Statistical Analysis and Yield Calculations

(Part 1)

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1

Outline

- Sources of design-performance uncertainty
- Design and development processes
- Design for manufacturability
- A general formulation to statistical analysis
- Tolerance and acceptability regions
- Probability distributions and tolerances
- Yield definitions

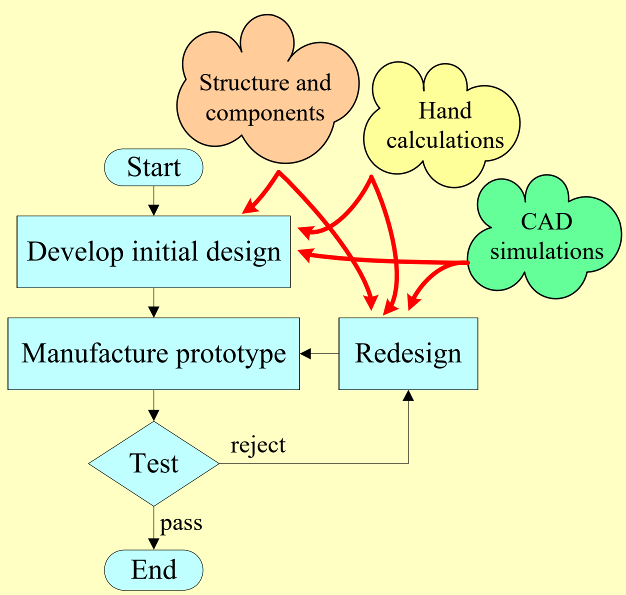
Dr. J. E. Rayas-Sánchez

2

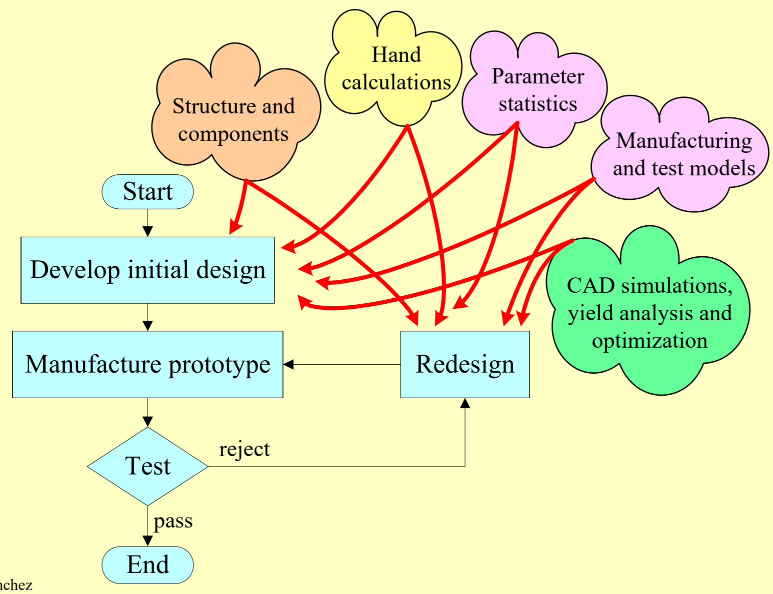
Sources of Design-Performance Uncertainty

- FROM THE DESIGN PROCESS
  Modeling errors
  Numerical errors (simulation errors)
- FROM THE MANUFACTURING PROCESS
  Environmental effects
  Component aging
  Fabrication imprecision
- FROM THE TESTING PROCESS
  Measurement noise
  Calibration errors
  Measurement errors

Typical Design and Development Process

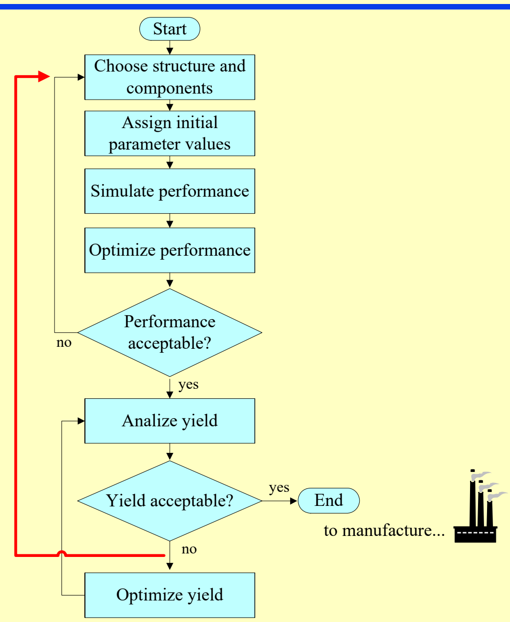


Improving the Design and Development Process



Dr. J. E. Rayas-Sánchez 5

Design for Manufacturability



Dr. J. E. Rayas-Sánchez 6

Achieving a High Yield

- By numerical methods (yield optimization): for a given structure, find a nominal solution that best suit the manufacturing tolerances - **$**
- By developing new structures: find a structure (topology, components, materials) less sensitive to manufacturing tolerances - **$$$$**
- By controlling (increasing precision of) the manufacturing process - **$$$$$$$$$$**

Dr. J. E. Rayas-Sánchez 7

A Formulation to Statistical Analysis

- It is assumed that the circuit topology and the component types are already selected by the designer and are fixed
- $\boldsymbol{y} \in \Re^t$ contains the t parameters of the electronic circuit that are subject to statistical fluctuations
- The parameters of the k-th manufactured device, outcome $\boldsymbol{y}_k$, are actually spread around the nominal point $\boldsymbol{y}$ according to their statistical distributions and tolerances
- These parameters can be represented as

$$\boldsymbol{y}_j = \boldsymbol{y} + \Delta\boldsymbol{y}_j \qquad j = 1, 2, \dots N$$

where N is the number of outcomes, and $\Delta\boldsymbol{y}_j$ represents a random variation for the j-th outcome

Dr. J. E. Rayas-Sánchez 8

A Formulation to Statistical Analysis (cont.)

- The circuit responses are denoted by $\boldsymbol{R}(\boldsymbol{y}) \in \Re^r$ where r is the number of responses of interest
- The statistical analysis of a circuit around a nominal point $\boldsymbol{y}$ consists of realizing N simulations of $\boldsymbol{R}(\boldsymbol{y}_j)$ for j = 1 to N (also called Monte Carlo analysis)
- The number of outcomes N must be sufficiently large to have statistical significance

Probability Distributions

- The circuit parameters for the j-th outcome at the nominal design $\boldsymbol{y}$ are given by $\boldsymbol{y}_j = \boldsymbol{y} + \Delta\boldsymbol{y}_j$, where $\Delta\boldsymbol{y}_j$ represents a random variation for the j-th outcome
- Each parameter in $\boldsymbol{y}$ follows some probability distribution function (PDF), typically uniform or normal (Gaussian)
- If the i-th parameter y_i follows a probability distribution function p_i, then the probability of y_i to have a value between a and b is

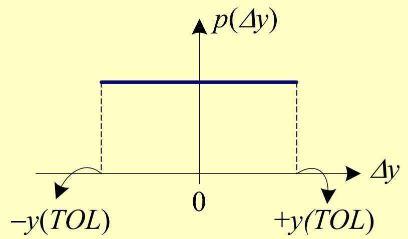
$$p(a \le y_i \le b) = \int_{y_i=a}^{y_i=b} p_i(y_i)dy_i$$

$$p(-\infty \le y_i \le +\infty) = \int_{-\infty}^{\infty} p_i(y_i)dy_i = 1$$

Probability Distributions and Tolerances

- Uniform distribution

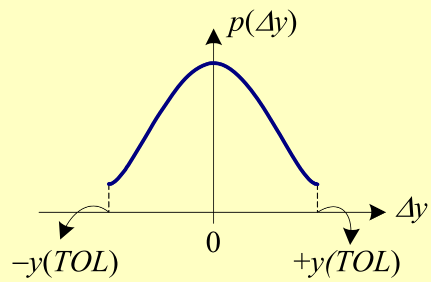
$$p(\Delta y) = C$$



- Gaussian distribution

$$p(\Delta y) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-\Delta y^2}{2\sigma^2}}$$

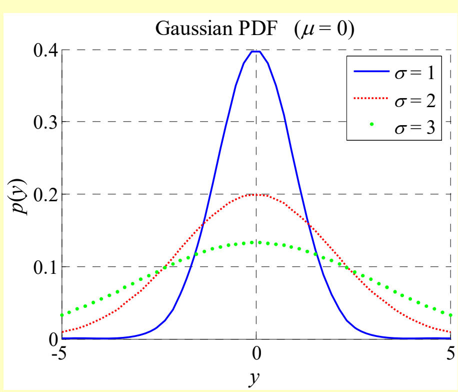
σ^2 : variance
σ : standard deviation

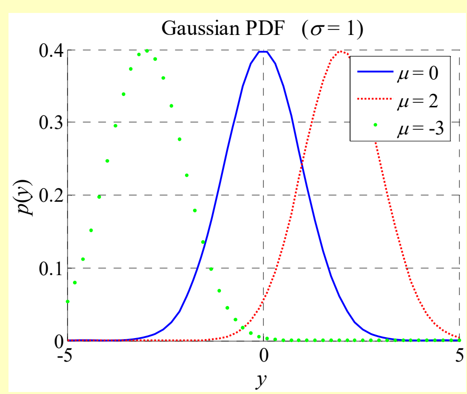


Normal Probability Distribution Function

$$p(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(y-\mu)^2}{2\sigma^2}}$$

μ : average
σ : standard deviation
σ^2 : variance



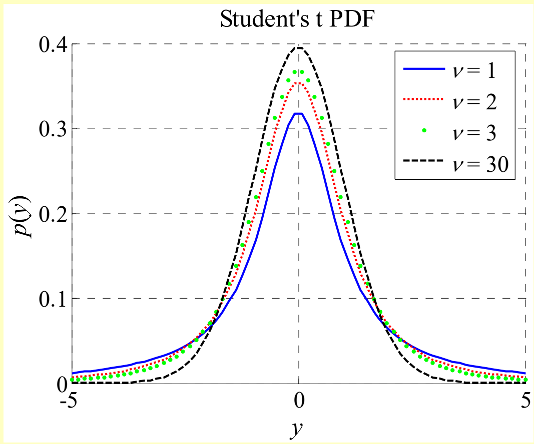


Student's t Probability Distribution Function

$$p(y) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)\left(1+\frac{y^2}{\nu}\right)^{-(\nu+1)/2}}{\sqrt{\nu\pi}\,\Gamma(\nu/2)}$$

ν : degrees of freedom

Γ : gamma function

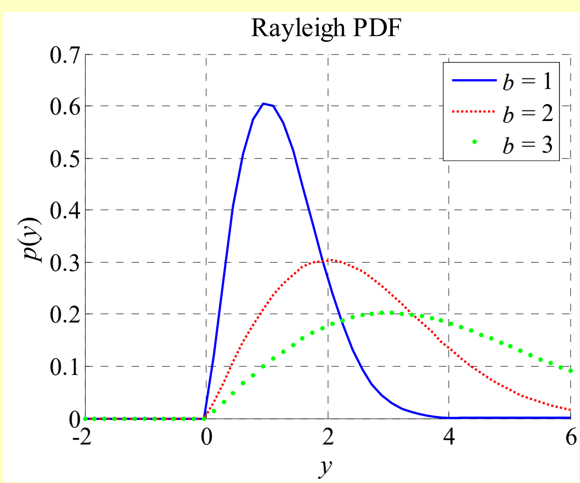


Dr. J. E. Rayas-Sánchez 13

Rayleigh Probability Distribution Function

$$p(y) = yb^{-2}e^{\frac{-y^2}{2b^2}}$$

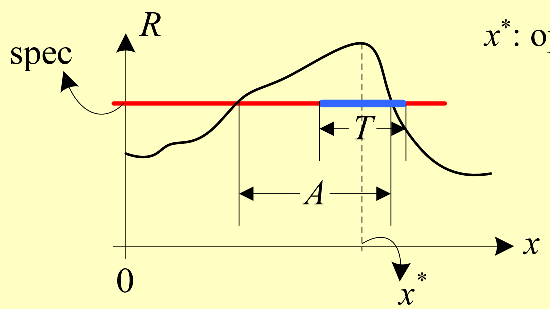
b : center (Rayleigh)



Dr. J. E. Rayas-Sánchez 14

Tolerance and Acceptability Regions

- *T*: Design tolerance region around a point $\boldsymbol{x}$ (depends on all the sources of uncertainty)
- *A*: Design acceptability region for the responses (depends on the design specifications)
- Example:



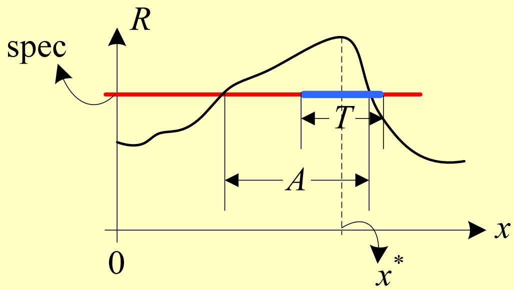
x^*: optimal nominal design

Yield Definitions

- The probability that a manufactured unit (outcome) will pass its performance test
- The probability that a manufactured unit will satisfy all its design specifications
- The ratio of the number of manufactured units which pass performance testing to the total number of units manufactured (in the limit, when the number of units tends to infinity)
- The intersection between the *A* and *T* regions, over *T*, where *A* is the acceptability region and *T* is the tolerance region

Yield Definitions (cont.)

- The intersection between the *A* and *T* regions, over *T*

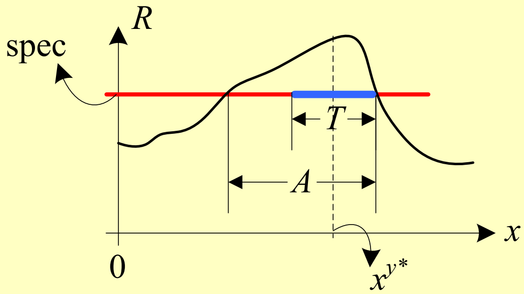


$$Y = \frac{A \cap T}{T}$$

x^* : optimal nominal solution: solution with maximum nominal performance, regardless of yield ($Y \approx 80\%$)

Yield Definitions (cont.)

- The intersection between the *A* and *T* regions, over *T*

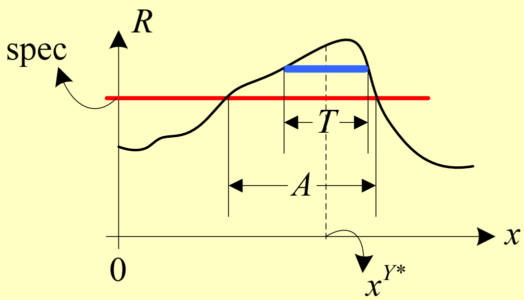


$$Y = \frac{A \cap T}{T}$$

x^{y*} : solution with maximum nominal performance and maximum yield ($Y = 100\%$); zero margin for specs fulfillment

Yield Definitions (cont.)

- The intersection between the *A* and *T* regions, over *T*



$$Y = \frac{A \cap T}{T}$$

x^{Y*} : optimal yield solution: solution with maximum performance on *T* and maximum yield (Y = 100%); non-zero margin for specs fulfillment

Dr. J. E. Rayas-Sánchez 19