

# Basics on High Frequency Circuit Analysis

**Dr. José Ernesto Rayas Sánchez**

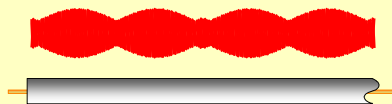
Most of the figures of this presentation were taken from Agilent Technologies Educator's Corner: 1999 RF Design and Measurement Seminar, David Ballo, Joe Civello, Ed Henicle, Sara Meszaros, Andy Potter, Boyd Shaw, My Le Truong

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## Electrical Size

### Low frequencies

- wavelengths  $\gg$  wire length
- current (I) travels down the wires easily for efficient power transmission
- measured voltage and current not dependent on position along the wire



### High frequencies

- wavelength  $\approx$  or  $<$  length of transmission medium
- need transmission lines for efficient power transmission
- matching to characteristic impedance ( $Z_0$ ) is very important for low reflection and maximum power transfer
- measured envelope voltage dependent on position along line

## The Need of Transmission Line Theory

- For analog circuits:

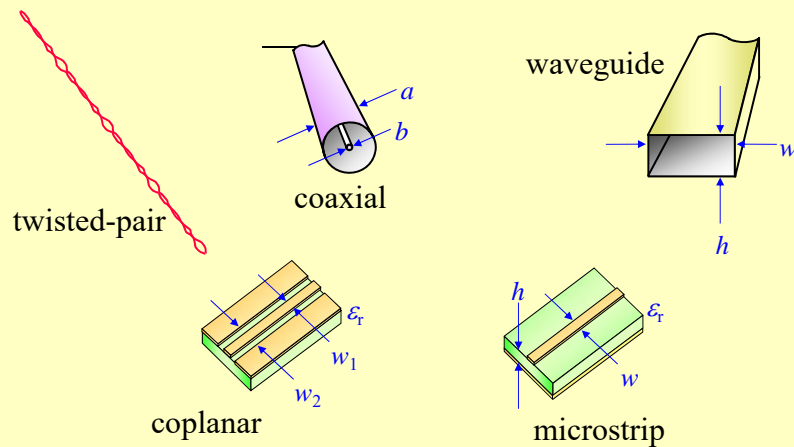
If the physical length of the transmission media is larger 10% of the wavelength of the highest frequency of interest

- For digital circuits:

If the propagation time in the longest transmission path is larger than 10% of the fastest transition time

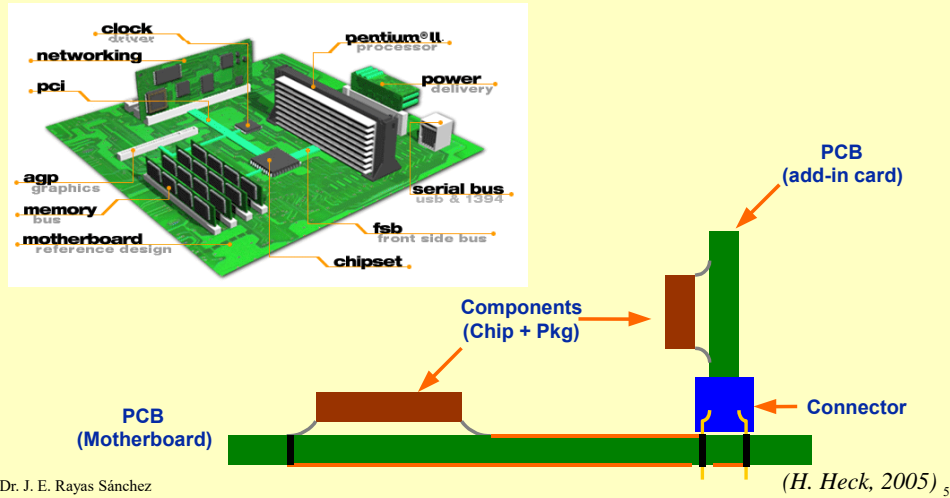
## Common Transmission Media

- Uniform Interconnects



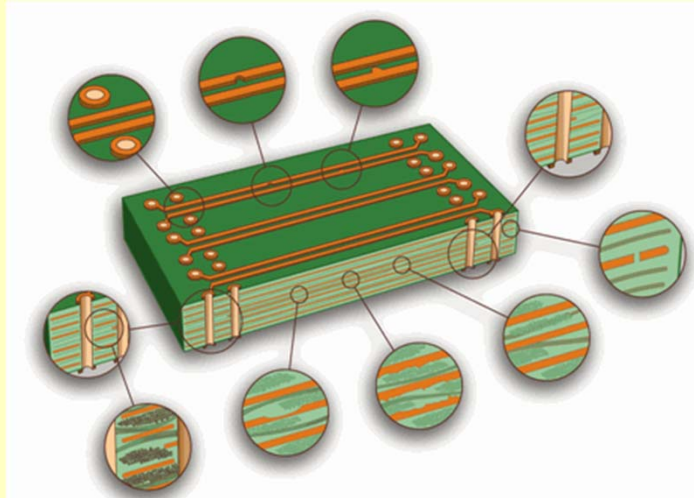
## Common Transmission Media (cont.)

- Practical interconnects can be decomposed in segments of uniform interconnects (usually necessary)



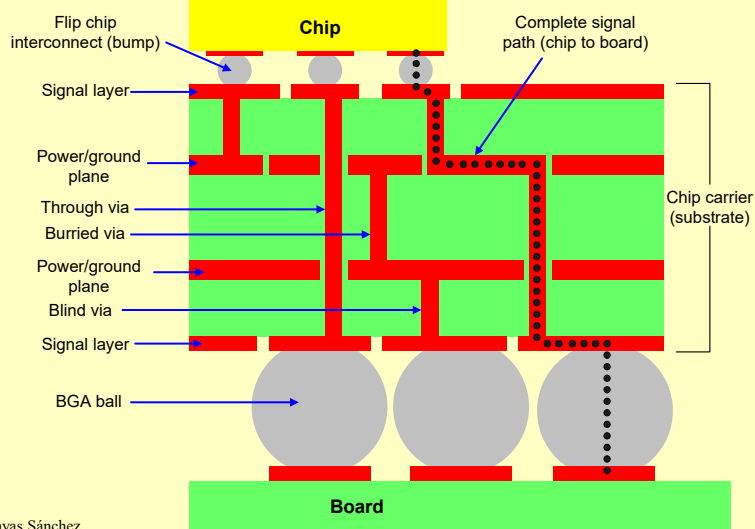
## Common Transmission Media (cont.)

- Practical interconnects have discontinuities and imperfections



## Common Transmission Media (cont.)

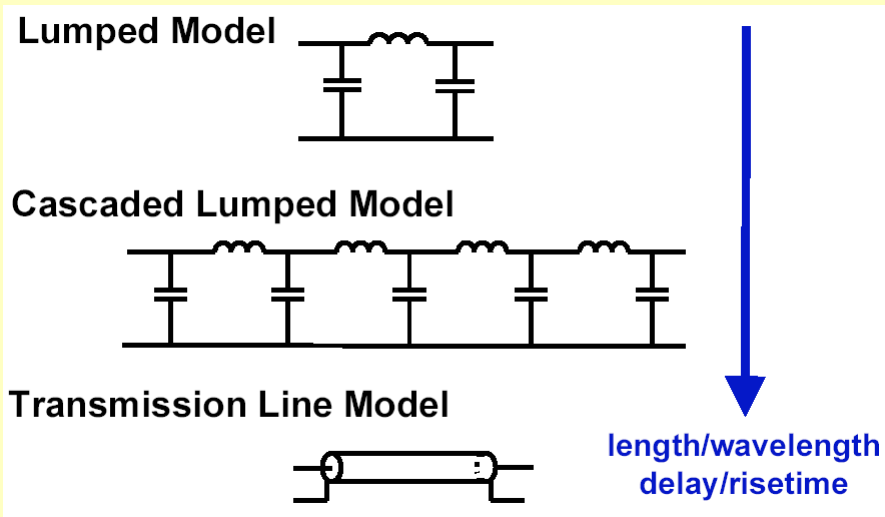
- High-speed BGA (Ball Grid Array) packages



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## From Lumped Circuits to Distributed Circuits

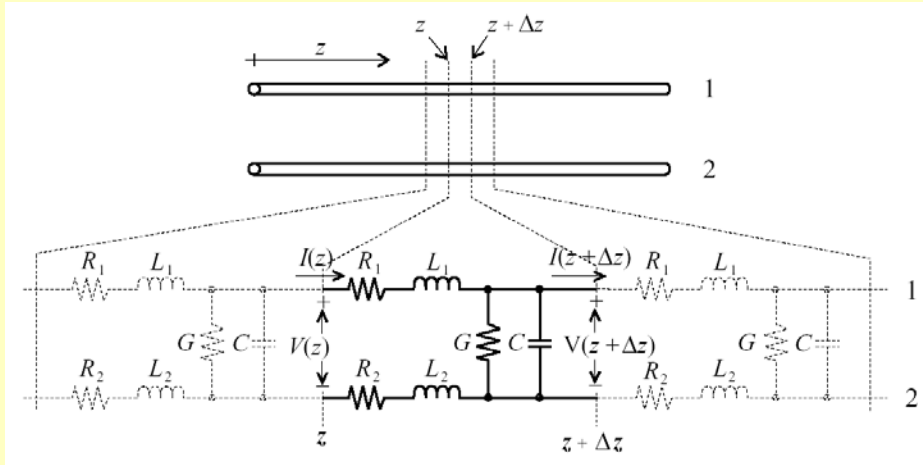


(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

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## Transmission Line Model

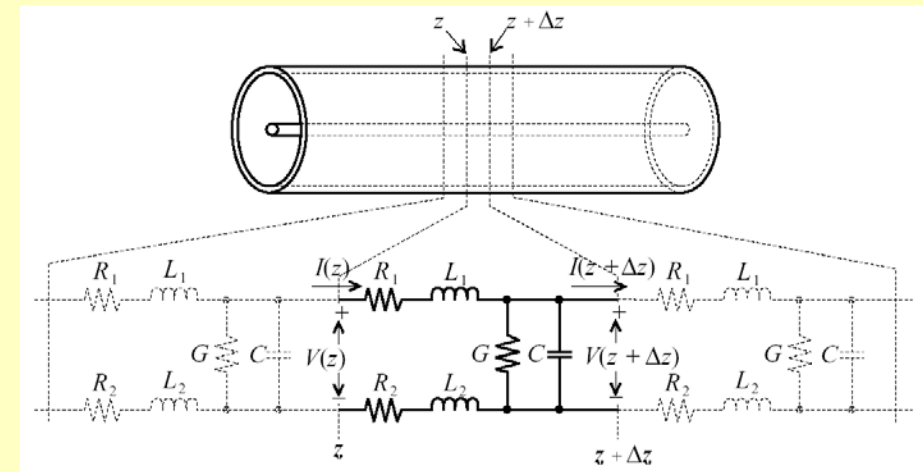


The interconnect is modeled using an infinite number of RLCG sections

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(R. Ludwig and P. Bretchko, *RF Circuit Design*, Prentice Hall, 2000)

## Transmission Line Model

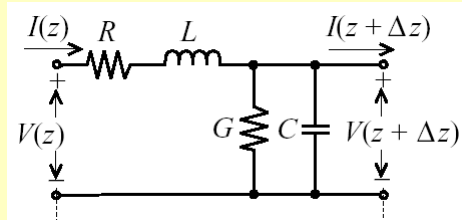


The interconnect is modeled using an infinite number of RLCG sections

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(R. Ludwig and P. Bretchko, *RF Circuit Design*, Prentice Hall, 2000)

## Transmission Line Model



- Generic equivalent circuit for each section ( $R$ ,  $L$ ,  $C$  and  $G$  are per unit length)
- The interconnect is modeled using an infinite number of these sections, making  $\Delta z \rightarrow 0$

## Transmission Line Equations

- Time-Domain (Telegrapher Equations)

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L \frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C \frac{\partial v(z,t)}{\partial t}$$

- Telegrapher Equations in Frequency-Domain

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z)$$

$$\frac{dI(z)}{dz} = -(G + j\omega C)V(z)$$

## Transmission Line Equations (cont.)

- Wave equation (frequency-domain)

$$\frac{d^2V(z)}{dz^2} - \gamma^2V(z) = 0 \qquad \frac{d^2I(z)}{dz^2} - \gamma^2I(z) = 0$$

where  $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \equiv \alpha + j\beta$   
is the complex propagation constant

- Solutions to the wave equation are

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \qquad I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

## Transmission Line Equations (cont.)

- Solutions in the frequency domain

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \qquad I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \equiv \alpha + j\beta$$

- Solutions in the time domain

$$v(z, t) = |V_0^+| \cos(\omega t - \beta z + \phi^+) e^{-\alpha z} + |V_0^-| \cos(\omega t + \beta z + \phi^-) e^{+\alpha z}$$

- Wavelength

$$\lambda = \frac{v_p}{f} = \frac{2\pi}{\beta}$$

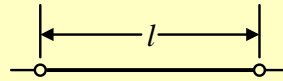
- Phase velocity, wave velocity or propagation speed

$$v_p = \frac{dz}{dt} = \frac{\omega}{\beta}$$

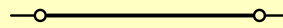
(speed at which a constant phase point travels down the line)

## Transmission Line Symbol

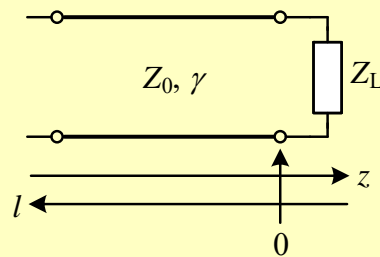
- (Lossy) Transmission line



$Z_0, \gamma$



- Length along the line



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## Characteristic Impedance

- Characteristic impedance of the TL

$$Z_0 \equiv \frac{V_0^+}{I_0^+} = \frac{V_0^-}{-I_0^-}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

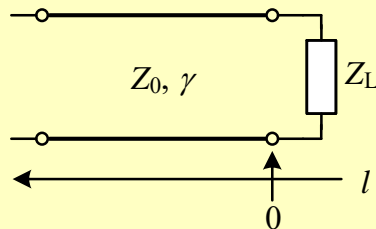
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## Reflection Coefficient

- Reflection coefficient along the line,  $\Gamma_l$



$$\Gamma_l(l) = \frac{V_0^- e^{-\gamma l}}{V_0^+ e^{+\gamma l}} = \frac{V_0^-}{V_0^+} e^{-2\gamma l}$$

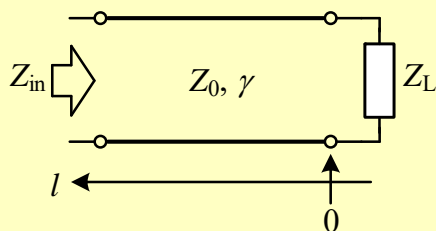
$$\Gamma_l(l) = \frac{-I_0^- e^{-\gamma l}}{I_0^+ e^{+\gamma l}} = \frac{-I_0^-}{I_0^+} e^{-2\gamma l}$$

- Reflection coefficient at the load,  $\Gamma$

$$\Gamma = \Gamma_l(l=0) = \frac{V_0^-}{V_0^+} = \frac{-I_0^-}{I_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

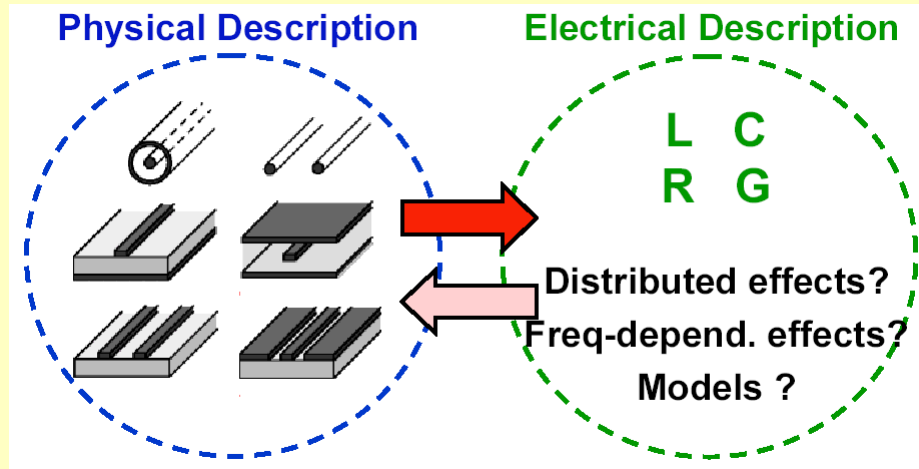
## Input Impedance

- Input impedance along the line



$$Z_{in}(l) = \frac{V(l)}{I(l)} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}$$

## Modeling Uniform Interconnects



(A. Weisshaar, *Tutorial on High-Speed Interconnects*, IMS June 2004, Fort Worth, TX)

## Modeling Uniform Interconnects (cont.)

**Cross-sectional** view of typical **uniform** interconnects:

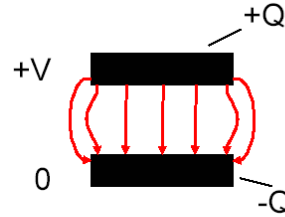


- Parasitic effects associated to each transmission media
  - Capacitance between conductors,  $C$
  - Resistance of conductors (conductor losses),  $R$
  - Inductance of conductor loops,  $L$
  - Dielectric conductivity (dielectric losses),  $G$
- $R$ ,  $C$ ,  $L$ , and  $G$  must be determined per unit length

## Interconnect Shunt Capacitance

- capacitance

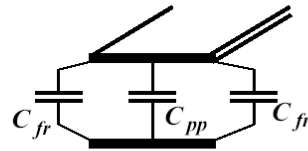
$$C = Q/V$$



- parallel-plate capacitor and field-fringing effect

$$C_{pp}/l = \epsilon_0 \epsilon_r \frac{w}{d}$$

( $\epsilon_0 = 8.854 \times 10^{-12}$  F/m)



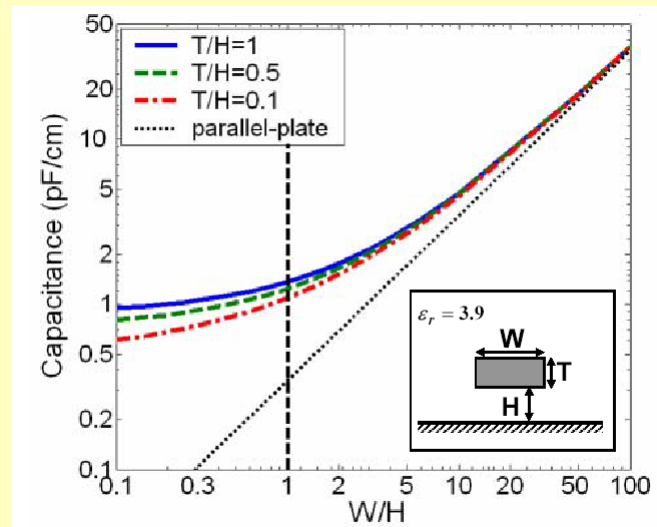
- Actual capacitance is **larger** than parallel-plate capacitance due to fringing fields

(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

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## Fringing Capacitance Effect



(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

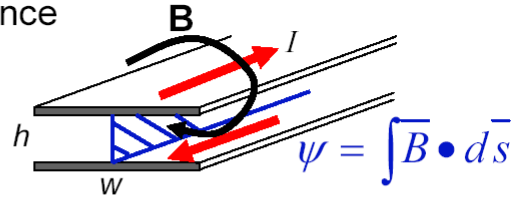
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## Interconnect Series Inductance

- **External Inductance**

$$\frac{L}{l} = \frac{\psi}{I}$$



- parallel-plate inductance  $\frac{L_{pp}}{l} = \mu_0 \frac{h}{w}$  ( $\mu_0 = 4\pi \times 10^{-7}$  H/m)
- closely spaced return path means smaller inductance
- **Internal Inductance**
  - associated with flux inside conductors

(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

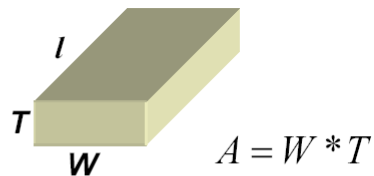
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## Interconnect Series DC Resistance

- DC resistance

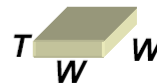
$$\frac{R}{l} = \frac{\rho}{A}$$



- Sheet resistance

$$R = \rho l / A = \rho \frac{W}{WT} = \rho / T$$

$$R_s = \rho / T \text{ (ohms per square)}$$



(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

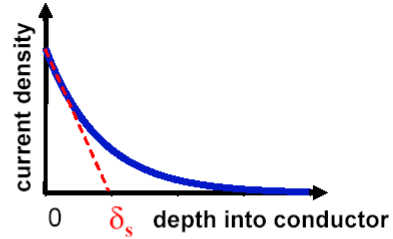
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## Skin Effect

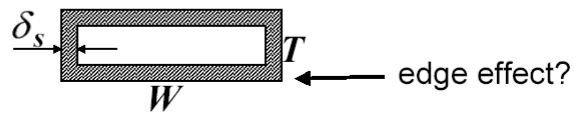
- Skin depth in conductor

$$\delta_s = \sqrt{\frac{\rho}{\pi f \mu_0}} = \frac{1}{\sqrt{\pi f \sigma \mu_0}}$$



→ for copper at 1 GHz:  $\delta_s \cong 2.1 \mu\text{m}$ ; at 10 GHz:  $\delta_s \cong 0.7 \mu\text{m}$

- High-frequency approximation ( $W, T \gg \delta_s$ )



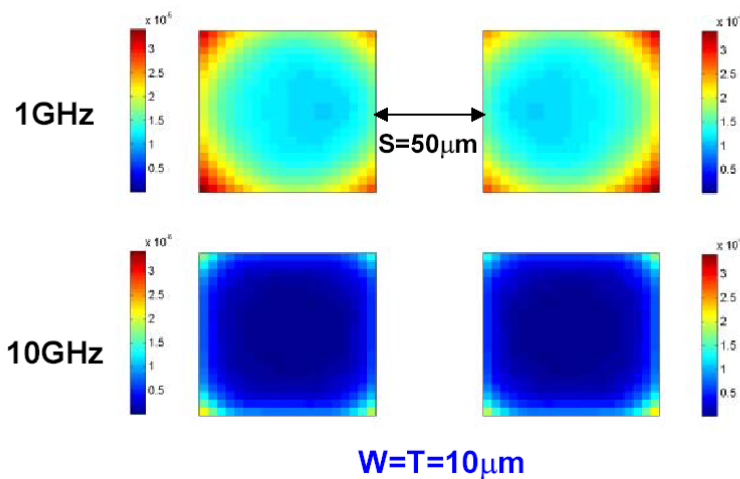
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## EM-Simulation of Conductor Current Distribution

(with currents in same direction)



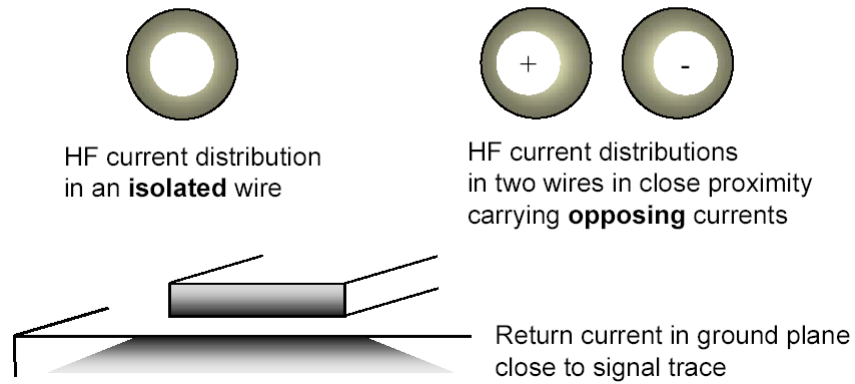
(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

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## Proximity Effect

- Opposing high frequency (HF) currents in close proximity are drawn to each other

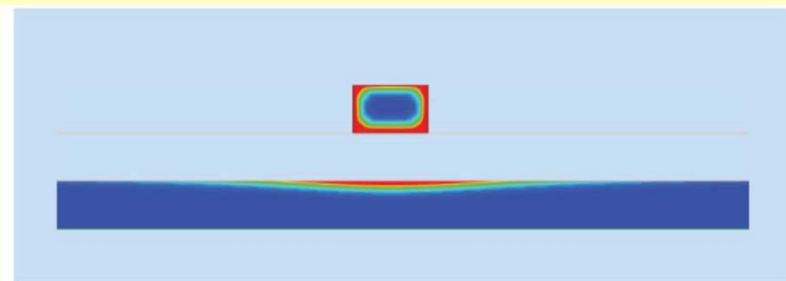


(A. Weisshaar, *Tutorial on High-Speed Interconnects*, IMS June 2004, Fort Worth, TX)

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## Proximity and Skin Effects



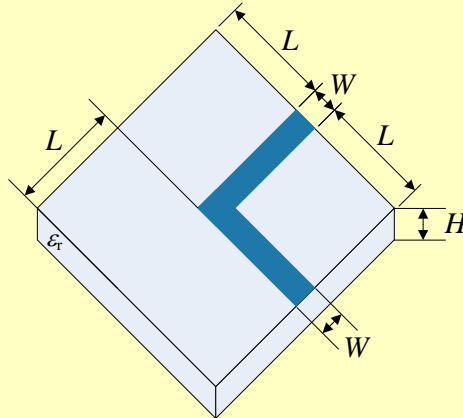
**Figure 9.** Current distribution in the signal and return conductors at 100 MHz for copper traces is driven by two forces: current within each conductor wants to spread out as much as possible, while the signal and return current want to get as close together as possible.

E. Bogatin, "Essential principles of signal integrity," *IEEE Microwave Magazine*, vol. 12, pp. 34-41, Aug. 2011.

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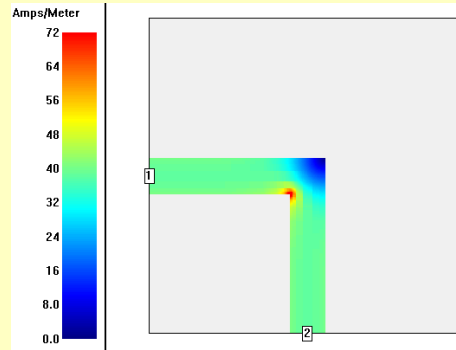
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## Edge and Indy Effects – Example

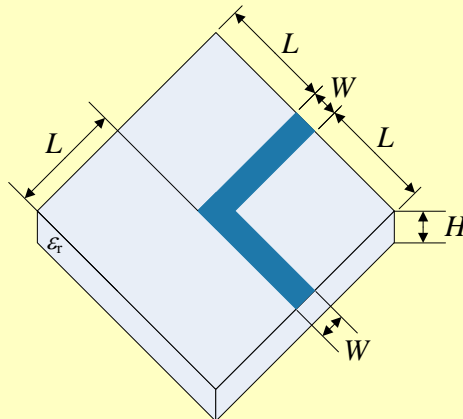


$W = 10 \text{ mil}$        $L = 40 \text{ mil}$   
 $H = 5 \text{ mil}$        $\epsilon_r = 4.2$   
dielectric loss  $\tan = 0.017$   
PCL-FR-226 Polyclad Laminates

EM simulation using Sonnet  
At 1 MHz:

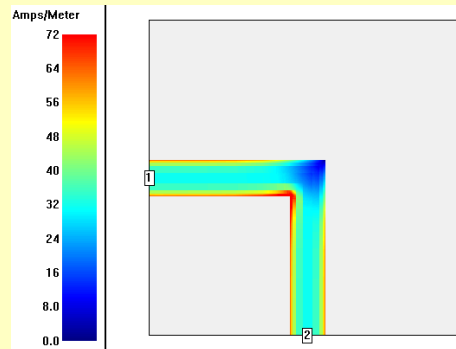


## Edge and Indy Effects – Example (cont.)

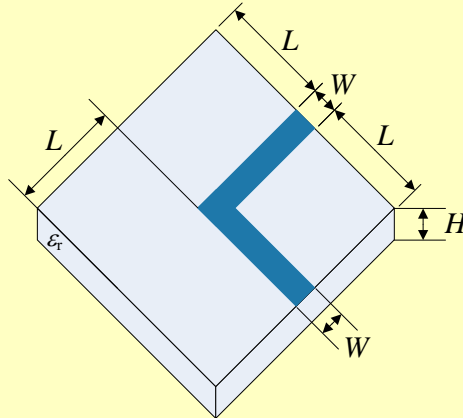


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EM simulation using Sonnet  
At 10 MHz:

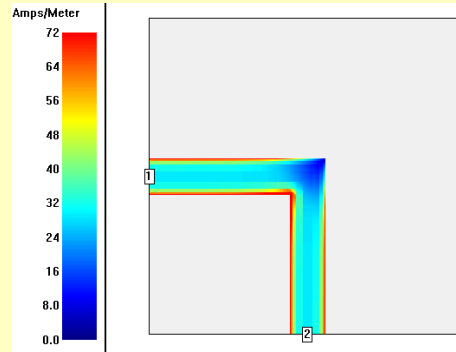


## Edge and Indy Effects – Example (cont.)

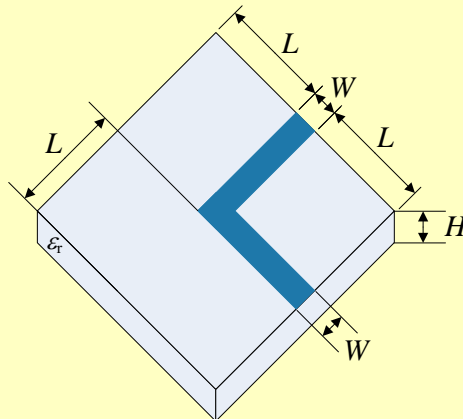


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PCL-FR-226 Polyclad Laminates

EM simulation using Sonnet  
At 100 MHz:

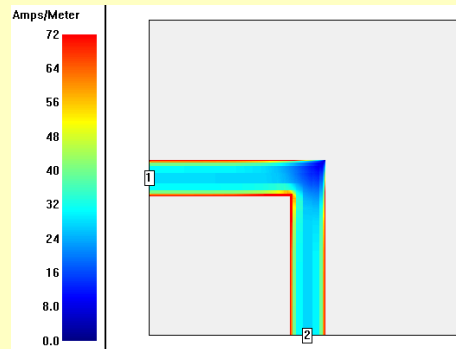


## Edge and Indy Effects – Example (cont.)



$W = 10 \text{ mil}$        $L = 40 \text{ mil}$   
 $H = 5 \text{ mil}$        $\epsilon_r = 4.2$   
dielectric loss  $\tan = 0.017$   
PCL-FR-226 Polyclad Laminates

EM simulation using Sonnet  
At 1 GHz:

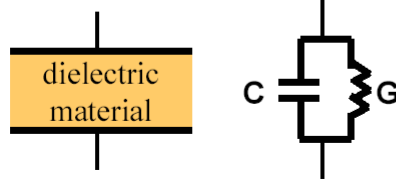




## Interconnect Shunt Conductance

- shunt loss due to
  - ohmic losses (free charge carriers)
  - out-of-phase polarization (power loss due to frictional damping forces)

- simple model



loss tangent:

$$\begin{aligned} \tan \delta_d &= G / \omega C \\ &= \epsilon'' / \epsilon' \\ \text{where } \epsilon &= \epsilon' - j\epsilon'' \end{aligned}$$

(A. Weisshaar, Tutorial on High-Speed Interconnects, IMS June 2004, Fort Worth, TX)

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## Loss Tangent of Typical Materials

Dielectric Material	Loss Tangent	Dielectric Constant
Ceramic (Alumina)	0.001	9.4
Glass-epoxy	0.03	4.0
Glass (Quartz)	0.00006	3.8
Polyimide	0.01	3.5
Silicon (100 Ω-cm)	0.51	11.8
Silicon (10 Ω-cm)	5.1	11.8
Teflon	0.00015	2.1

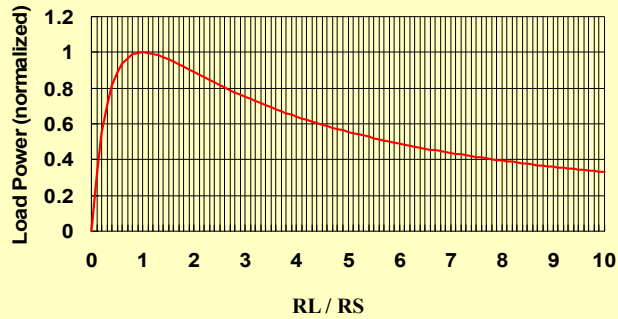
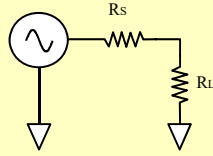
at 3GHz

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## Power Transfer Efficiency



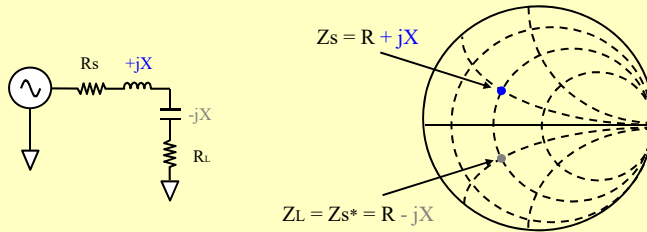
Maximum power is transferred when  $R_L = R_S$

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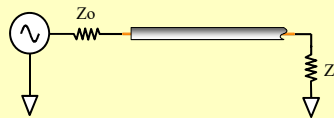
(Hewlett-Packard's RF Design and Measurement Seminar, 2000)<sub>35</sub>

## Power Transfer Efficiency (cont.)

For complex impedances, maximum power transfer occurs when  $Z_L = Z_S^*$  (conjugate match)



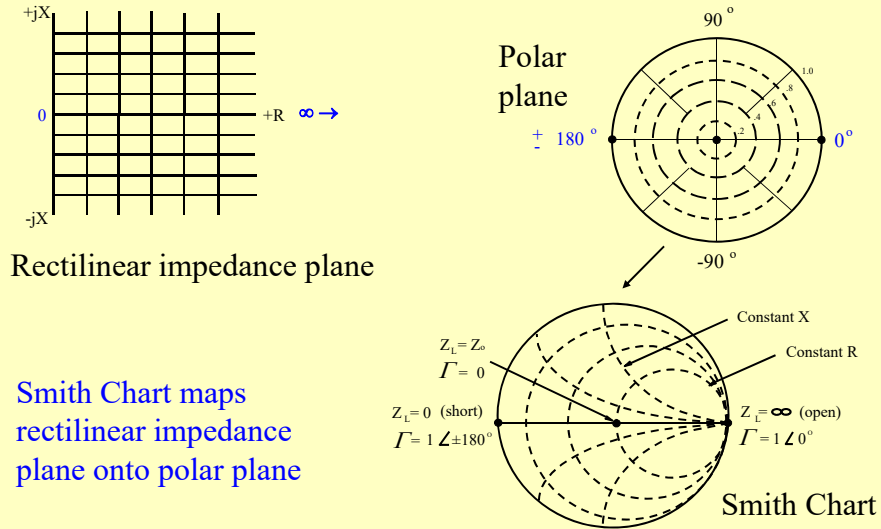
At high frequencies, maximum power transfer occurs when  $R_S = R_L = Z_0$



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(Hewlett-Packard's RF Design and Measurement Seminar, 2000)<sub>36</sub>

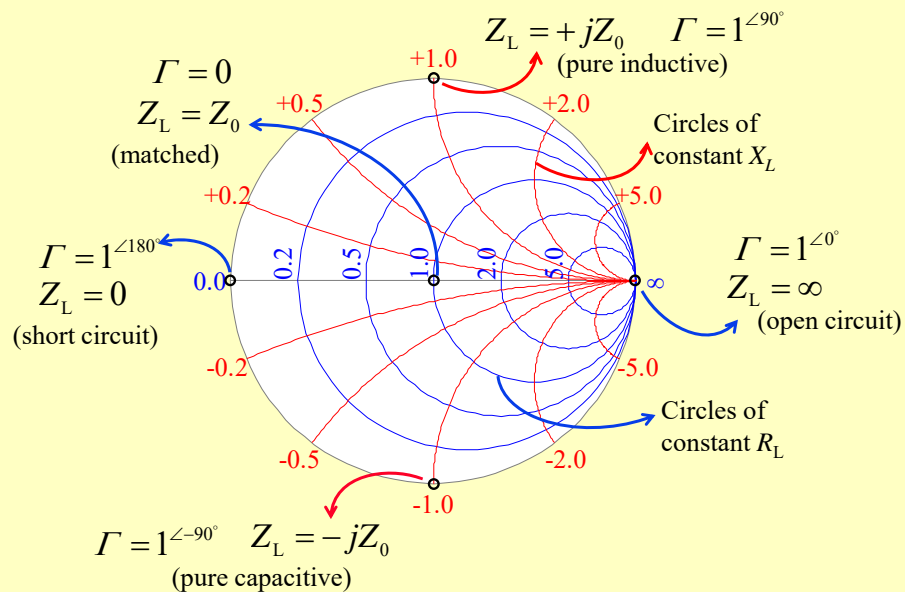
## The Smith Chart



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(Hewlett-Packard's RF Design and Measurement Seminar, 2000)<sub>37</sub>

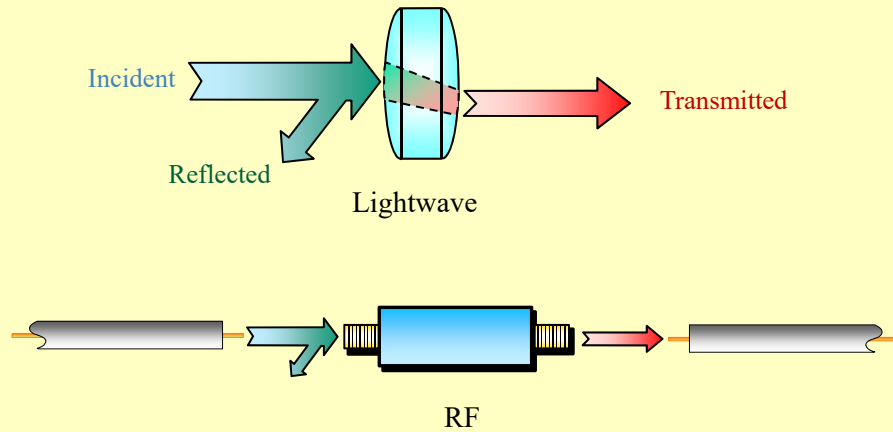
## The Smith Chart – Interpretation



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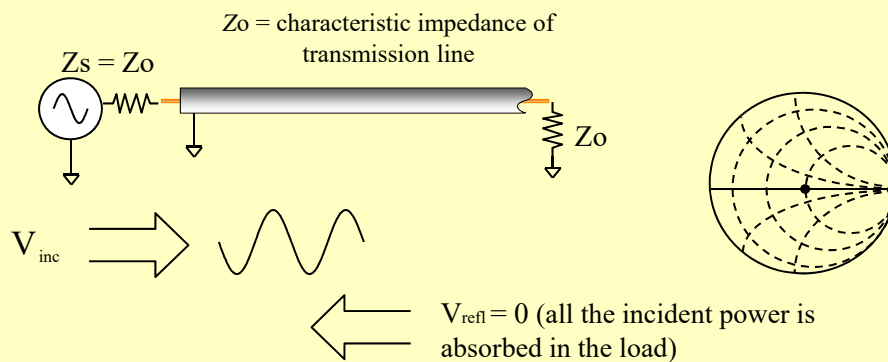
## Lightwave Analogy to RF Energy



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(Hewlett-Packard's RF Design and Measurement Seminar, 2000)<sub>39</sub>

## Transmission Line Terminated with $Z_0$

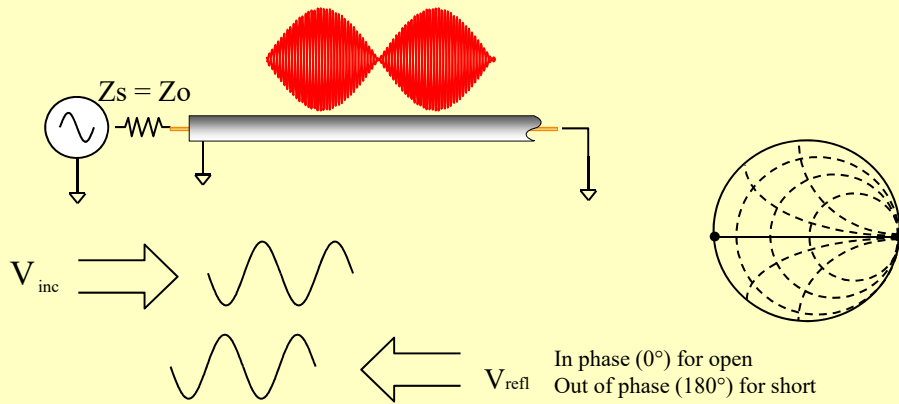


For reflection, a transmission line terminated in  $Z_0$  behaves like an infinitely long transmission line

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(Hewlett-Packard's RF Design and Measurement Seminar, 2000)<sub>40</sub>

## Transmission Line Terminated with Short, Open

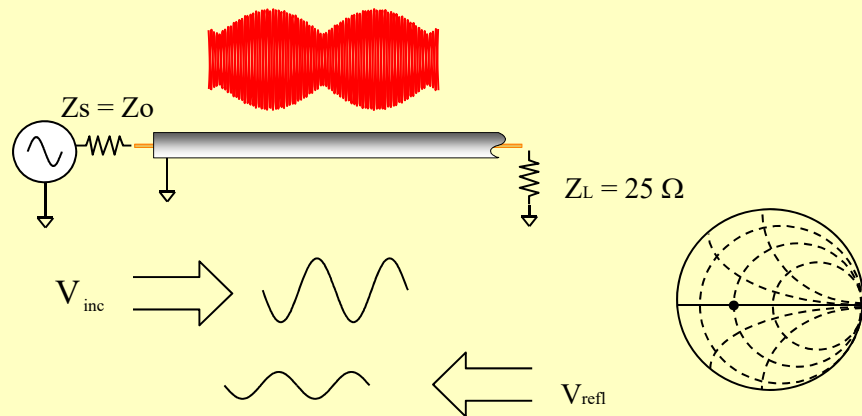


For reflection, a transmission line terminated in a short or open reflects all power back to source

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(Hewlett-Packard's RF Design and Measurement Seminar, 2000)<sub>41</sub>

## Transmission Line Terminated with $25 \Omega$



Standing wave pattern does not go to zero as with short or open

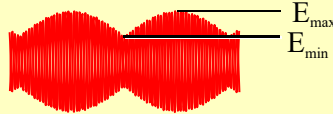
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(Hewlett-Packard's RF Design and Measurement Seminar, 2000)<sub>42</sub>

## Reflection Parameters

Reflection Coefficient  $\Gamma = \frac{V_{\text{reflected}}}{V_{\text{incident}}} = \rho \angle \Phi = \frac{Z_L - Z_0}{Z_L + Z_0}$

Return loss =  $-20 \log(\rho)$ ,  $\rho = |\Gamma|$



Standing Wave Ratio

$$SWR = \frac{E_{\text{max}}}{E_{\text{min}}} = \frac{1 + \rho}{1 - \rho}$$

No reflection  
( $Z_L = Z_0$ )

Full reflection  
( $Z_L = \text{open, short}$ )

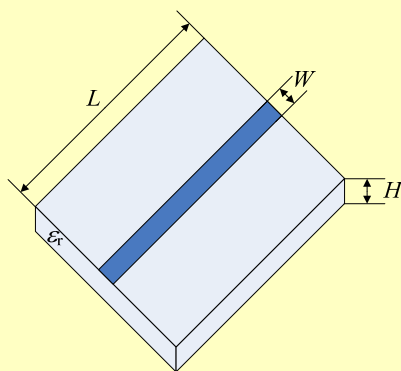
0	$\rho$	1
$\infty$ dB	RL	0 dB
1	SWR	$\infty$

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(Hewlett-Packard's RF Design and Measurement Seminar, 2000)<sub>43</sub>

## Standing Wave Ratio – EM Simulation

Microstrip line



Laminate: Rogers RO3003

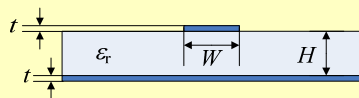
$H = 5$  mil

$\epsilon_r = 3$

$\tan(\delta) = 0.0013$  at 10 GHz

Lossy metals:  $\sigma_{\text{Cu}} = 5.8 \times 10^7$  S/m

$t = 0.6$  mil (half-ounce copper)



$W = 12.5$  mil

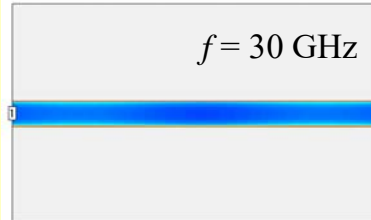
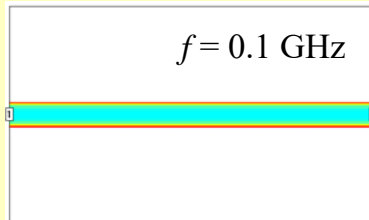
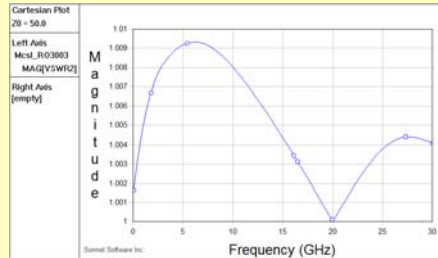
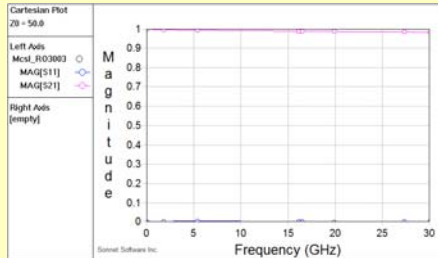
$L = 187.5$  mil

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## Standing Wave Ratio – EM Simulation (cont.)

Using  $Z_{port1} = Z_{port2} = 50 \Omega$

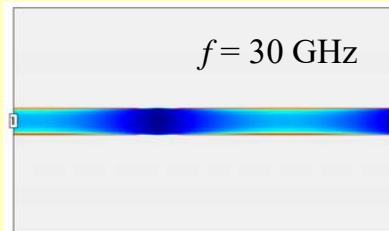
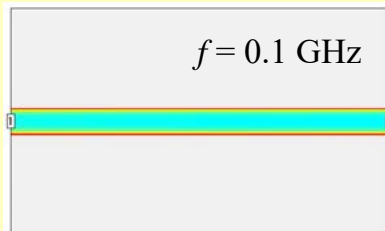
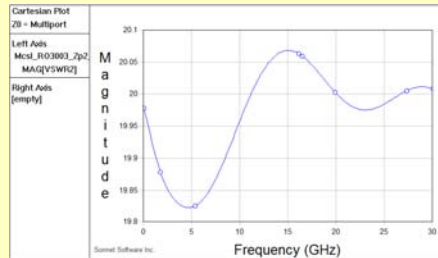
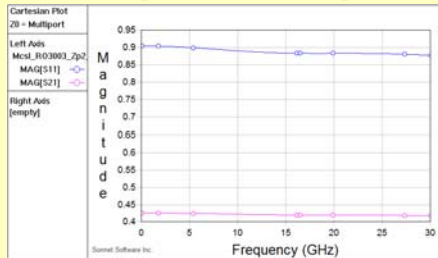


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## Standing Wave Ratio – EM Simulation (cont.)

Using  $Z_{port1} = 50 \Omega$ ,  $Z_{port2} = 1 \text{ K}\Omega$



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## Transmission Parameters



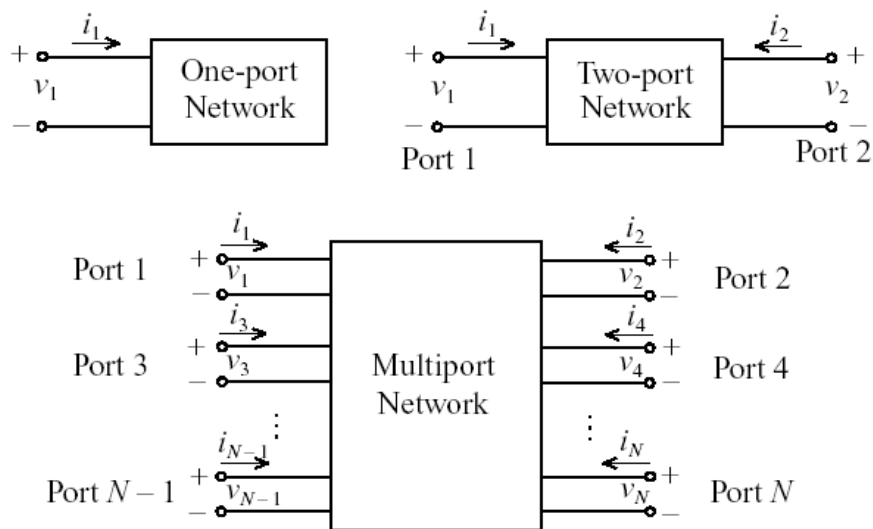
$$\text{Transmission Coefficient} = T = \frac{V_{\text{Transmitted}}}{V_{\text{Incident}}} = \tau \angle \phi$$

$$\text{Insertion Loss (dB)} = -20 \text{ Log} \left| \frac{V_{\text{Trans}}}{V_{\text{Inc}}} \right| = -20 \log \tau$$

$$\text{Gain (dB)} = 20 \text{ Log} \left| \frac{V_{\text{Trans}}}{V_{\text{Inc}}} \right| = 20 \log \tau$$

$$\text{Insertion Phase (deg)} = \angle \frac{V_{\text{Trans}}}{V_{\text{Inc}}} = \phi$$

## N-Ports Networks (Linear Circuits)





## Impedance Matrix Representation ( $\mathbf{Z}$ )

$$\mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \quad \mathbf{I} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} \quad \mathbf{V} = \mathbf{Z}\mathbf{I}$$

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1N} \\ Z_{21} & Z_{22} & \dots & Z_{2N} \\ \vdots & & & \\ Z_{N1} & Z_{N2} & \dots & Z_{NN} \end{bmatrix}$$

Each element of matrix  $\mathbf{Z}$  is given by

$$Z_{ij} = \left. \frac{V_i}{I_j} \right|_{I_k=0 \text{ for } k \neq j}$$

## Admittance Matrix Representation ( $\mathbf{Y}$ )

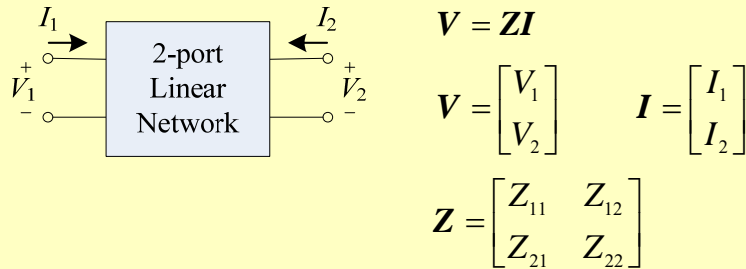
$$\mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \quad \mathbf{I} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} \quad \mathbf{I} = \mathbf{Y}\mathbf{V}$$

$$\mathbf{Y} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1N} \\ Y_{21} & Y_{22} & \dots & Y_{2N} \\ \vdots & & & \\ Y_{N1} & Y_{N2} & \dots & Y_{NN} \end{bmatrix}$$

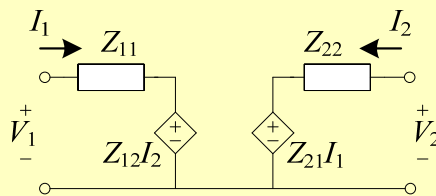
Each element of matrix  $\mathbf{Y}$  is given by

$$Y_{ij} = \left. \frac{I_i}{V_j} \right|_{V_k=0 \text{ for } k \neq j}$$

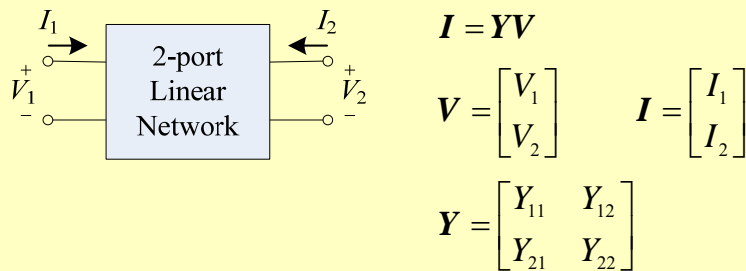
## Z-Parameters for 2-Port Networks



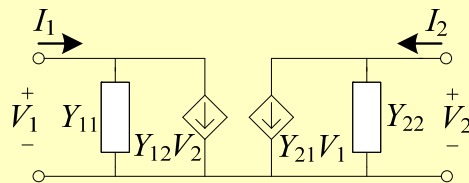
Equivalent circuit:



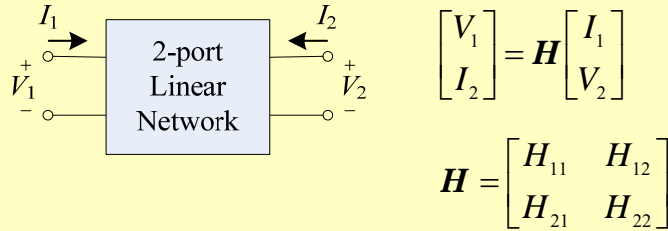
## Y-Parameters for 2-Port Networks



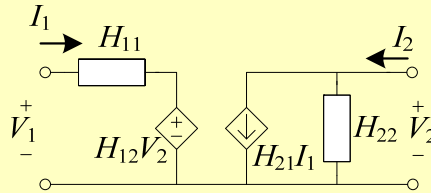
Equivalent circuit:



## H-Parameters (Hybrid) for 2-Port Networks



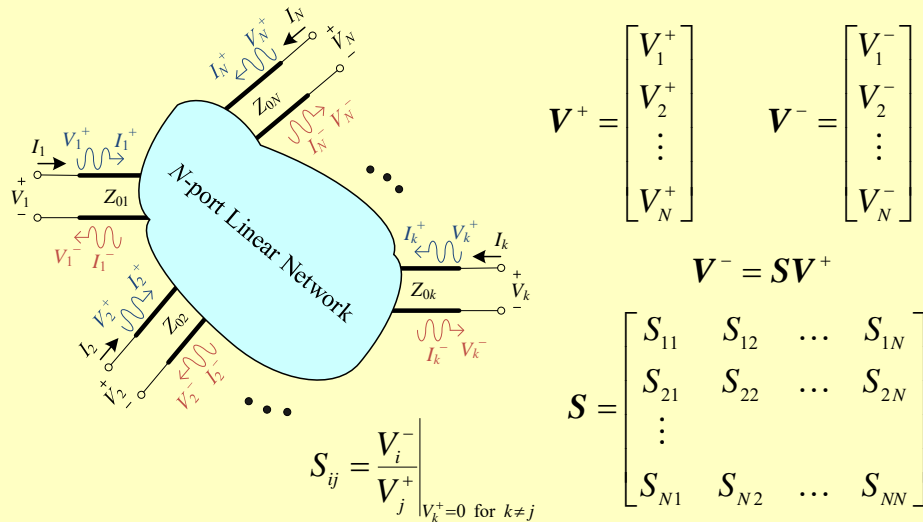
Equivalent circuit:



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## The Scattering Matrix (S)



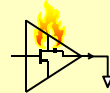
$V_k^+ = 0$  if we terminate port  $k$  with a matched load ( $Z_{Lk} = Z_{0k}$ )

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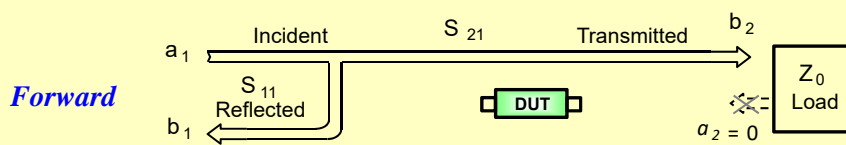
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## The Scattering Matrix Representation (S)

- They can be more easily obtained at high frequencies:
  - Incident and reflected waves can be measured using a Vector Network Analyzer (VNA)
  - They do not require “shorts” or “opens” (active devices might oscillate or self-destroy)
- They are more directly related to high-frequency effects ( $\Gamma$ ,  $T$ ,  $IL$ ,  $RL$ ,  $SWR$ , etc.)
- We can convert back and forth between S, Y and Z parameters



## Measuring S-Parameters

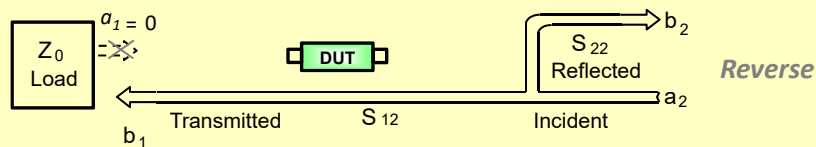


$$S_{11} = \frac{\text{Reflected}}{\text{Incident}} = \frac{b_1}{a_1} \Big|_{a_2 = 0}$$

$$S_{21} = \frac{\text{Transmitted}}{\text{Incident}} = \frac{b_2}{a_1} \Big|_{a_2 = 0}$$

$$S_{22} = \frac{\text{Reflected}}{\text{Incident}} = \frac{b_2}{a_2} \Big|_{a_1 = 0}$$

$$S_{12} = \frac{\text{Transmitted}}{\text{Incident}} = \frac{b_1}{a_2} \Big|_{a_1 = 0}$$



## Meaning of the S-parameters (cont.)

$S_{11}$  : forward reflection coefficient (input match)

$S_{22}$  : reverse reflection coefficient (output match)

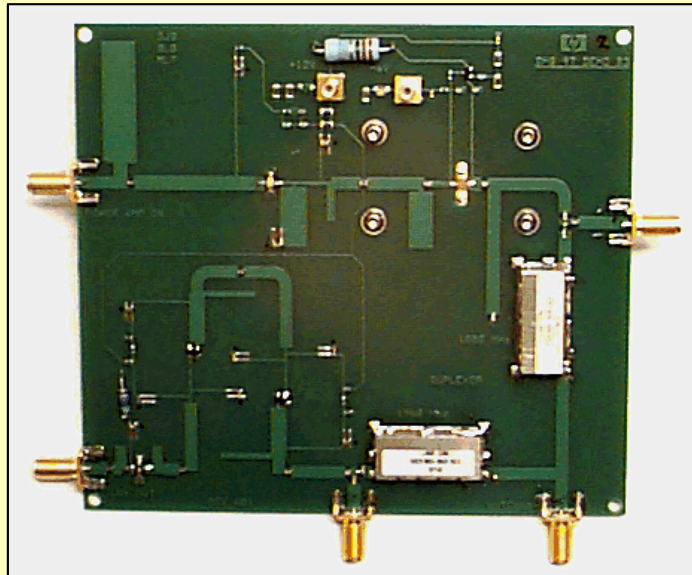
$S_{21}$  : forward transmission coefficient (gain or loss)

$S_{12}$  : reverse transmission coefficient (isolation)

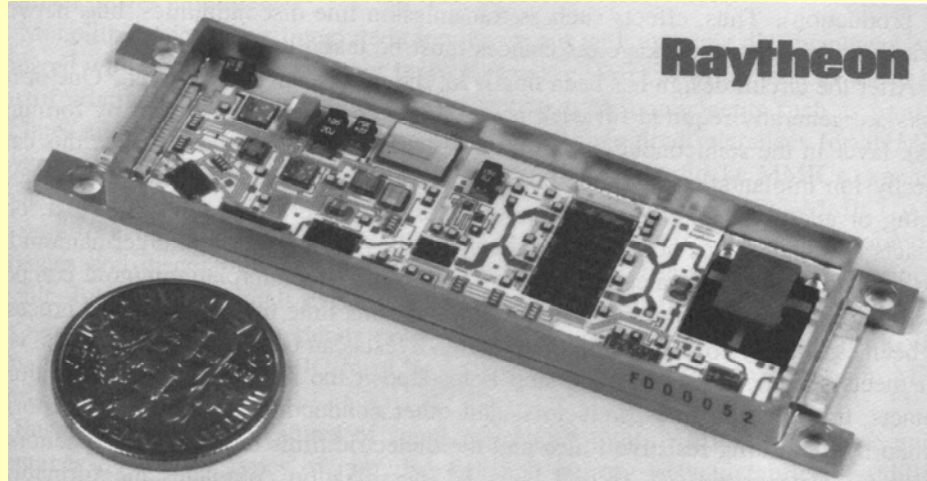
$$S_{ii} \neq \Gamma_i \quad S_{ii} = \left. \frac{V_i^-}{V_i^+} \right|_{V_k^+ = 0 \text{ for } k \neq i} = \Gamma_i \Big|_{V_k^+ = 0 \text{ for } k \neq i}$$

$$S_{ij} \neq T_{ji} \quad S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0 \text{ for } k \neq j} = T_{ji} \Big|_{V_k^+ = 0 \text{ for } k \neq j}$$

## An RF Prototype

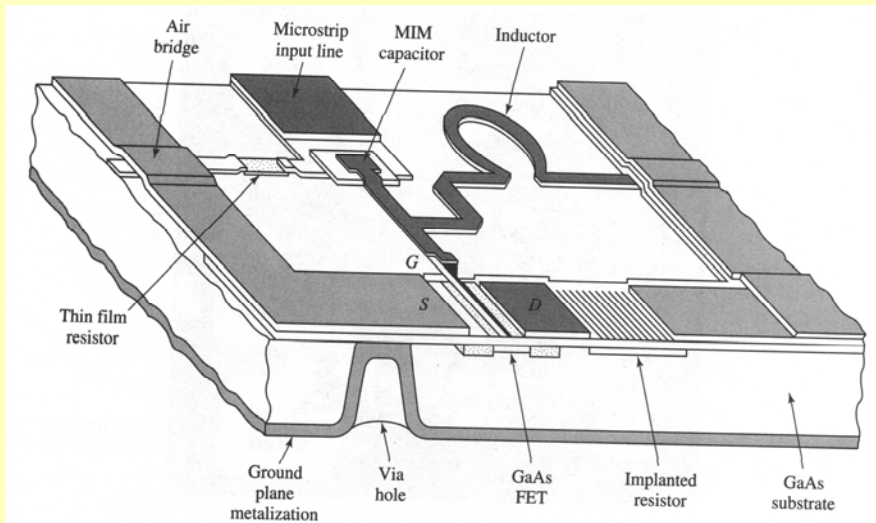


## Hybrid Microwave Integrated Circuits (cont.)



M. Pozar (1998), *Microwave Engineering*. Amherst, MA: John Wiley and Sons.

## Monolithic Microwave Integrated Circuits (MMIC)



M. Pozar (1998), *Microwave Engineering*. Amherst, MA: John Wiley and Sons.

## High Speed Interconnects

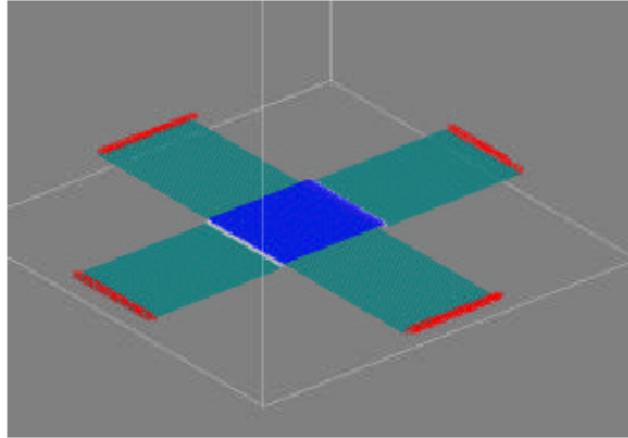


Figure 13. Two 32-bit buses crossing over each other yield a complex 128 port circuit.

J.C. Rautio, *Rigorous Evaluation of Worst Case Total Crosstalk in the Time Domain Using Frequency Domain Scattering Parameters*, 2001 High-Performance System Design Conference

## High Speed Interconnects (cont.)

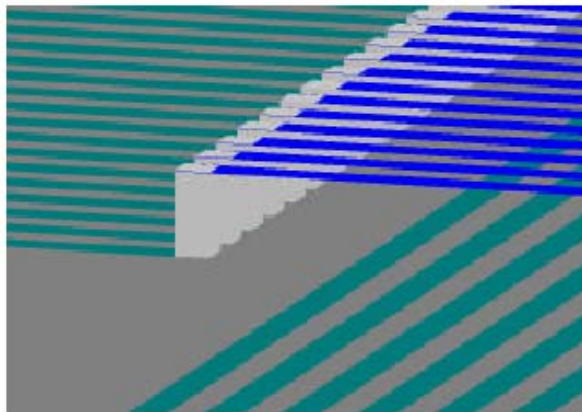


Figure 14. Close-up of the cross-over region of the 32-bit bus cross-over circuit. The Z-axis (vertical) is exaggerated.

J.C. Rautio, *Rigorous Evaluation of Worst Case Total Crosstalk in the Time Domain Using Frequency Domain Scattering Parameters*, 2001 High-Performance System Design Conference