} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

Equivalent circuit:



Y-Parameters for 2-Port Networks

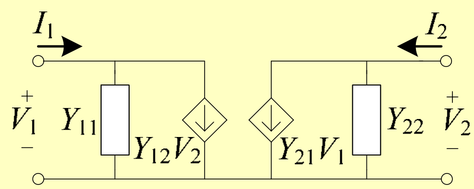


$$\boldsymbol{I} = \boldsymbol{YV}$$

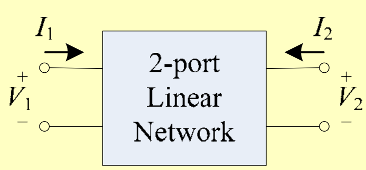
$$\boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \boldsymbol{I} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\boldsymbol{Y} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

Equivalent circuit:



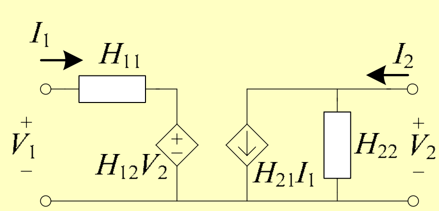
H-Parameters (Hybrid) for 2-Port Networks



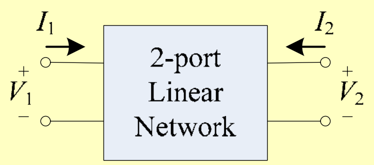
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \boldsymbol{H} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

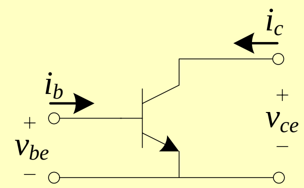
$$\boldsymbol{H} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

Equivalent circuit:



H-Parameters for a BJT

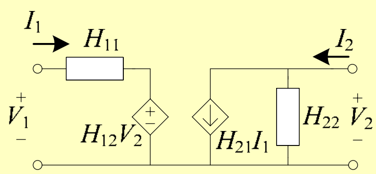




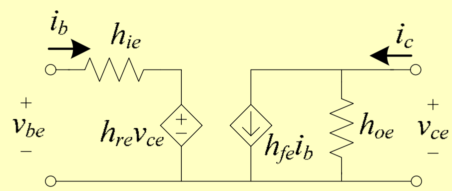
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} v_{eb} \\ i_c \end{bmatrix} = \begin{bmatrix} h_{ie} & h_{re} \\ h_{fe} & h_{oe} \end{bmatrix} \begin{bmatrix} i_b \\ v_{ce} \end{bmatrix}$$

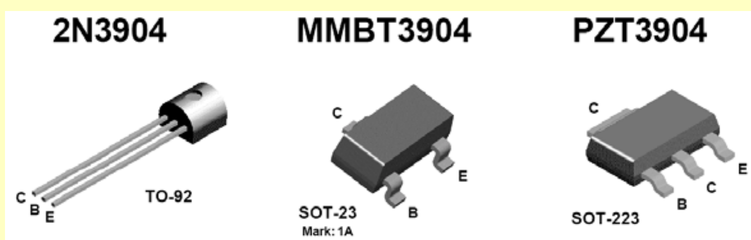
Equivalent circuit:



Equivalent circuit at low freq.:



BJT Manufacturing Data Sheets



FAIRCHILD SEMICONDUCTOR™

NPN General Purpose Amplifier

This device is designed as a general purpose amplifier and switch. The useful dynamic range extends to 100 mA as a switch and to 100 MHz as an amplifier.

Absolute Maximum Ratings* T_A = 25°C unless otherwise noted

Symbol	Parameter	Value	Units
V_{CEO}	Collector-Emitter Voltage	40	V
V_{CBO}	Collector-Base Voltage	60	V
V_{EBO}	Emitter-Base Voltage	6.0	V
I_C	Collector Current - Continuous	200	mA
T_J, T_{stg}	Operating and Storage Junction Temperature Range	-55 to +150	°C

BJT Manufacturing Data Sheets (cont)

Electrical Characteristics $T_A = 25°C$ unless otherwise noted

Symbol	Parameter	Test Conditions	Min	Max	Units
OFF CHARACTERISTICS					
$V_{(BR)CEO}$	Collector-Emitter Breakdown Voltage	I_C = 1.0 mA, I_B = 0	40		V
$V_{(BR)CBO}$	Collector-Base Breakdown Voltage	I_C = 10 μA, I_E = 0	60		V
$V_{(BR)EBO}$	Emitter-Base Breakdown Voltage	I_E = 10 μA, I_C = 0	6.0		V
I_{BL}	Base Cutoff Current	V_{CE} = 30 V, V_{EB} = 3V		50	nA
I_{CEX}	Collector Cutoff Current	V_{CE} = 30 V, V_{EB} = 3V		50	nA
ON CHARACTERISTICS*					
h_{FE}	DC Current Gain	I_C = 0.1 mA, V_{CE} = 1.0 V I_C = 1.0 mA, V_{CE} = 1.0 V I_C = 10 mA, V_{CE} = 1.0 V I_C = 50 mA, V_{CE} = 1.0 V I_C = 100 mA, V_{CE} = 1.0 V	40 70 100 60 30	 300 	
$V_{CE(sat)}$	Collector-Emitter Saturation Voltage	I_C = 10 mA, I_B = 1.0 mA I_C = 50 mA, I_B = 5.0 mA		0.2 0.3	V V
$V_{BE(sat)}$	Base-Emitter Saturation Voltage	I_C = 10 mA, I_B = 1.0 mA I_C = 50 mA, I_B = 5.0 mA	0.65	0.85 0.95	V V

Dr. J. E. Rayas Sánchez 23

BJT Manufacturing Data Sheets (cont)

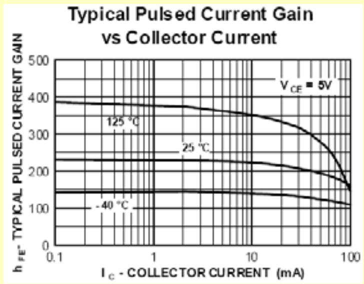
SMALL SIGNAL CHARACTERISTICS					
f_T	Current Gain - Bandwidth Product	I_C = 10 mA, V_{CE} = 20 V, f = 100 MHz	300		MHz
C_{obo}	Output Capacitance	V_{CB} = 5.0 V, I_E = 0, f = 1.0 MHz		4.0	pF
C_{ibo}	Input Capacitance	V_{EB} = 0.5 V, I_C = 0, f = 1.0 MHz		8.0	pF
NF	Noise Figure	I_C = 100 μA, V_{CE} = 5.0 V, R_S =1.0kΩ,f=10 Hz to 15.7kHz		5.0	dB
SWITCHING CHARACTERISTICS					
t_d	Delay Time	V_{CC} = 3.0 V, V_{BE} = 0.5 V,		35	ns
t_r	Rise Time	I_C = 10 mA, I_{B1} = 1.0 mA		35	ns
t_s	Storage Time	V_{CC} = 3.0 V, I_C = 10mA		200	ns
t_f	Fall Time	I_{B1} = I_{B2} = 1.0 mA		50	ns

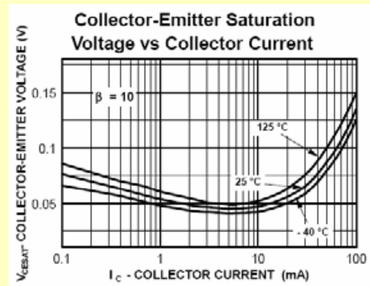
Spice Model

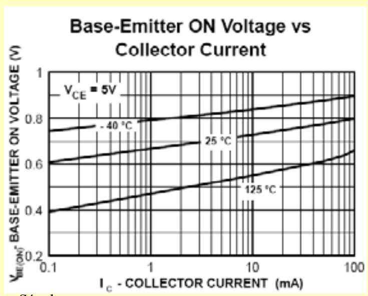
NPN (Is=6.734f Xti=3 Eg=1.11 Vaf=74.03 Bf=416.4 Ne=1.259 Ise=6.734 Ikf=66.78m Xtb=1.5 Br=.7371 Nc=2 Isc=0 Ikr=0 Rc=1 Cjc=3.638p Mjc=.3085 Vjc=.75 Fc=.5 Cje=4.493p Mje=.2593 Vje=.75 Tr=239.5n Tf=301.2p Itf=.4 Vtf=4 Xtf=2 Rb=10)

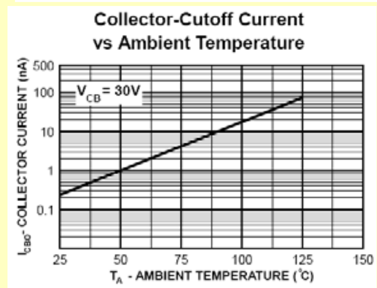
Dr. J. E. Rayas Sánchez 24

BJT Manufacturing Data Sheets (cont)

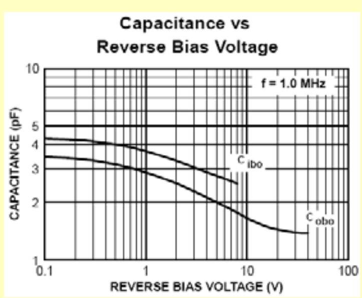


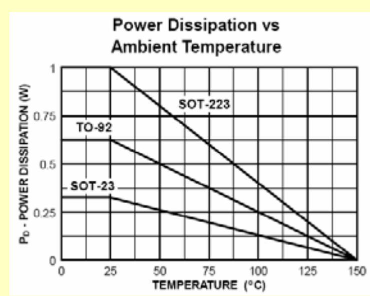




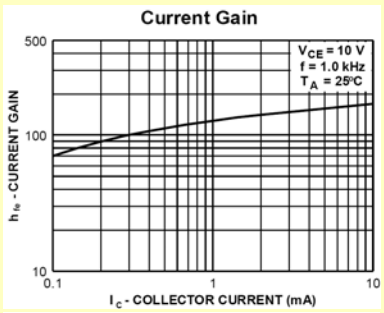


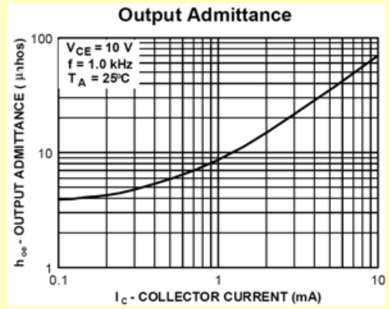
BJT Manufacturing Data Sheets (cont)

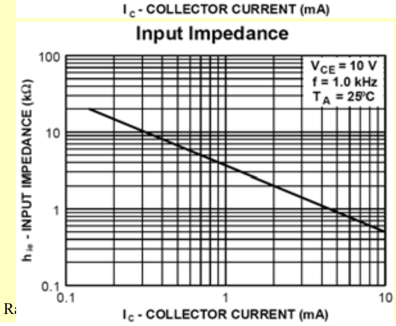


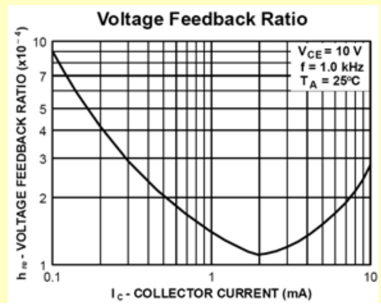


BJT Manufacturing Data Sheets (cont)



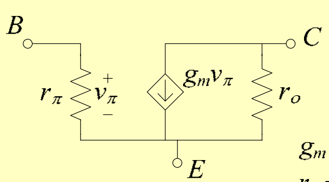






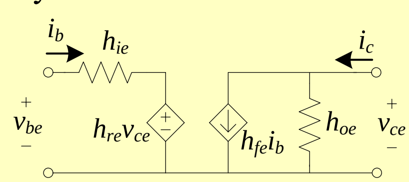
Getting π Parameters from H-Parameters

Hybrid π Model:



$g_m = I_{CQ}/\eta V_T$
$r_\pi = \beta/g_m$
$r_o = V_A/I_{CQ}$

Hybrid-Parameters Model:



$$r_\pi \approx h_{ie}$$

$$g_m \approx h_{fe} / h_{ie}$$

$$r_o = 1/h_{oe} \qquad V_A = I_{CQ}/h_{oe}$$