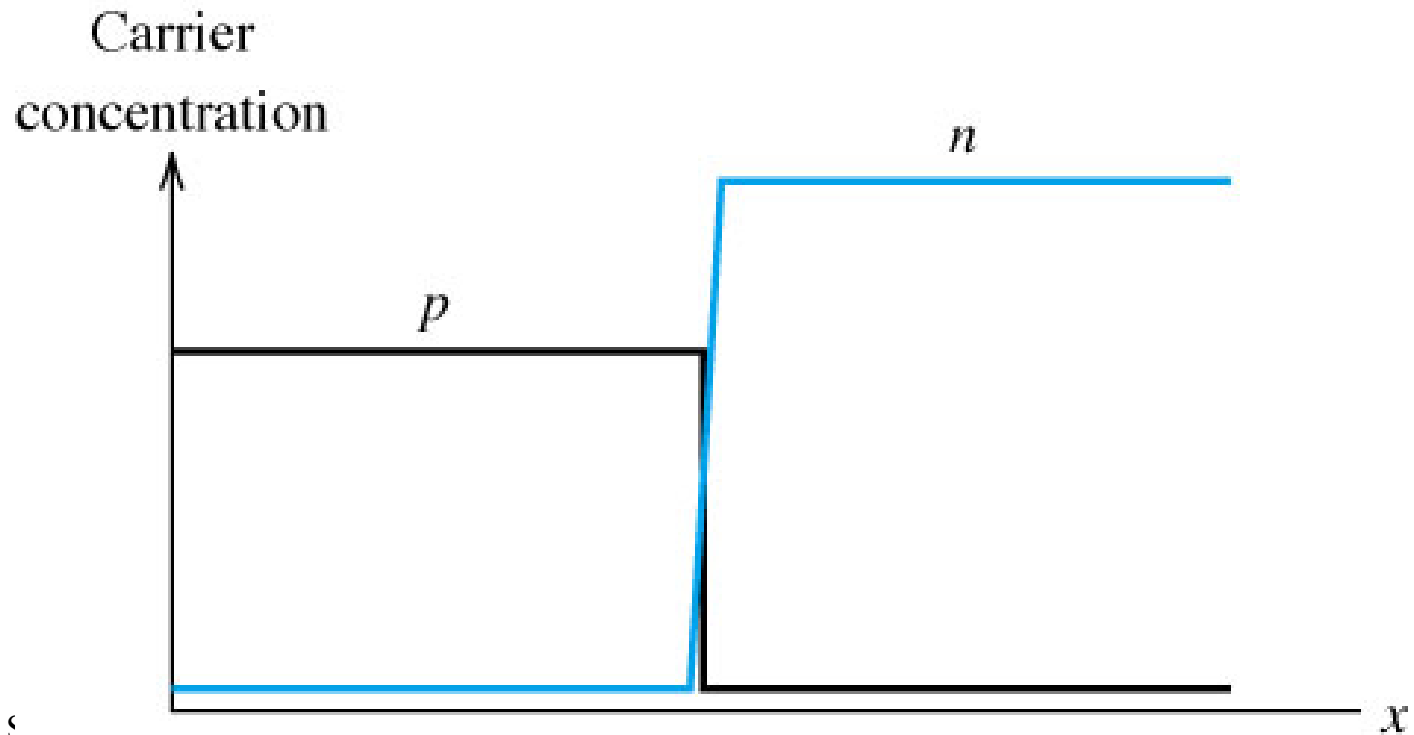
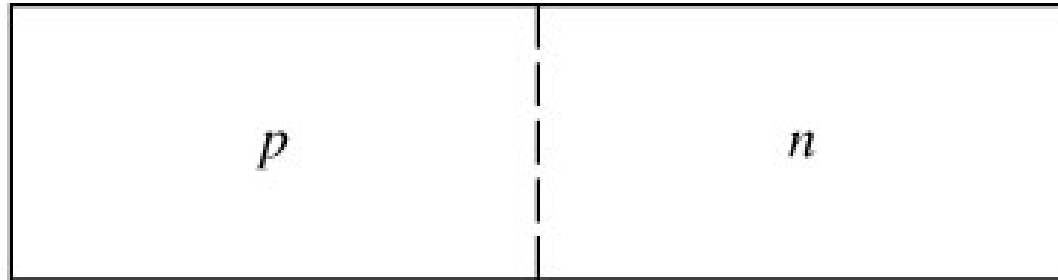

Modelando la Unión P-N

Algunas de las figuras de esta presentación fueron tomadas de las páginas de internet de los autores de los textos:

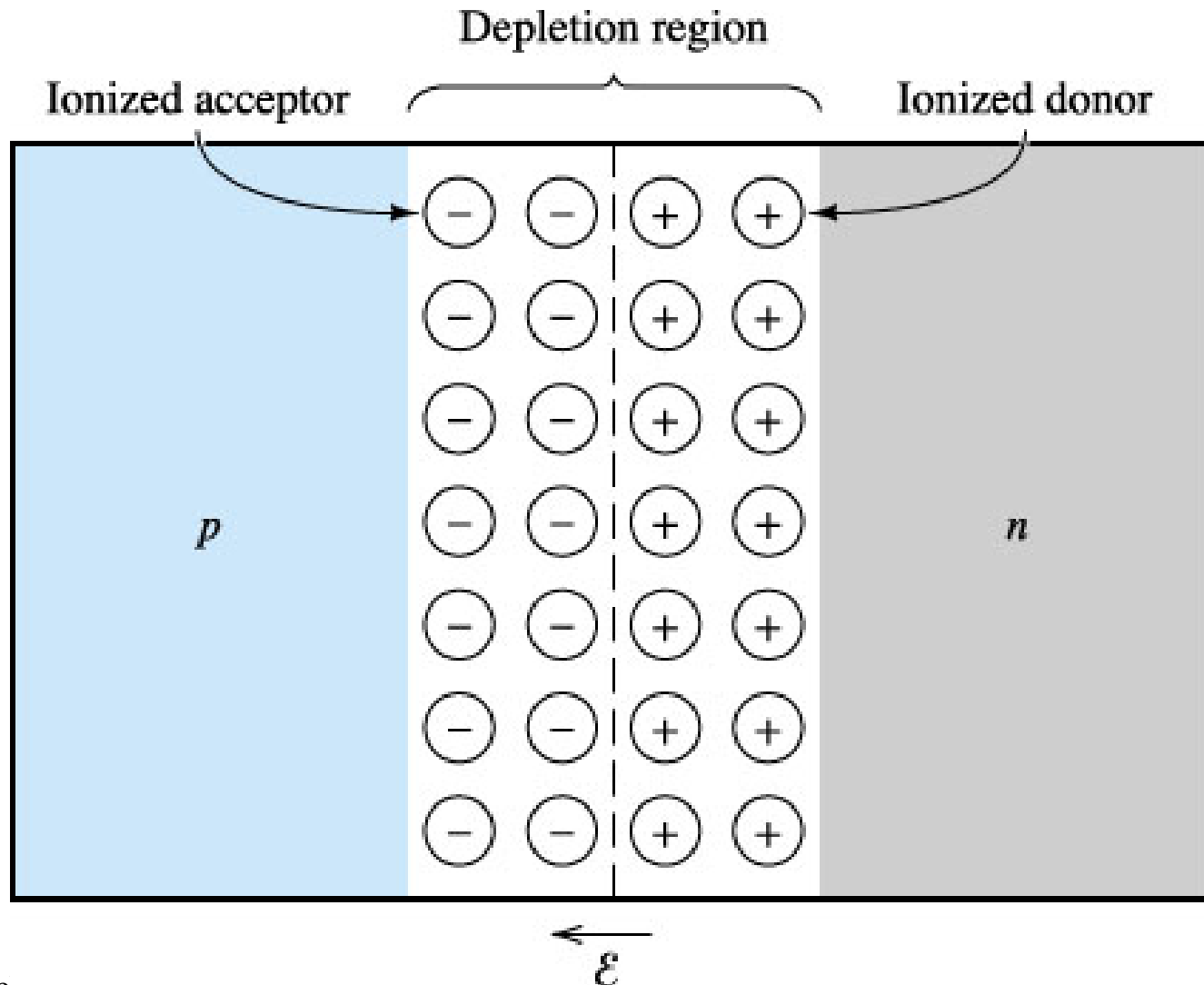
A.S. Sedra and K.C. Smith, *Microelectronic Circuits*. New York, NY: Oxford University Press, 1998.

A.R. Hambley, *Electronics: A Top-Down Approach to Computer-Aided Circuit Design*. Englewood Cliffs, NJ: Prentice Hall, 2000.

La Unión P-N sin Polarizar

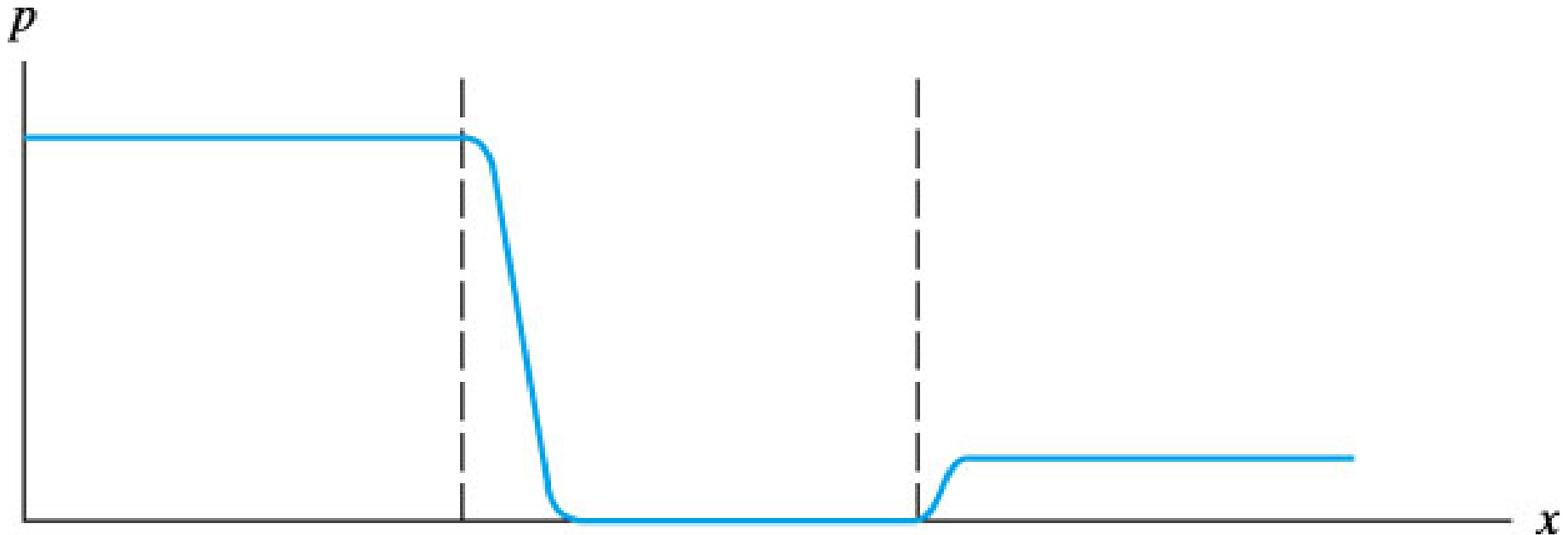


La Unión P-N sin Polarizar (cont.)



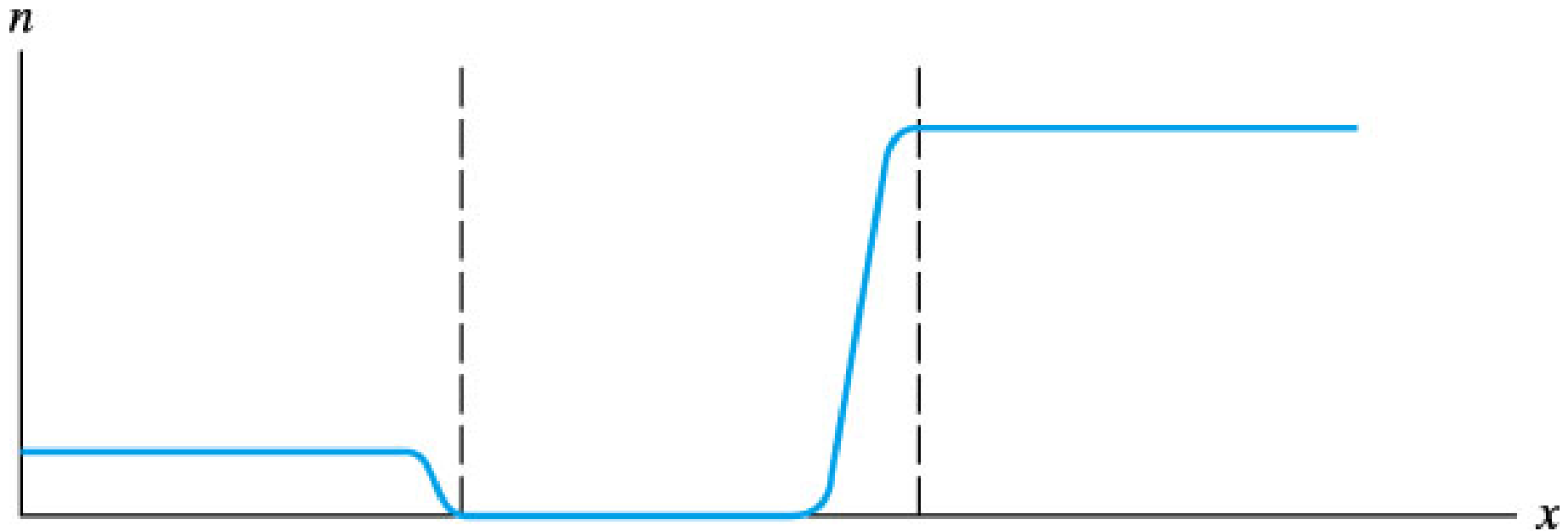
La Unión P-N sin Polarizar (cont.)

Concentración de huecos cerca de la unión

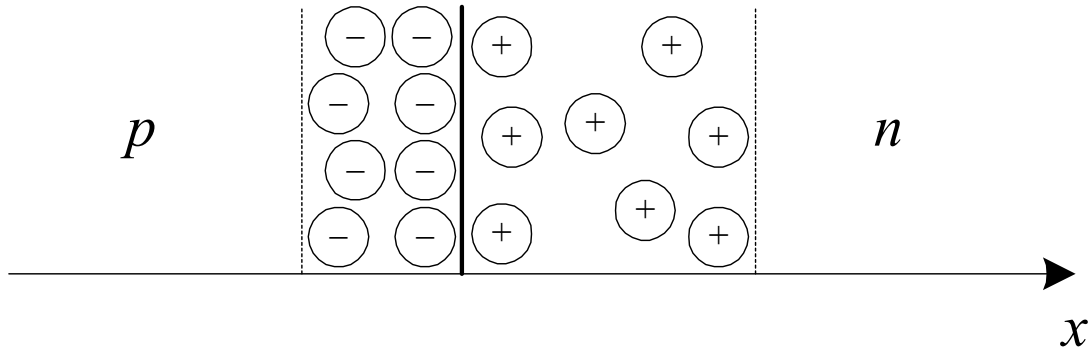


La Unión P-N sin Polarizar (cont.)

Concentración de electrones cerca de la unión



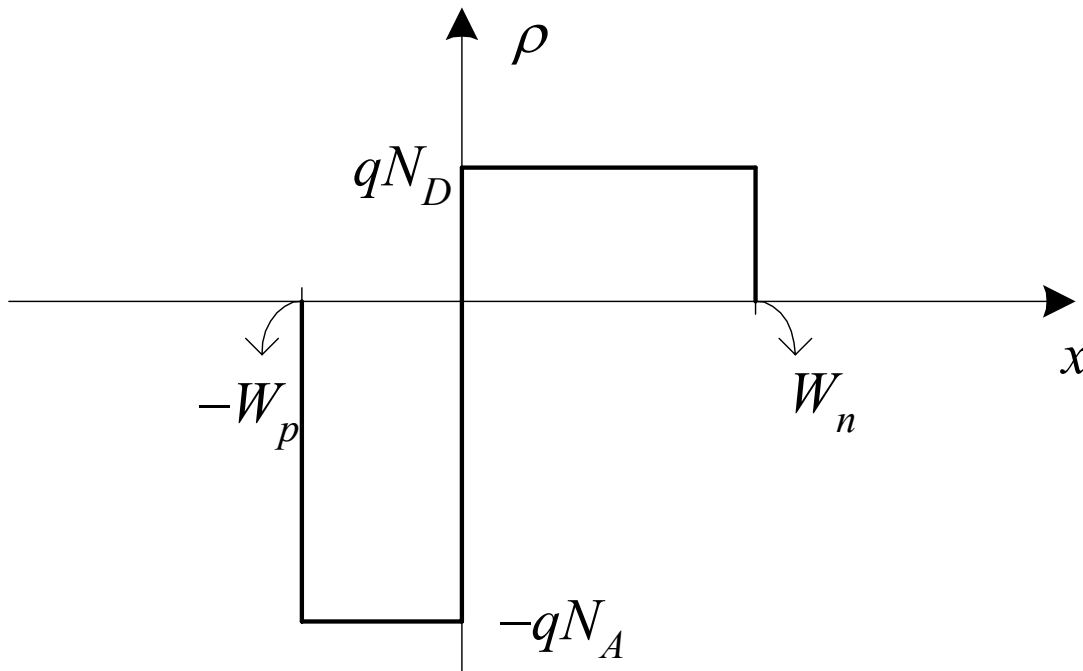
La Unión P-N sin Polarizar (cont.)



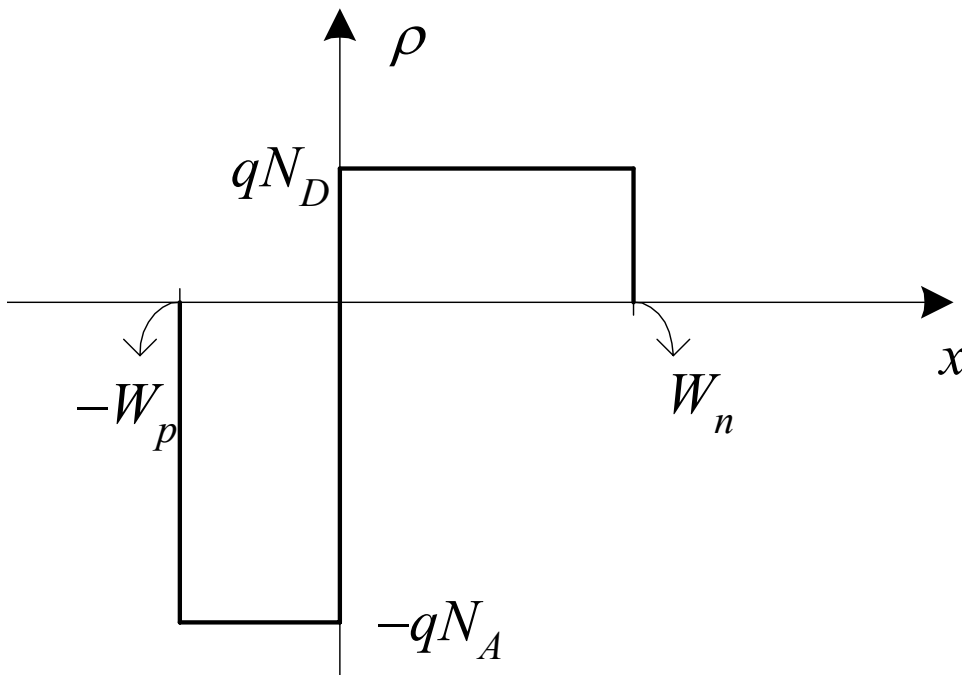
Como la región de agotamiento es eléctricamente neutra:

$$W_p N_A = W_n N_D$$

$$\frac{W_n}{W_p} = \frac{N_A}{N_D}$$



La Unión P-N sin Polarizar (cont.)



Como $\nabla \circ \mathbf{D} = \rho$ $\mathbf{D} = \epsilon \mathbf{E}$
 ρ densidad de carga, \mathbf{D}
densidad de campo eléctrico,
 ϵ constante dieléctrica)

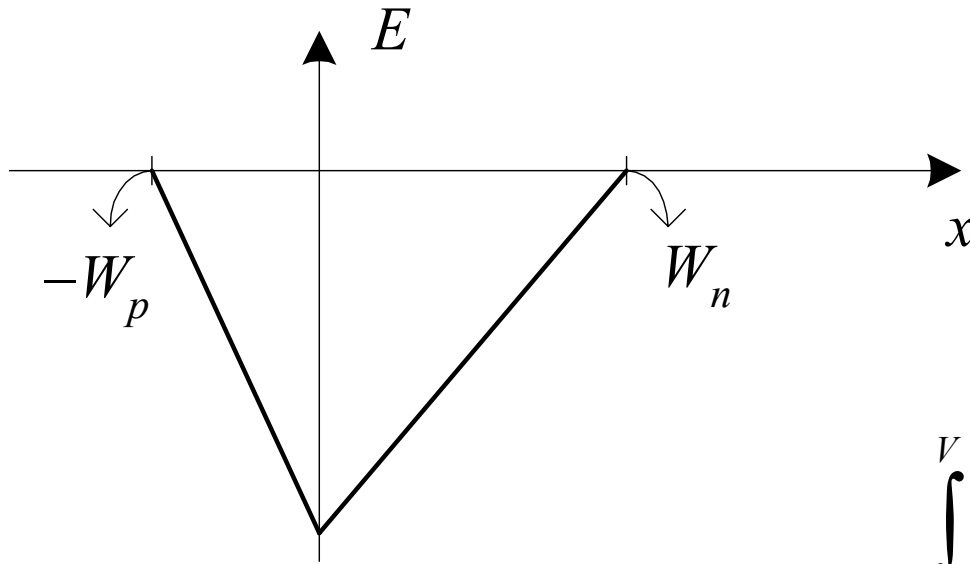
$$\nabla \circ \mathbf{E} = \frac{\rho}{\epsilon}$$

En una dimensión: $\frac{dE}{dx} = \frac{\rho}{\epsilon}$

$$\int_0^E dE = \int_{-W_p}^x \frac{\rho}{\epsilon} dx$$

$$E = \frac{-qN_A}{\epsilon} (x + W_p)$$

La Unión P-N sin Polarizar (cont.)



$$E = \frac{-qN_A}{\epsilon} (x + W_p) \quad -W_p \leq x \leq 0$$

Como $E = -\nabla V$

En una dimensión: $E = -\frac{dV}{dx}$

$$\int_0^V dV = \frac{qN_A}{\epsilon} \int_{-W_p}^x (x + W_p) dx$$

$$V = \frac{qN_A}{\epsilon} \left(\frac{x^2}{2} + W_p x + \frac{W_p^2}{2} \right), \quad -W_p \leq x \leq 0$$

La Unión P-N sin Polarizar (cont.)

$$V = \frac{qN_A}{\epsilon} \left(\frac{x^2}{2} + W_p x + \frac{W_p^2}{2} \right), \quad -W_p \leq x \leq 0$$

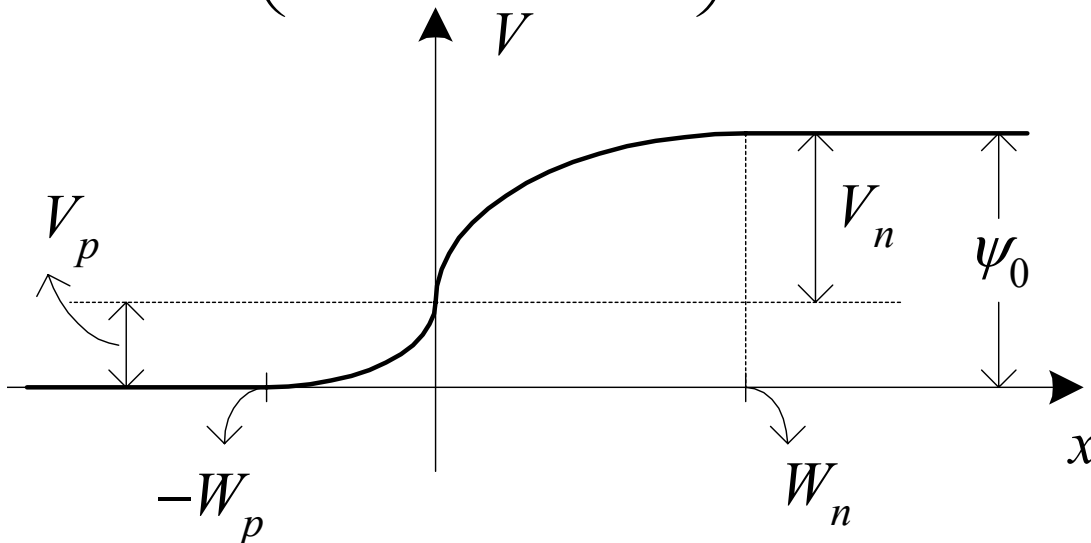
$$V_p = V(x=0) = \frac{qN_A}{\epsilon} \frac{W_p^2}{2}$$

Análogamente:

$$V_n = V(x=W_n) = \frac{qN_D}{\epsilon} \frac{W_n^2}{2}$$

Como $\psi_0 = V_n + V_p$

$$\psi_0 = \frac{q}{2\epsilon} (N_A W_p^2 + N_D W_n^2)$$



Análogamente:

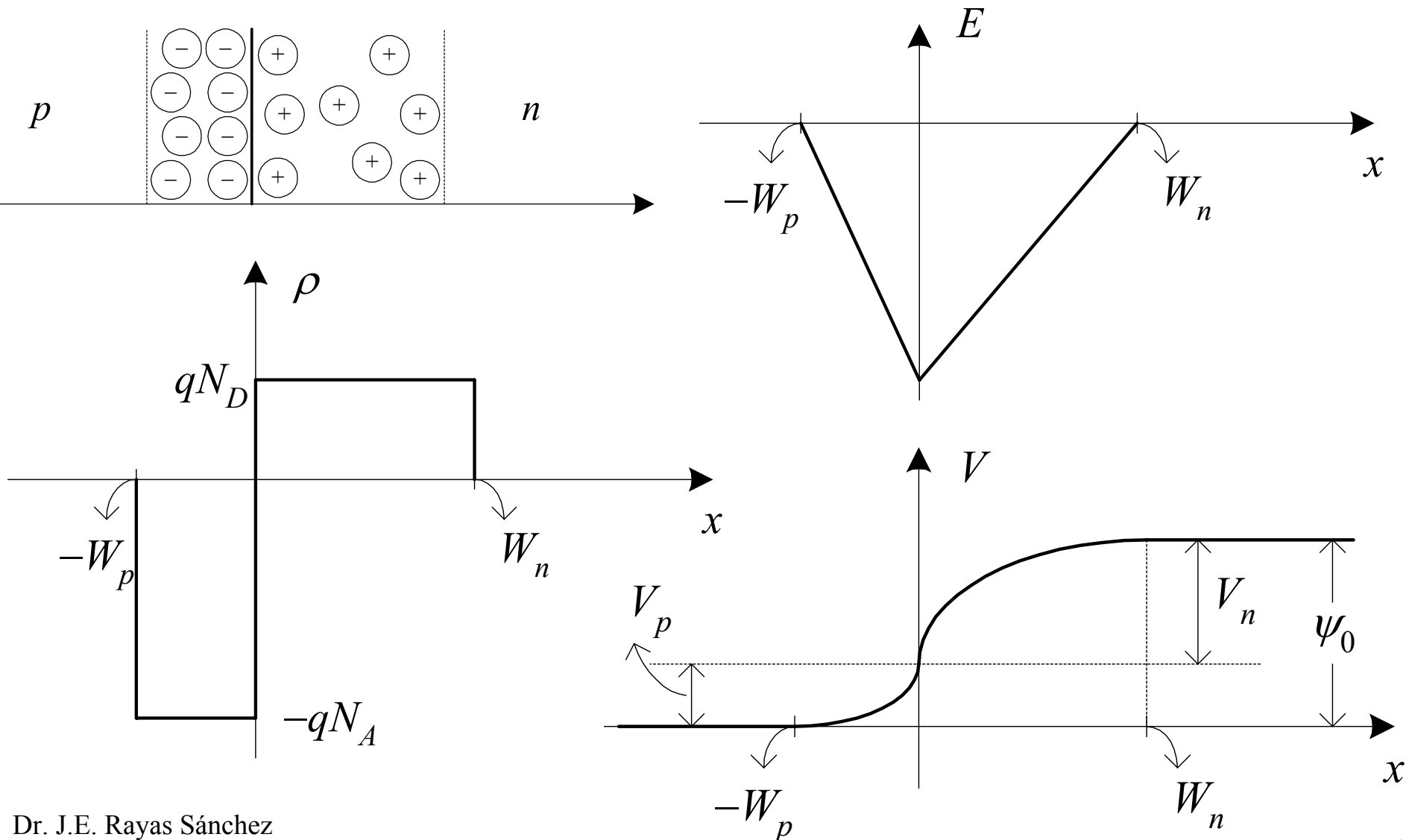
y como $\frac{W_n}{W_p} = \frac{N_A}{N_D}$

$$W_p = \sqrt{\frac{2\epsilon\psi_0}{qN_A \left(1 + \frac{N_A}{N_D} \right)}}$$

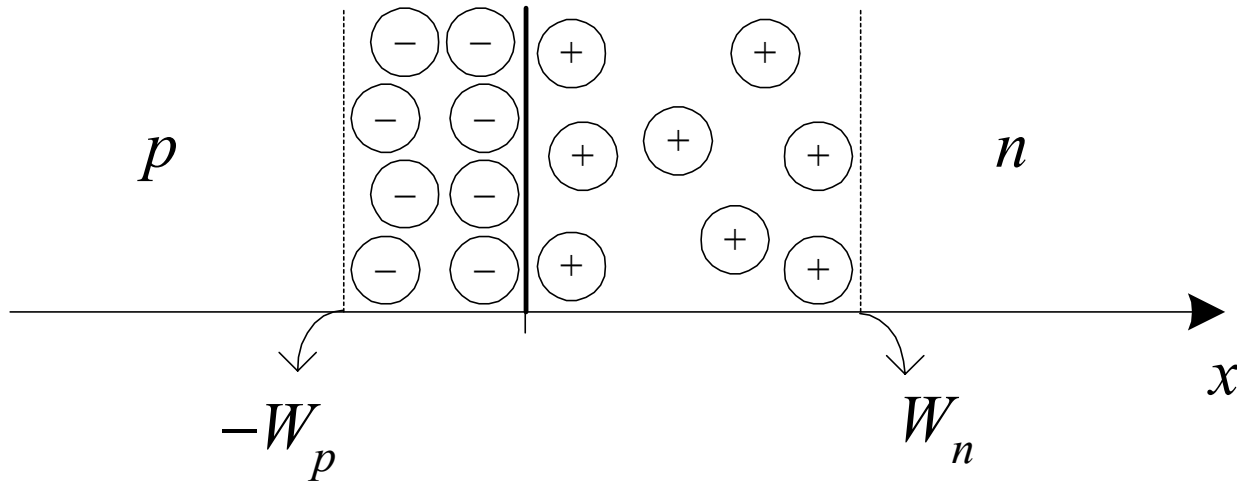
$$W_n = \sqrt{\frac{2\epsilon\psi_0}{qN_D \left(1 + \frac{N_D}{N_A} \right)}}$$

$$\psi_0 = \frac{qN_A W_p^2}{2\epsilon} \left(1 + \frac{N_A}{N_D} \right)$$

La Unión P-N sin Polarizar - Resumen



La Unión P-N sin Polarizar – Resumen (cont.)

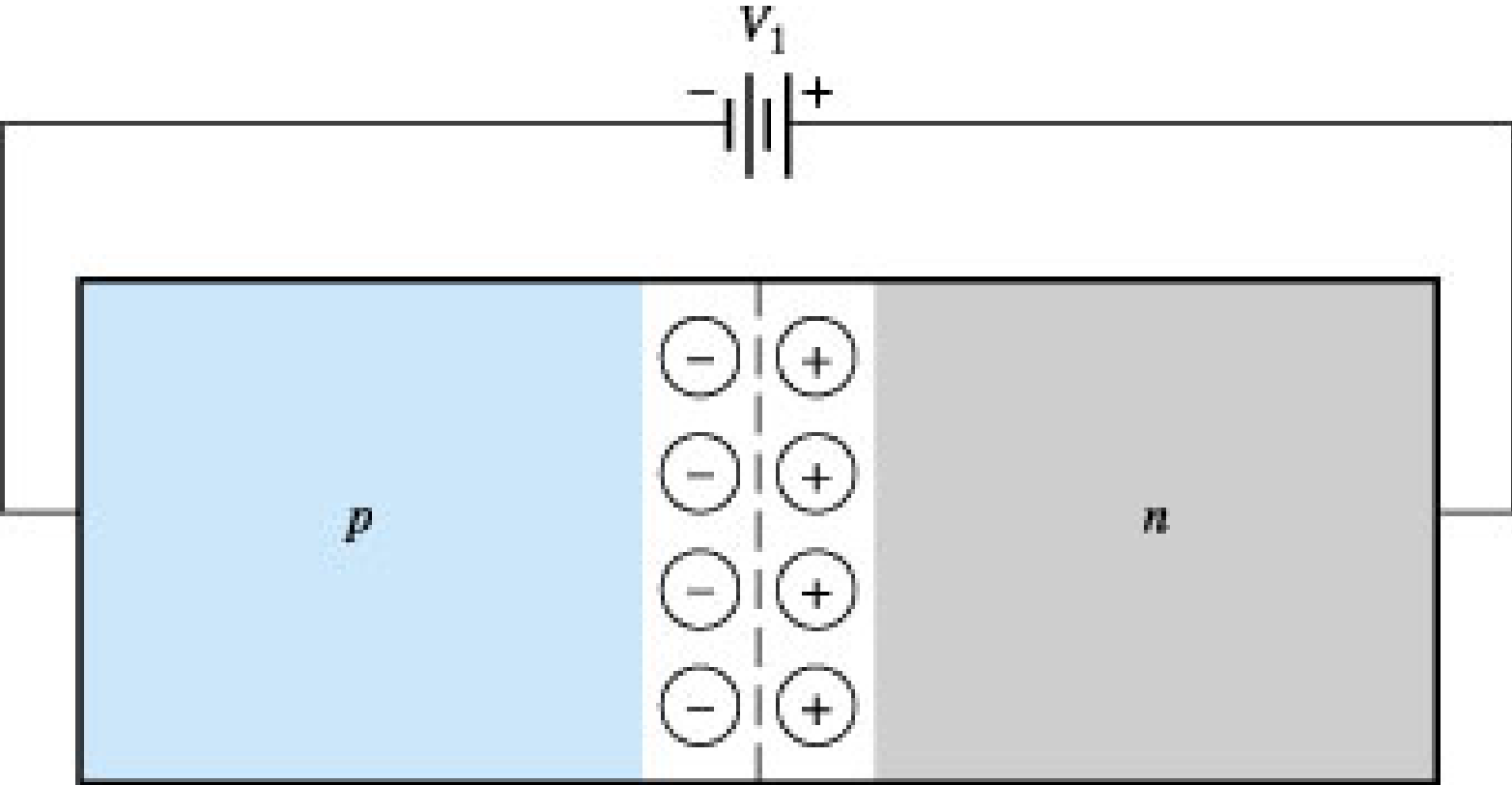


$$W_p = \sqrt{\frac{2\varepsilon\psi_0}{qN_A \left(1 + \frac{N_A}{N_D}\right)}}$$

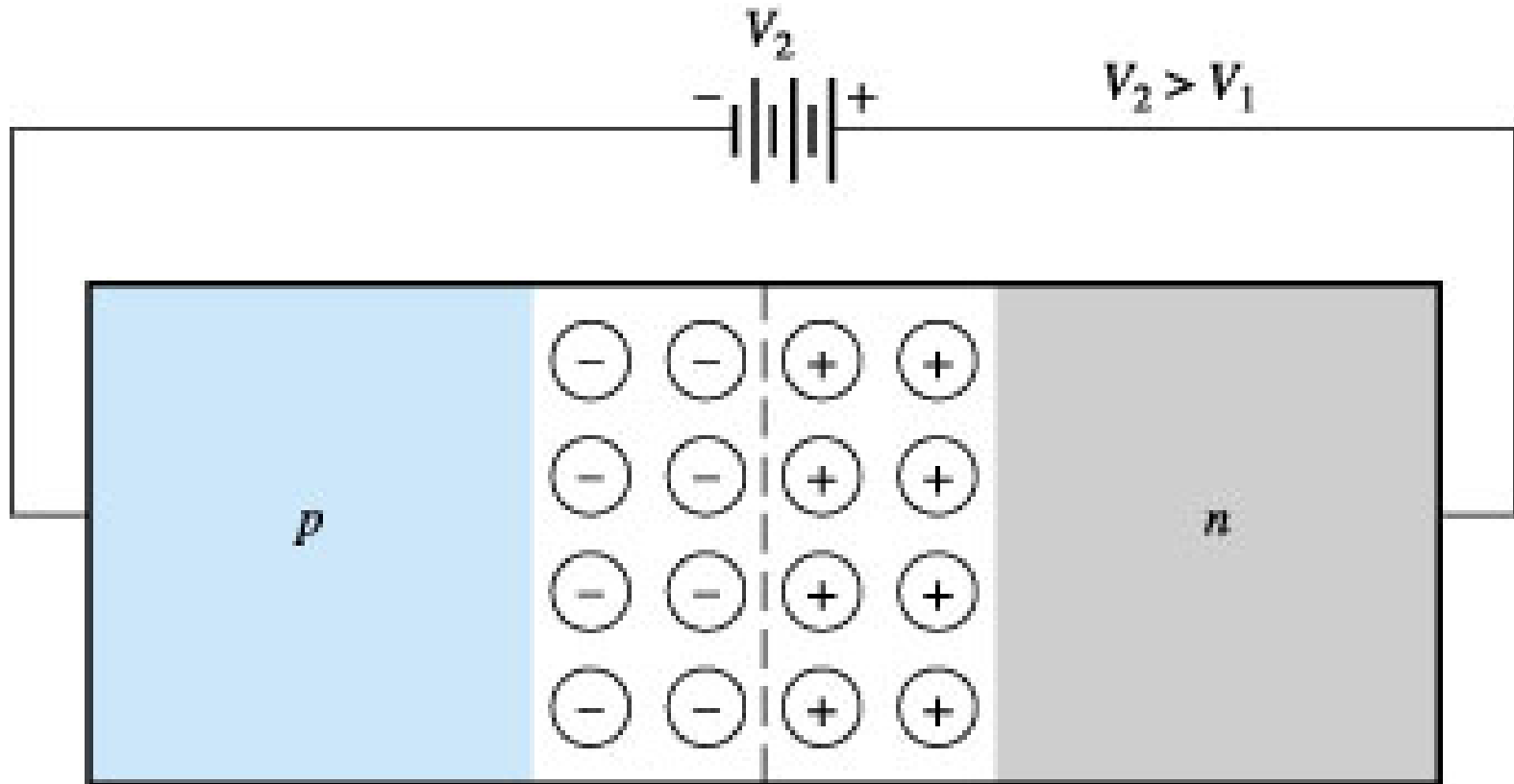
$$W_n = \sqrt{\frac{2\varepsilon\psi_0}{qN_D \left(1 + \frac{N_D}{N_A}\right)}}$$

$$\psi_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

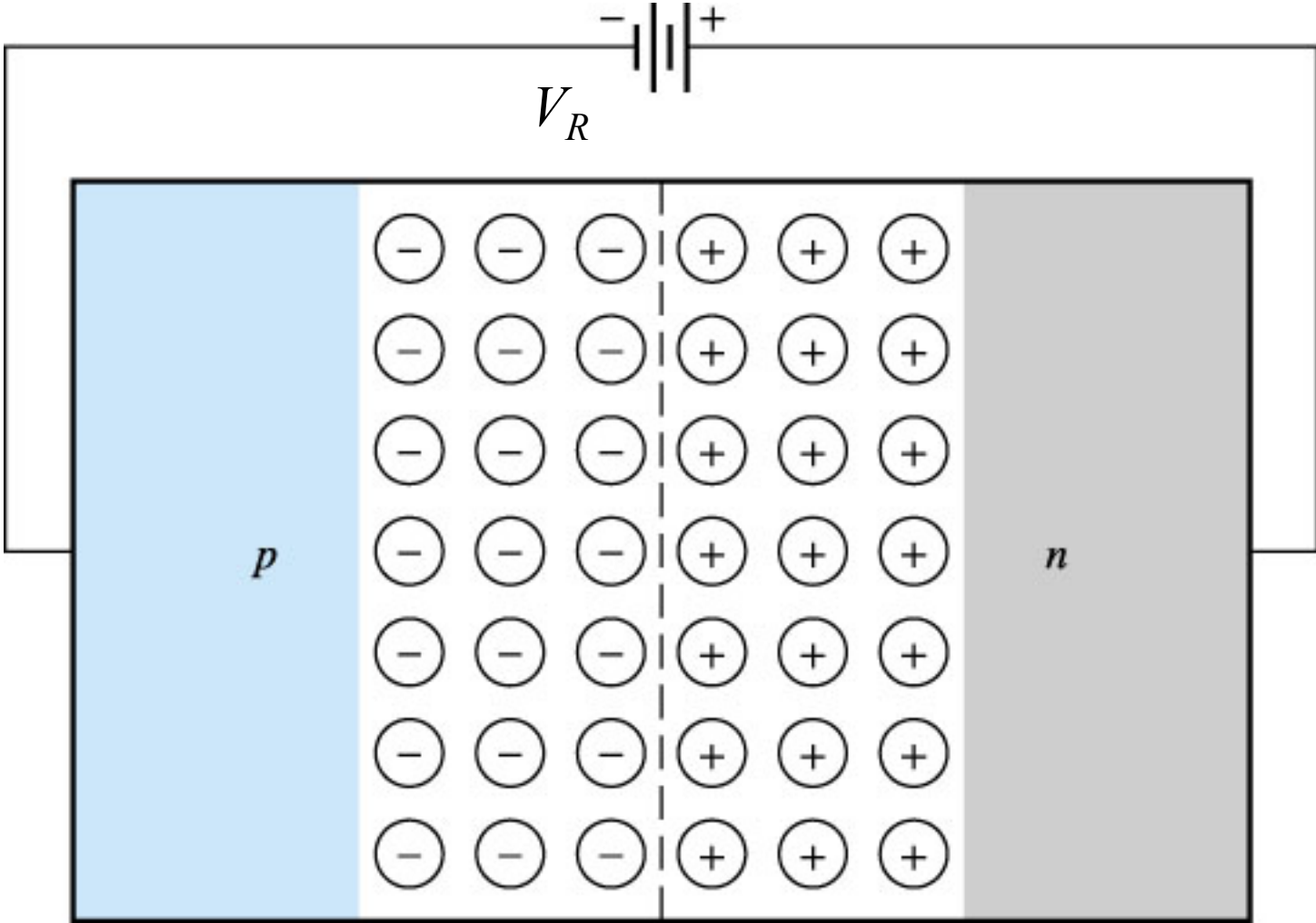
La Unión P-N Polarizada Inversamente



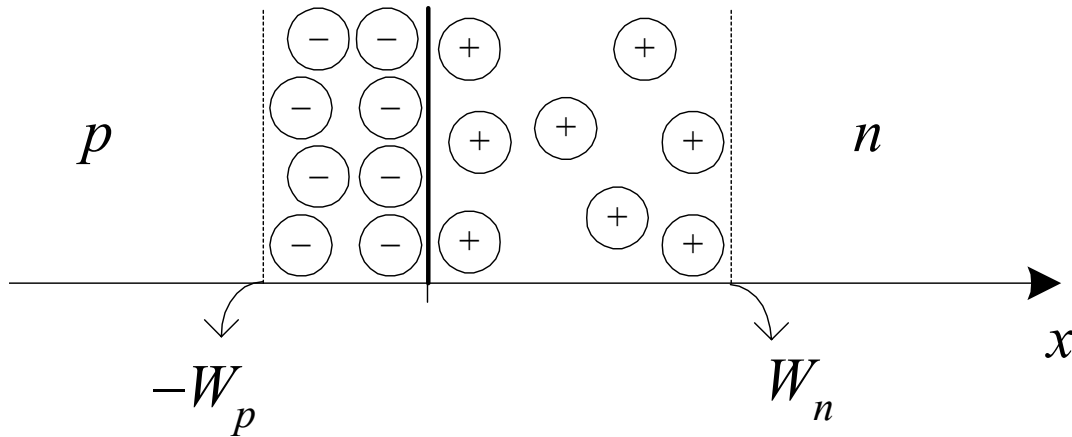
La Unión P-N Polarizada Inversamente (cont.)



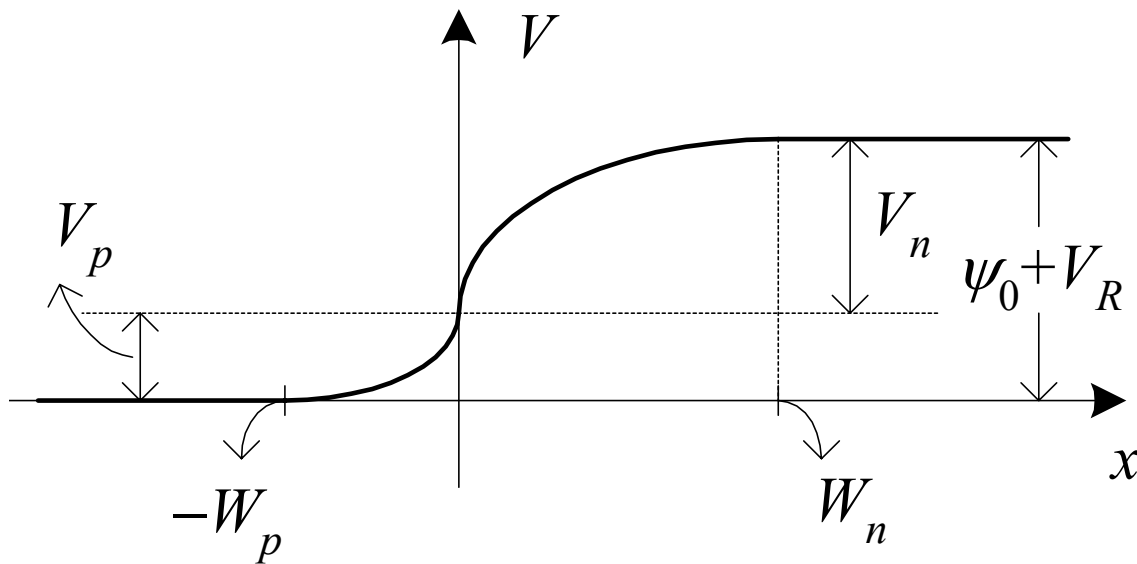
La Unión P-N Polarizada Inversamente (cont.)



La Unión P-N Polarizada Inversamente (cont.)



$$W_p = \sqrt{\frac{2\varepsilon(\psi_0 + V_R)}{qN_A \left(1 + \frac{N_A}{N_D}\right)}}$$



$$W_n = \sqrt{\frac{2\varepsilon(\psi_0 + V_R)}{qN_D \left(1 + \frac{N_D}{N_A}\right)}}$$

$$\psi_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Ejemplo

Calcular la penetración de la región desértica a $T = 300$ K en una unión p-n con $N_A = 10^{17}/\text{cm}^3$ y $N_D = 10^{15}/\text{cm}^3$, cuando el voltaje de polarización inversa es

a) $V_R = 0\text{V}$

$$\psi_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = 0.69\text{V}$$

$$\epsilon = \epsilon_r \epsilon_0 = 12(8.854 \times 10^{-12} \text{F/m})$$

$$\epsilon = 106.25 \times 10^{-12} \text{F/m}$$

$$W_p = \sqrt{\frac{2\epsilon\psi_0}{qN_A\left(1 + \frac{N_A}{N_D}\right)}} = 9.52\text{nm}$$

$$W_n = \sqrt{\frac{2\epsilon\psi_0}{qN_D\left(1 + \frac{N_D}{N_A}\right)}} = 957.29\text{nm}$$

Ejemplo

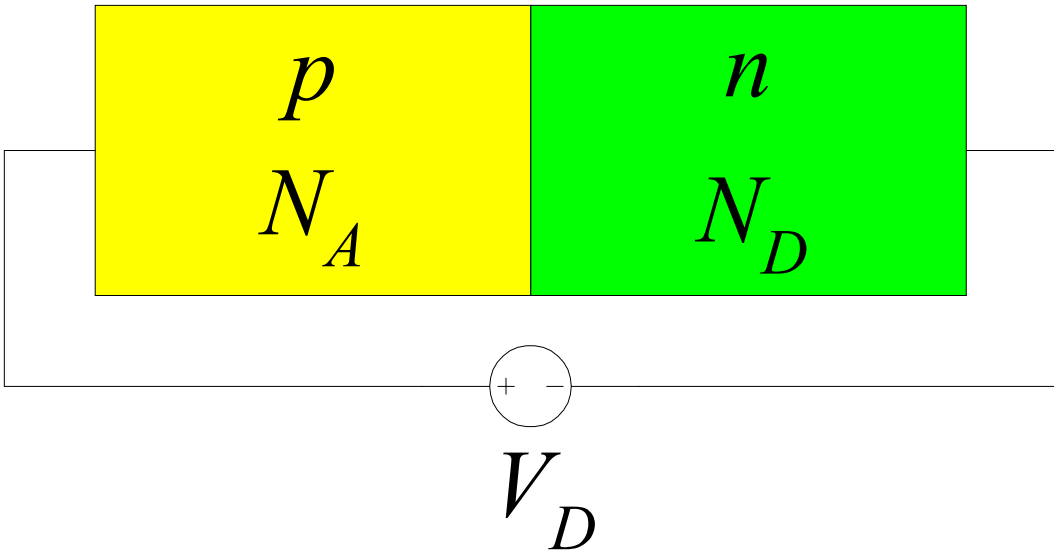
Calcular la penetración de la región desértica a $T = 300$ K en una unión p-n con $N_A = 10^{17}/\text{cm}^3$ y $N_D = 10^{15}/\text{cm}^3$, cuando el voltaje de polarización inversa es

b) $V_R = 25\text{V}$

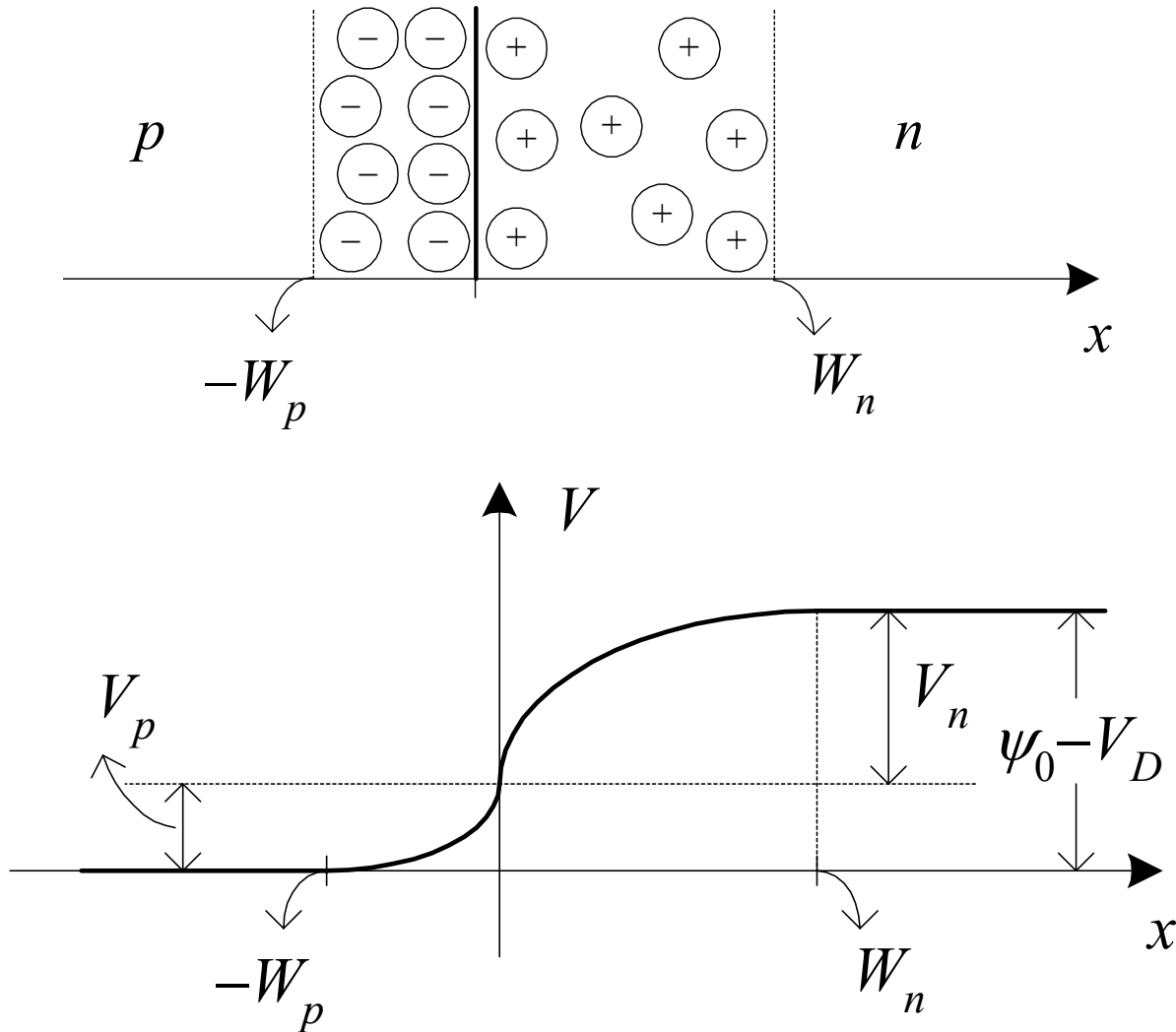
$$W_p = \sqrt{\frac{2\varepsilon(\psi_0 + V_R)}{qN_A\left(1 + \frac{N_A}{N_D}\right)}} = 58.12\text{nm}$$

$$W_n = \sqrt{\frac{2\varepsilon(\psi_0 + V_R)}{qN_D\left(1 + \frac{N_D}{N_A}\right)}} = 5.84\mu\text{m}$$

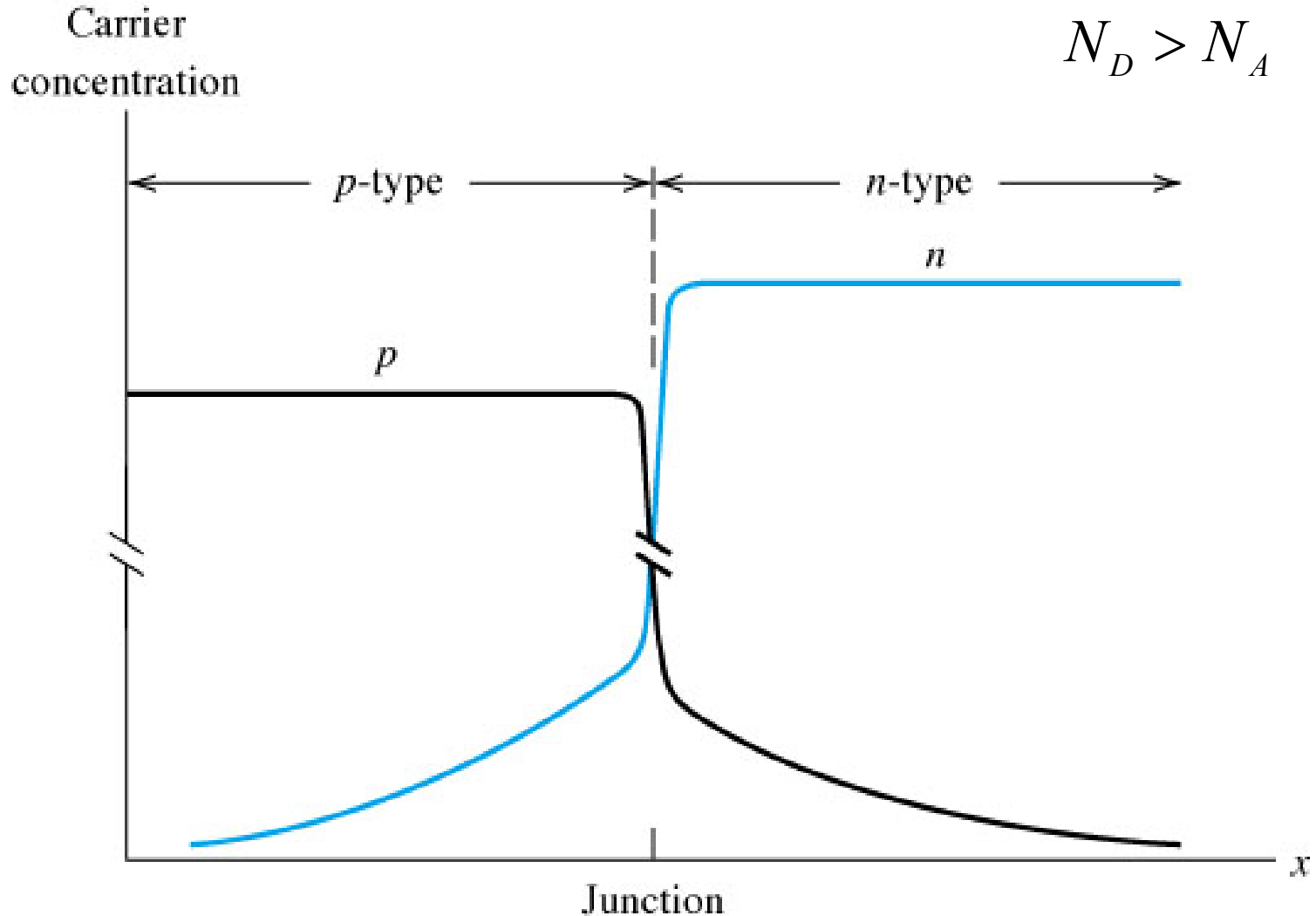
Curva Característica de la Unión P-N



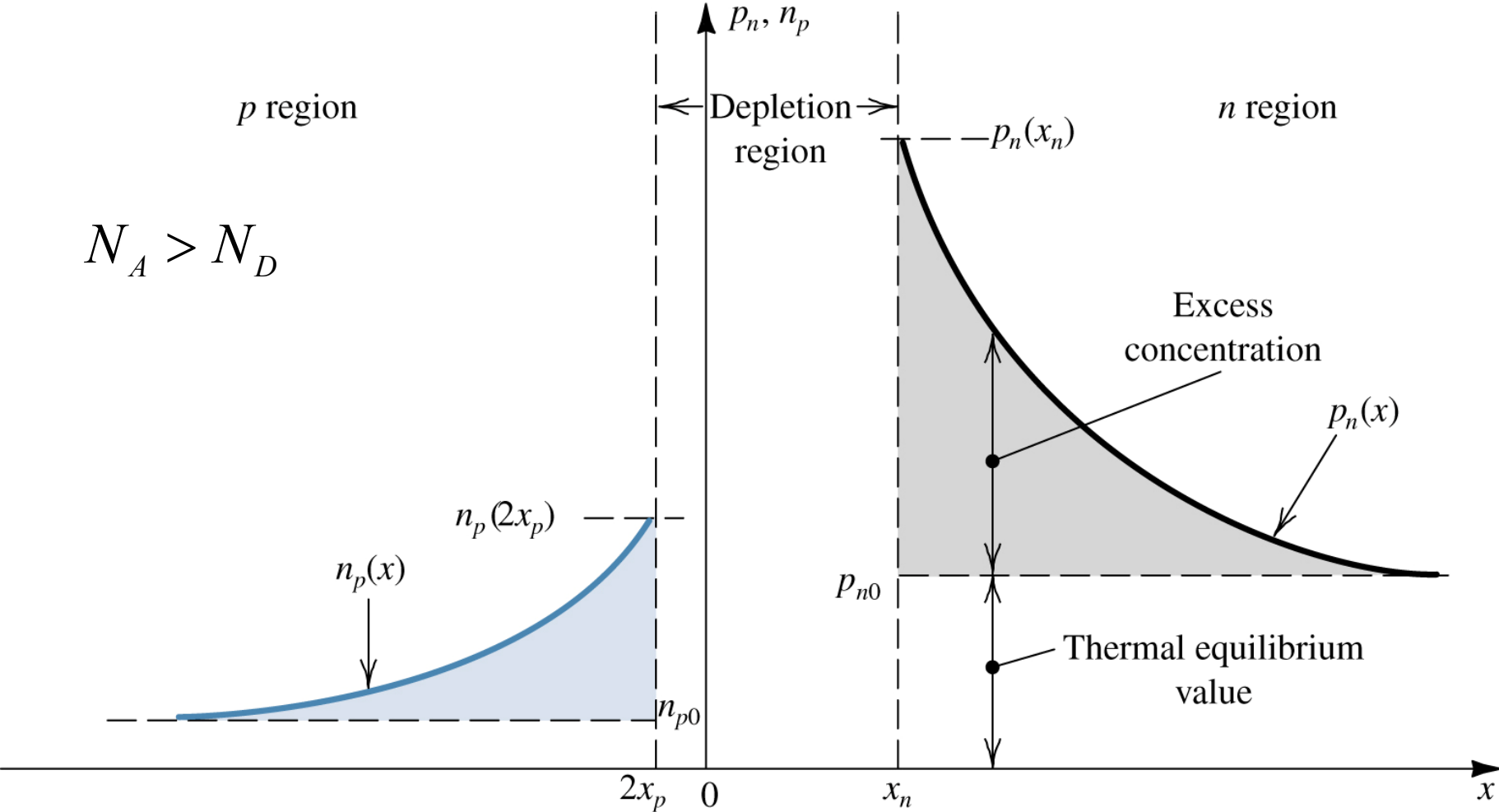
Curva Característica de la Unión P-N (cont.)



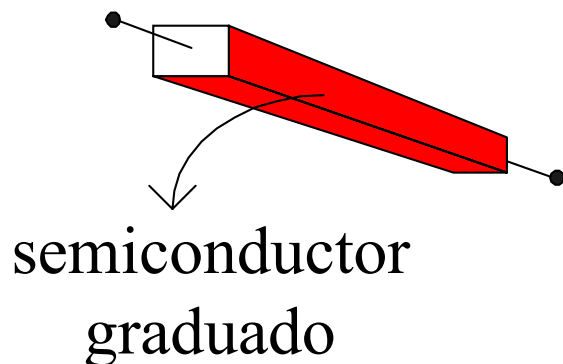
La Unión P-N Polarizada Directamente (cont.)



Portadores Minoritarios en la Unión



Potencial Interno (repasso)



$$qn\mu_n E = -qD_n \frac{dn}{dx}$$

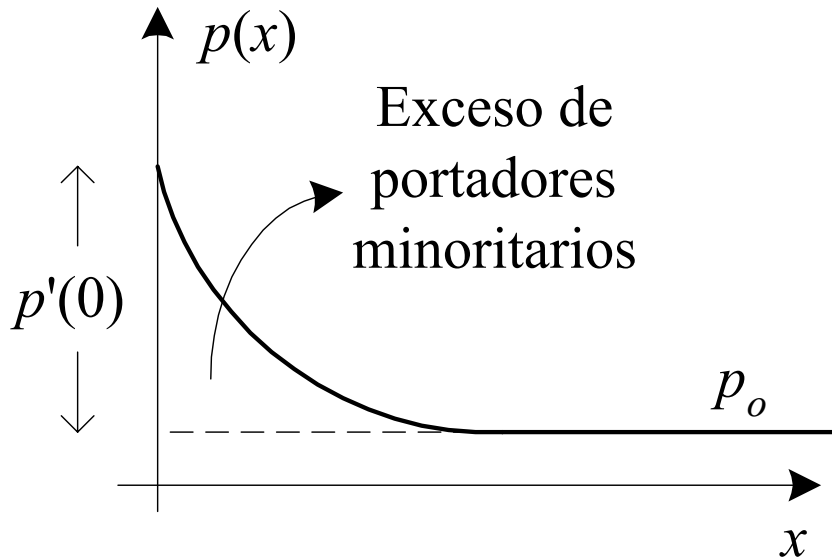
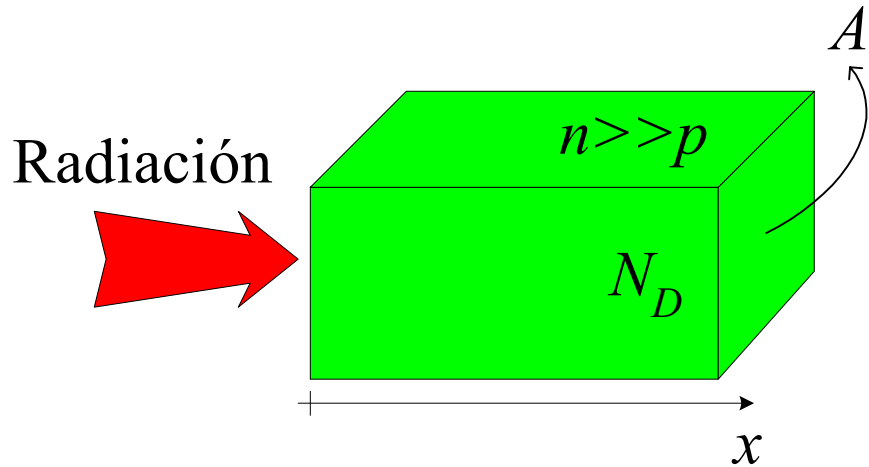
$$E = \frac{-D_n}{n\mu_n} \frac{dn}{dx} = \frac{-V_T}{n} \frac{dn}{dx}$$

Como $E = -\frac{dV}{dx}$

$$\int_{V_1}^{V_2} dV = V_T \int_{n_1}^{n_2} \frac{dn}{n}$$

$$V_2 - V_1 = V_T \ln\left(\frac{n_2}{n_1}\right) \quad \text{análogamente} \quad V_2 - V_1 = V_T \ln\left(\frac{p_1}{p_2}\right)$$

Inyección de Portadores Minoritarios (repass)



$$\frac{d^2 p}{dx^2} = \frac{p - p_o}{L_p^2}$$

$$p(x) = K_1 e^{-x/L_p} + K_2 e^{x/L_p} + p_o$$

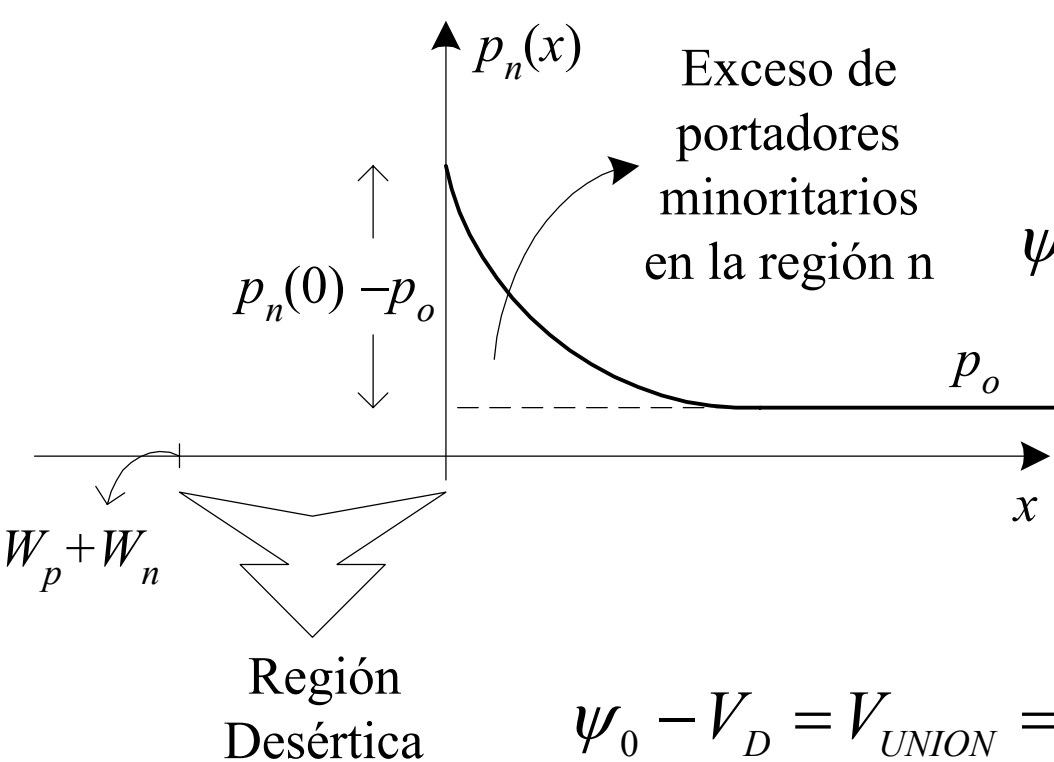
$$K_2 = 0$$

$$K_1 = p'(0)$$

$$p(x) = p'(0) e^{-x/L_p} + p_o$$

$$L_p \equiv \sqrt{D_p \tau_p}$$

Portadores Minoritarios en la Unión



$$p(x) = (p_n(0) - p_o)e^{-x/L_p} + p_o$$

$$\psi_0 = V_T \ln\left(\frac{p_p}{p_n}\right)\Big|_{\text{s.p.}} = V_T \ln\left(\frac{N_A}{p_o}\right)$$

$$p_o = N_A e^{-\left(\frac{\psi_0}{V_T}\right)}$$

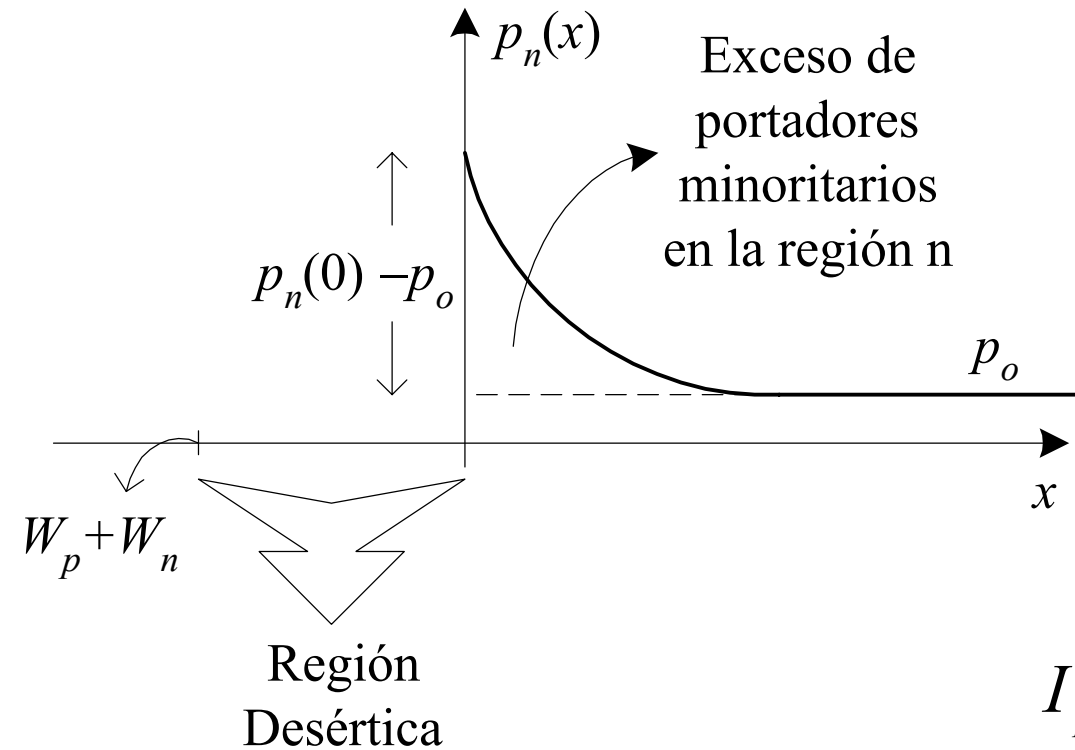
$$\psi_0 - V_D = V_{UNION} = V_T \ln\left(\frac{p_p}{p_n(0)}\right) = V_T \ln\left(\frac{N_A}{p_n(0)}\right)$$

$$p_n(0) = N_A e^{-\left(\frac{\psi_0 - V_D}{V_T}\right)} = p_o e^{\frac{V_D}{V_T}}$$

Luego

$$p(x) = p_o \left(e^{\frac{V_D}{V_T}} - 1\right) e^{-x/L_p} + p_o$$

Portadores Minoritarios en la Unión (cont.)



$$p(x) = p_o \left(e^{\frac{V_D}{V_T}} - 1 \right) e^{-x/L_p} + p_o$$

$$I_{pn} = J_p A$$

$$J_p = -qD_p \frac{dp}{dx}$$

$$I_{pn} = \frac{qAD_p p_o}{L_p} \left(e^{\frac{V_D}{V_T}} - 1 \right) e^{-x/L_p}$$

Análogamente

$$I_{np}(0) = \frac{qAD_n n_o}{L_n} \left(e^{\frac{V_D}{V_T}} - 1 \right)$$

$$I_{pn}(0) = \frac{qAD_p p_o}{L_p} \left(e^{\frac{V_D}{V_T}} - 1 \right)$$

Curva Característica de la Unión P-N

$$I_{pn}(0) = \frac{qAD_p p_o}{L_p} \left(e^{\frac{V_D}{V_T}} - 1 \right) \quad I_{np}(0) = \frac{qAD_n n_o}{L_n} \left(e^{\frac{V_D}{V_T}} - 1 \right)$$

$$I_D = I_{pn}(0) + I_{np}(0)$$

$$I_D = I_S \left(e^{\frac{V_D}{V_T}} - 1 \right)$$

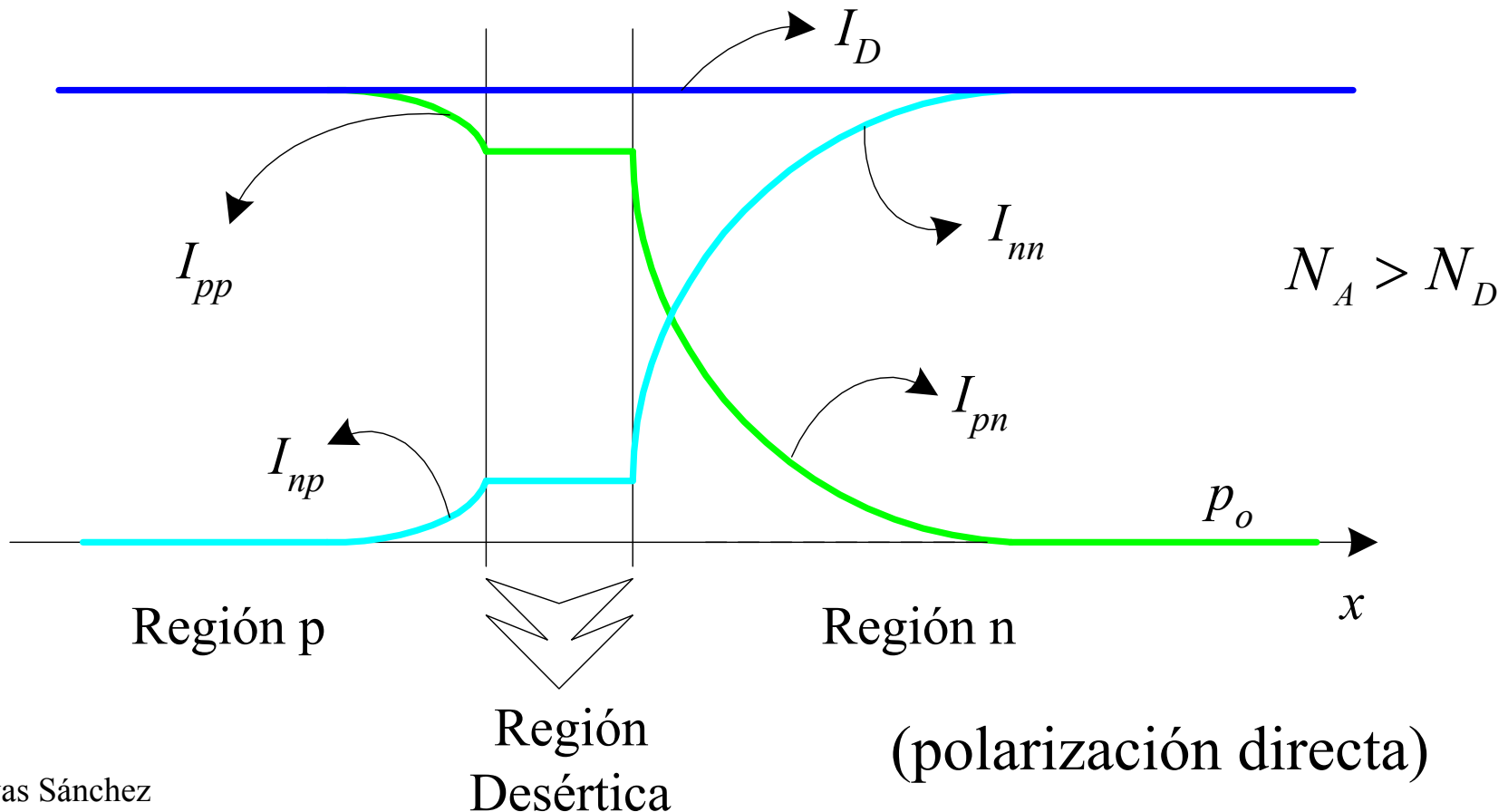
$$\text{donde} \quad I_S = qA \left(\frac{D_p p_o}{L_p} + \frac{D_n n_o}{L_n} \right)$$

I_S Corriente de saturación, de fuga,
o de escalamiento

Curva Característica de la Unión P-N

$$I_D = I_{pn}(0) + I_{np}(0)$$

$$I_D = I_S \left(e^{\frac{V_D}{V_T}} - 1 \right)$$



Ecuación de Shockley

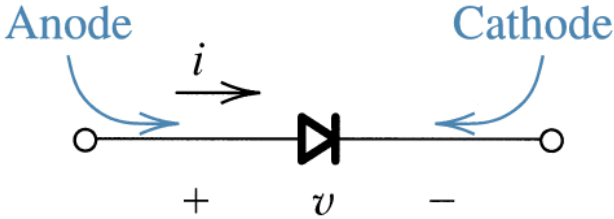
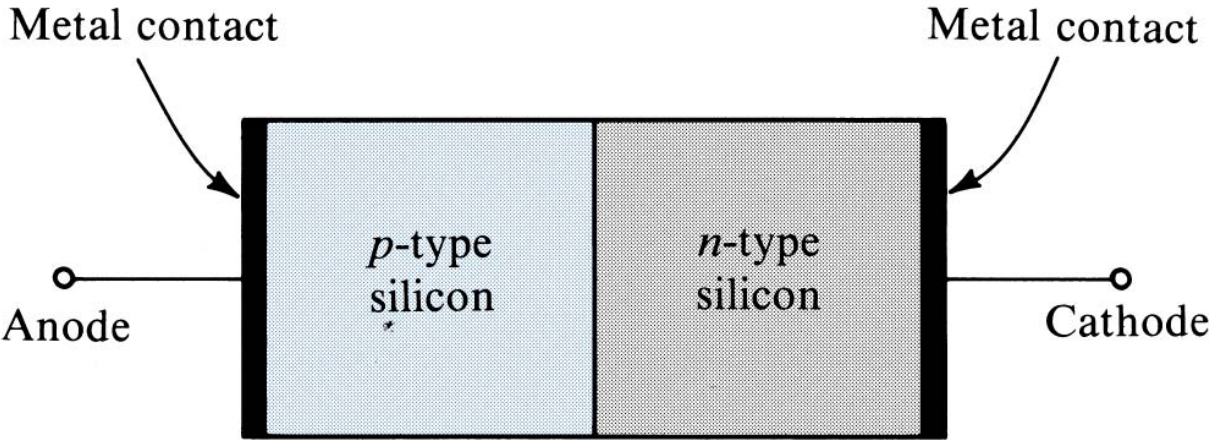
$$I_D = I_S \left(e^{\frac{V_D}{\eta V_T}} - 1 \right)$$

η Coeficiente de emisión $1 \leq \eta \leq 2$

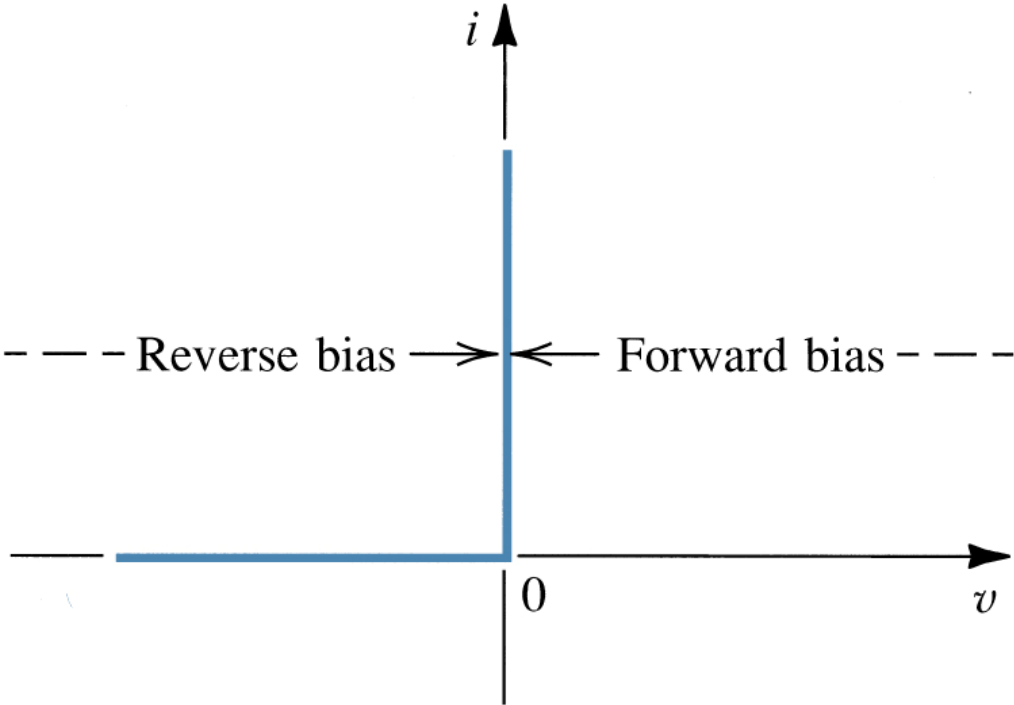
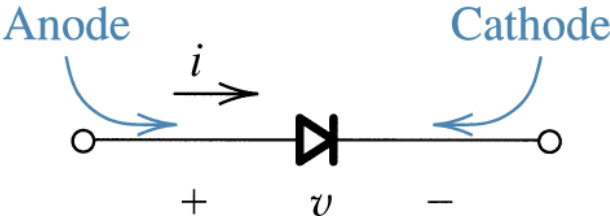
$\eta \approx 1$ Uniones p-n de alta difusión (diodos en C.I.)

$\eta \approx 2$ Uniones p-n de alta recombinación (diodos discretos)

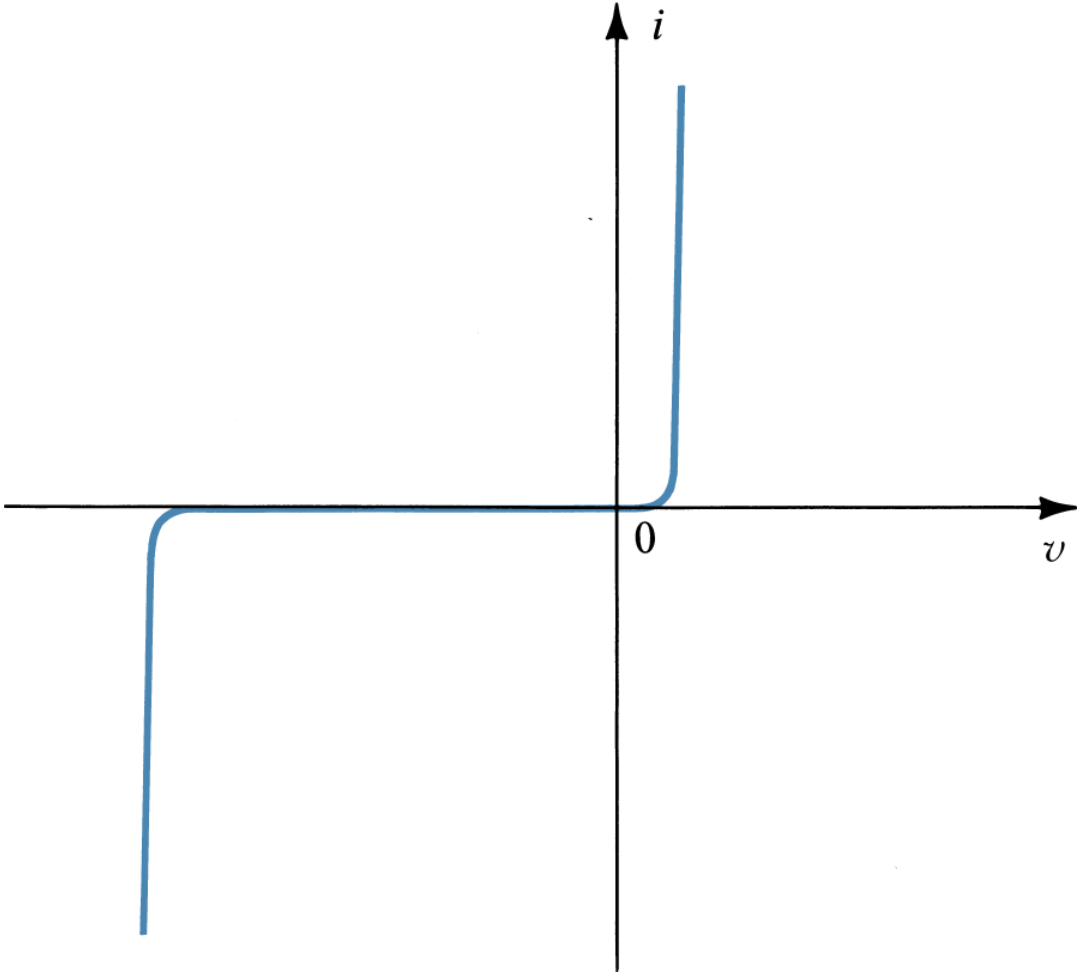
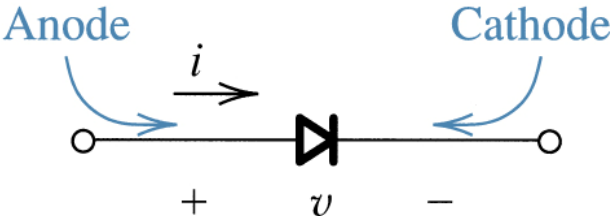
El Diodo



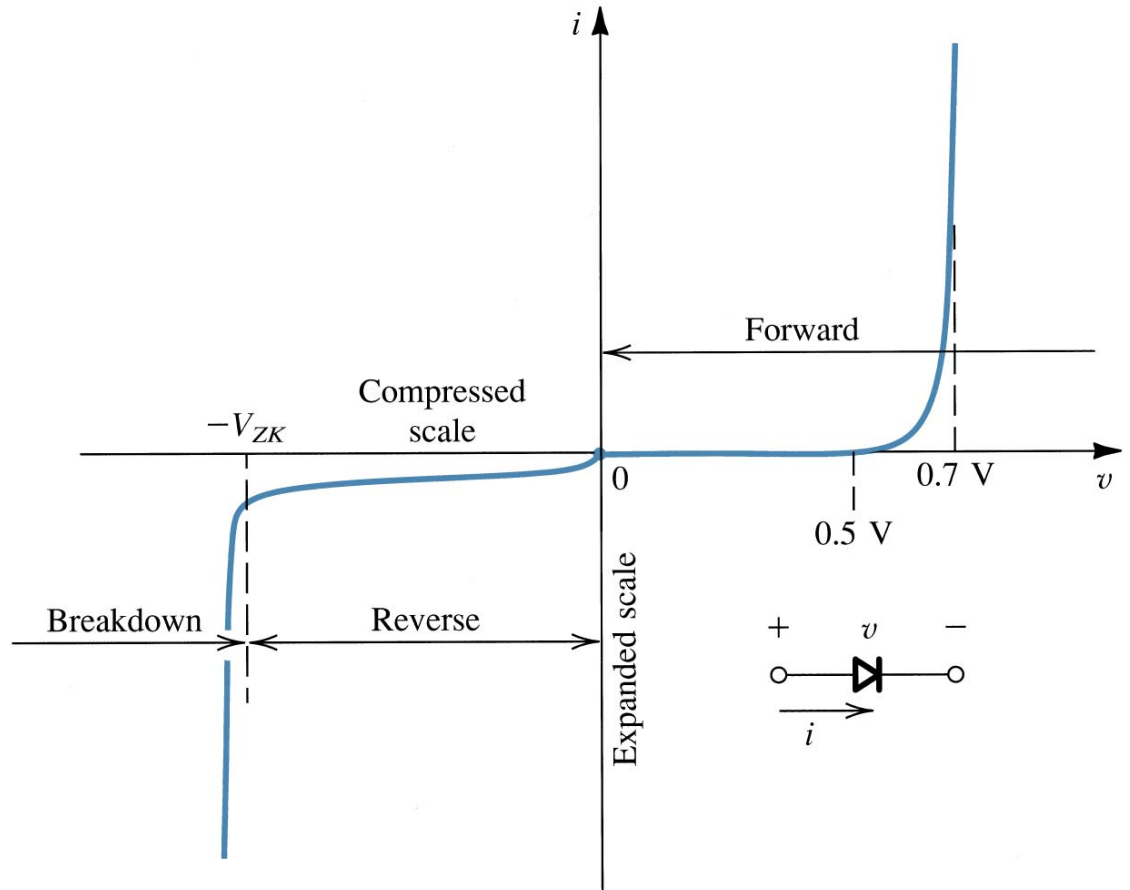
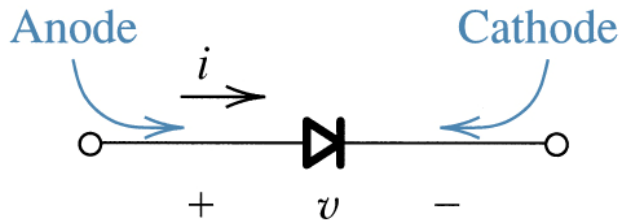
El Diodo Ideal



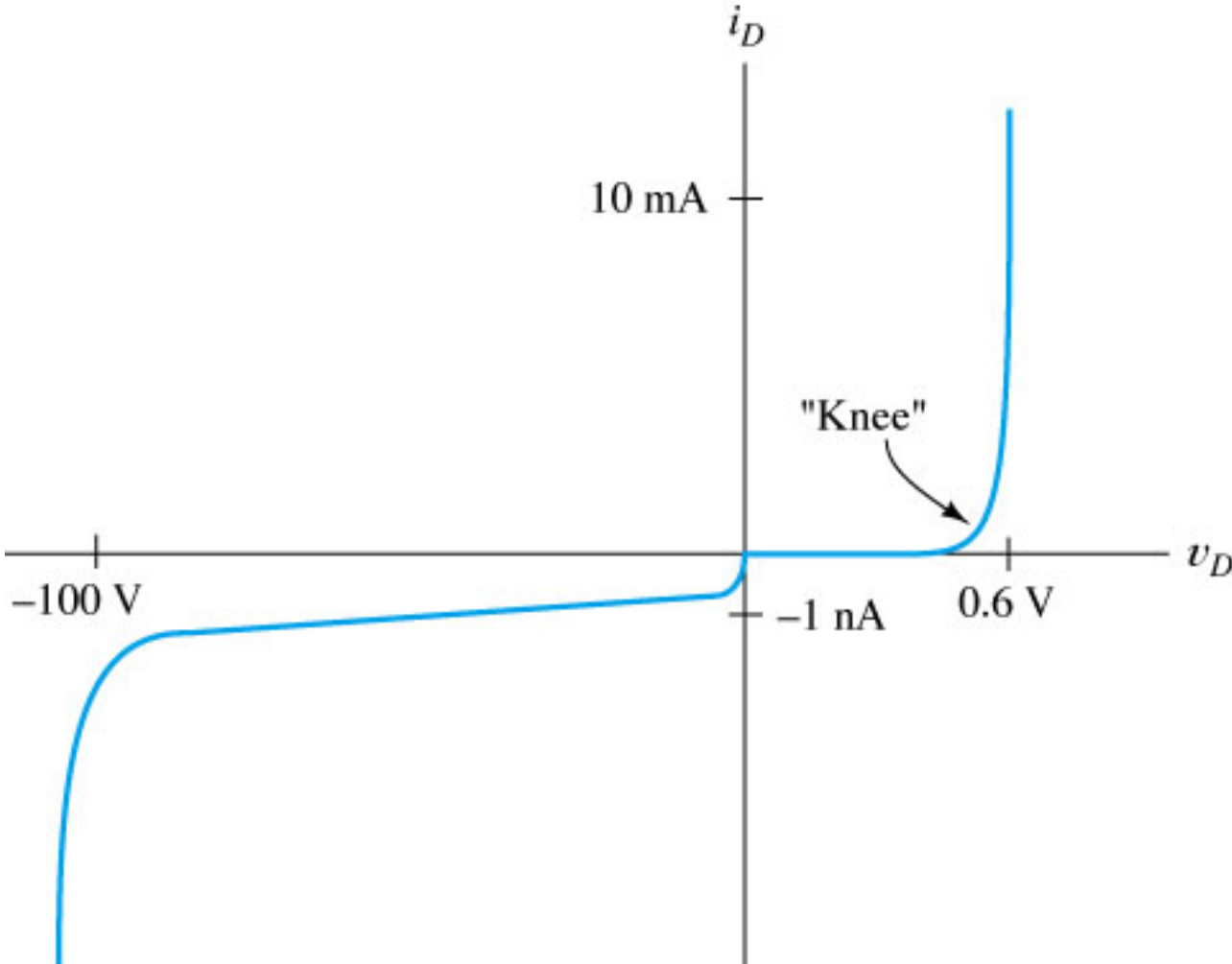
Curva Característica del Diodo



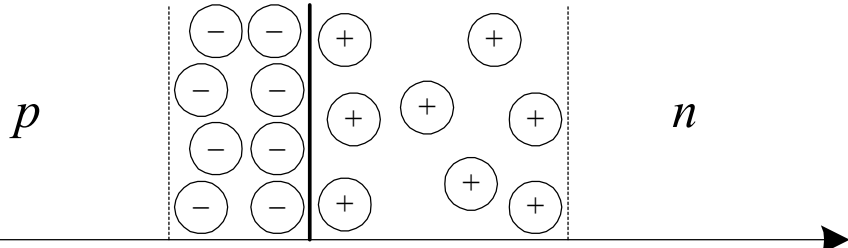
Curva Característica del Diodo (cont.)



Curva Característica del Diodo (cont.)

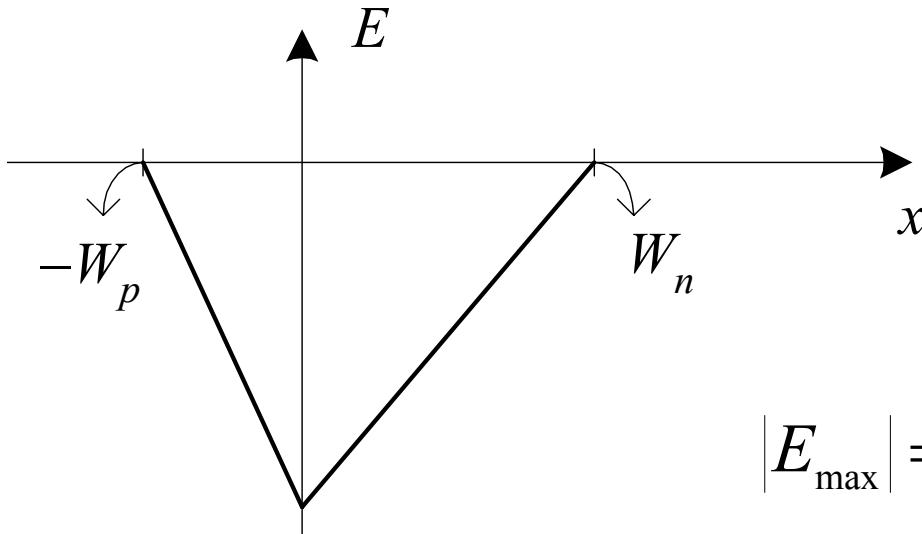


Zona de Rompimiento en la Unión P-N



$$E = -\frac{qN_A}{\epsilon}(x + W_p)$$

$$|E_{\max}| = \frac{qN_A W_p}{\epsilon}$$



Como $W_p = \sqrt{\frac{2\epsilon(\psi_0 + V_R)}{qN_A \left(1 + \frac{N_A}{N_D}\right)}}$

$$|E_{\max}| = \sqrt{\frac{2qN_A N_D V_R}{\epsilon(N_D + N_A)}} \quad (V_R \gg \psi_0)$$

Cuando $|E_{\max}| \rightarrow E_{\text{crítico}}$ se produce el efecto “avalancha”

Zona de Rompimiento en la Unión P-N (cont.)

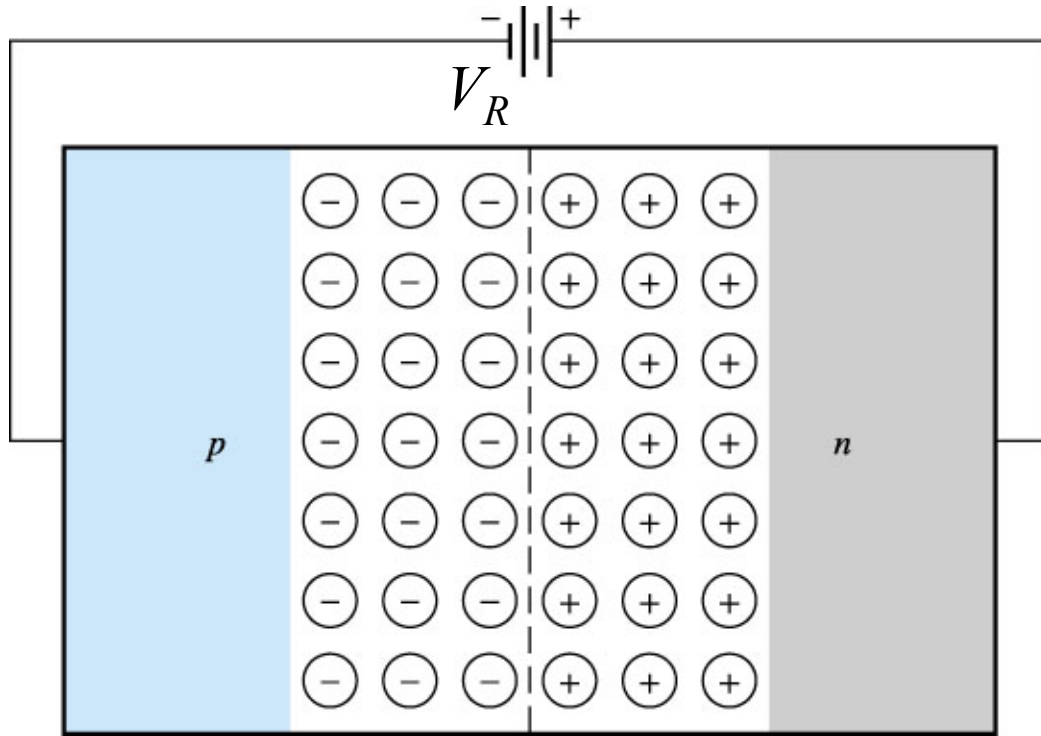
$$|E_{\max}| = \sqrt{\frac{2qN_A N_D V_R}{\epsilon(N_D + N_A)}}$$

$$BV = \frac{\epsilon(N_D + N_A)}{2qN_A N_D} (E_{\text{crítico}})^2$$

$E_{\text{crítico}}$ depende de los niveles de contaminación. Para uniones con $N_A, N_D \approx 10^{15}-10^{16}$ átomos/cm³, $E_{\text{crítico}} \approx 3 \times 10^5$ V/cm

Para uniones altamente contaminadas, el rompimiento se produce por el efecto “tunel” (zener), y no por el efecto avalancha. En este caso, $BV \approx 6V$.

Capacitancia de la Región de Desértica



$$C_j = \frac{dQ}{dV_R}$$

$$C_j = \frac{dQ}{dW_p} \frac{dW_p}{dV_R}$$

$$W_p = \sqrt{\frac{2\varepsilon(\psi_0 + V_R)}{qN_A \left(1 + \frac{N_A}{N_D}\right)}}$$

$$Q = AW_p qN_A$$

$$\frac{dQ}{dW_p} = AqN_A$$

$$\frac{dW_p}{dV_R} = \sqrt{\frac{\varepsilon}{2qN_A \left(1 + \frac{N_A}{N_D}\right) (\psi_0 + V_R)}}$$

Capacitancia de la Región de Desértica (cont.)

$$C_j = \frac{dQ}{dW_p} \frac{dW_p}{dV_R}$$

$$\frac{dQ}{dW_p} = AqN_A$$

$$\frac{dW_p}{dV_R} = \sqrt{\frac{\varepsilon}{2qN_A \left(1 + \frac{N_A}{N_D}\right) (\psi_0 + V_R)}}$$

$$C_j = A \sqrt{\frac{\varepsilon q N_A N_D}{2(N_D + N_A)}} \frac{1}{\sqrt{\psi_0 + V_R}}$$

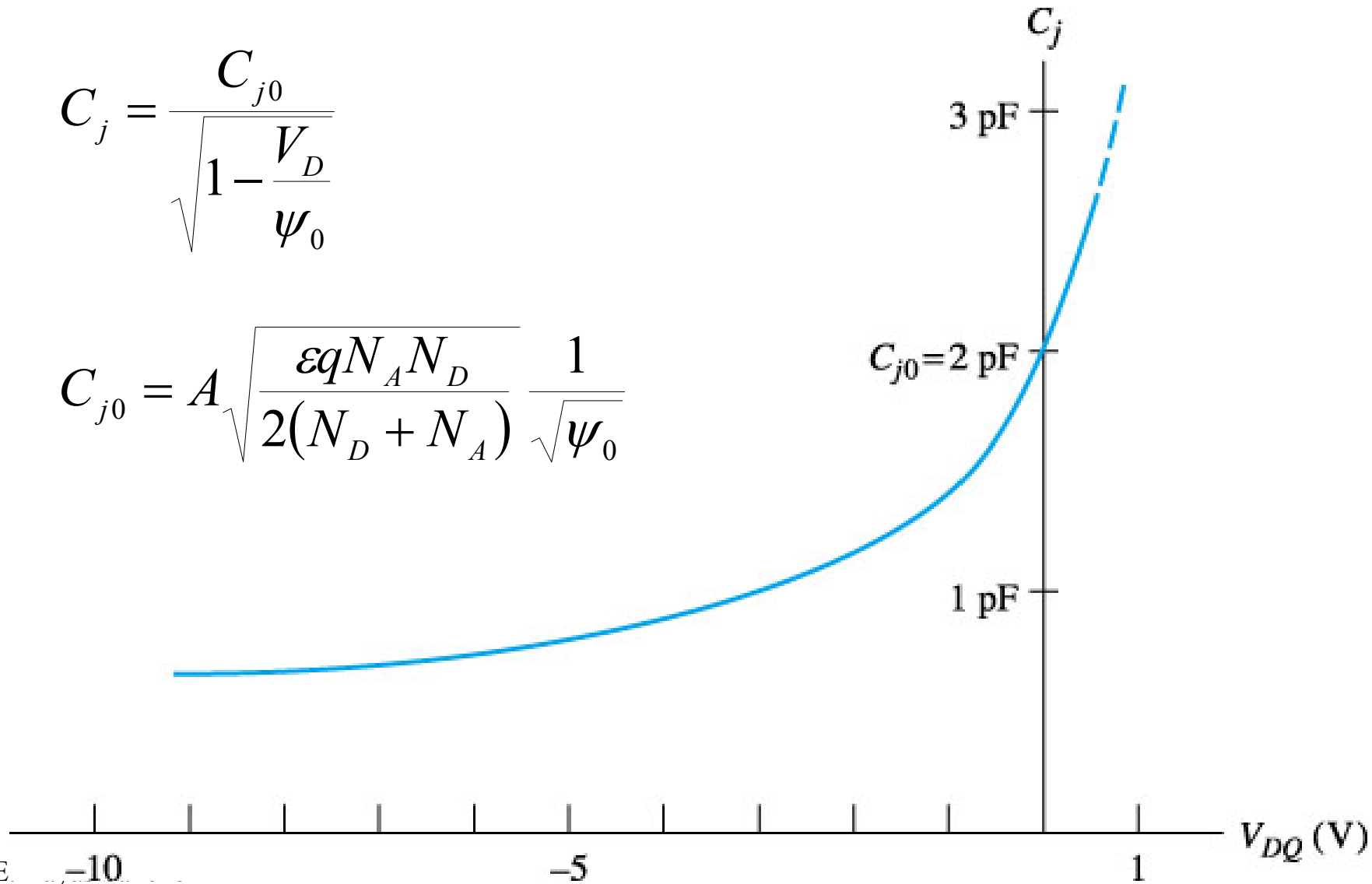
$$C_{j0} = A \sqrt{\frac{\varepsilon q N_A N_D}{2(N_D + N_A)}} \frac{1}{\sqrt{\psi_0}}$$

$$C_j = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\psi_0}}}$$

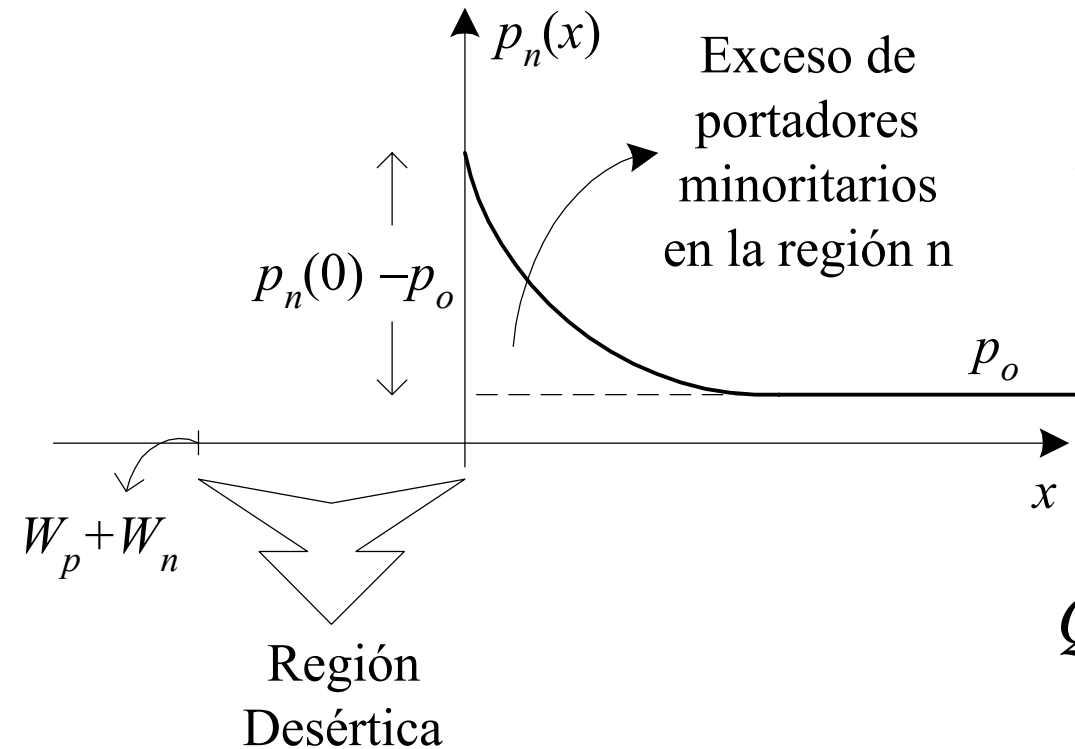
Capacitancia de la Región de Desértica (cont.)

$$C_j = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\psi_0}}}$$

$$C_{j0} = A \sqrt{\frac{\epsilon q N_A N_D}{2(N_D + N_A)}} \frac{1}{\sqrt{\psi_0}}$$



Capacitancia de Difusión



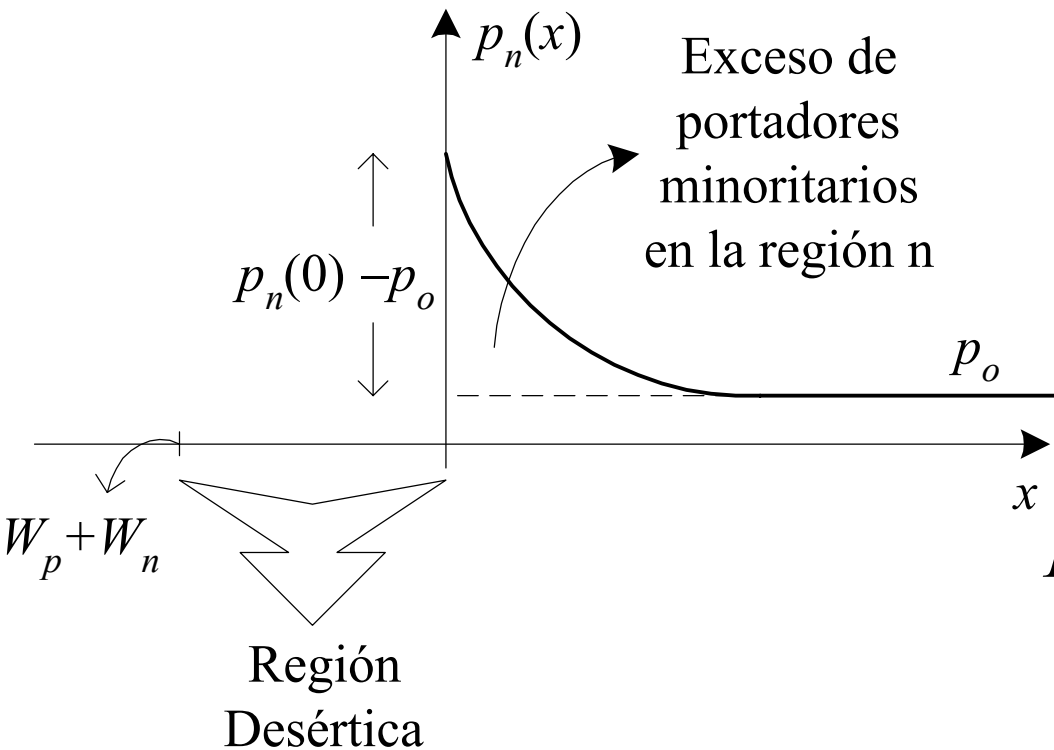
$$p(x) = (p_n(0) - p_o)e^{-x/L_p} + p_o$$

Q_p carga de los portadores minoritarios en exceso en la región n

$$Q_p = \int_0^{\infty} Aq(p_n(0) - p_o)e^{-x/L_p} dx$$

$$Q_p = AqL_p[p_n(0) - p_o]$$

Capacitancia de Difusión (cont.)



$$p(x) = [p_n(0) - p_o]e^{-x/L_p} + p_o$$

$$I_{pn} = J_p A$$

$$J_p = -qD_p \frac{dp}{dx}$$

$$I_{pn} = \frac{AqD_p}{L_p} [p_n(0) - p_o]e^{-x/L_p}$$

$$I_{pn}(0) = \frac{AqD_p}{L_p} [p_n(0) - p_o]$$

Como $Q_p = AqL_p [p_n(0) - p_o]$

\implies

$$I_{pn}(0) = \frac{Q_p D_p}{L_p^2}$$

y como

$$L_p \equiv \sqrt{D_p \tau_p} \implies I_{pn}(0) = \frac{Q_p}{\tau_p}$$

Capacitancia de Difusión (cont.)

$$I_{pn}(0) = \frac{Q_p}{\tau_p}$$

Similarmente $I_{np}(0) = \frac{Q_n}{\tau_n}$

$Q = Q_p + Q_n$ Carga del total de los portadores minoritarios en exceso en ambas regiones

$$Q = I_{pn}(0)\tau_p + I_{np}(0)\tau_n$$

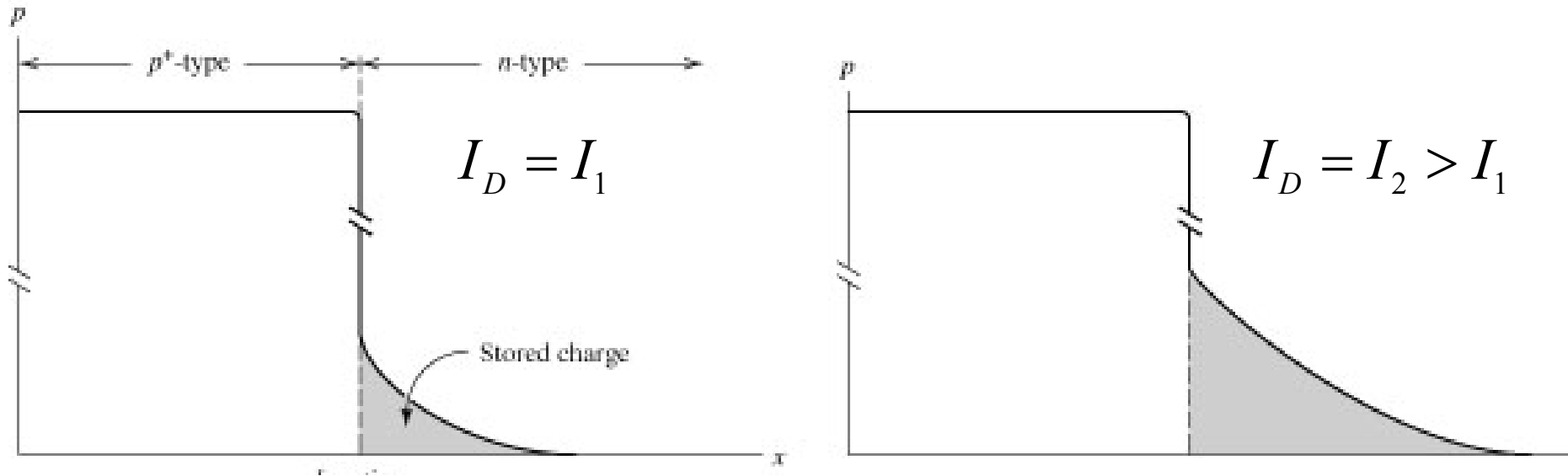
$$I_D = I_{pn}(0) + I_{np}(0)$$

$$\tau_p \approx \tau_n = \tau$$

$$C_d = \frac{dQ}{dV_D} = \tau \frac{dI_D}{dV_D} = \tau \frac{d}{dV_D} I_S e^{\frac{v_d}{V_T}} = \frac{\tau}{V_T} I_D$$

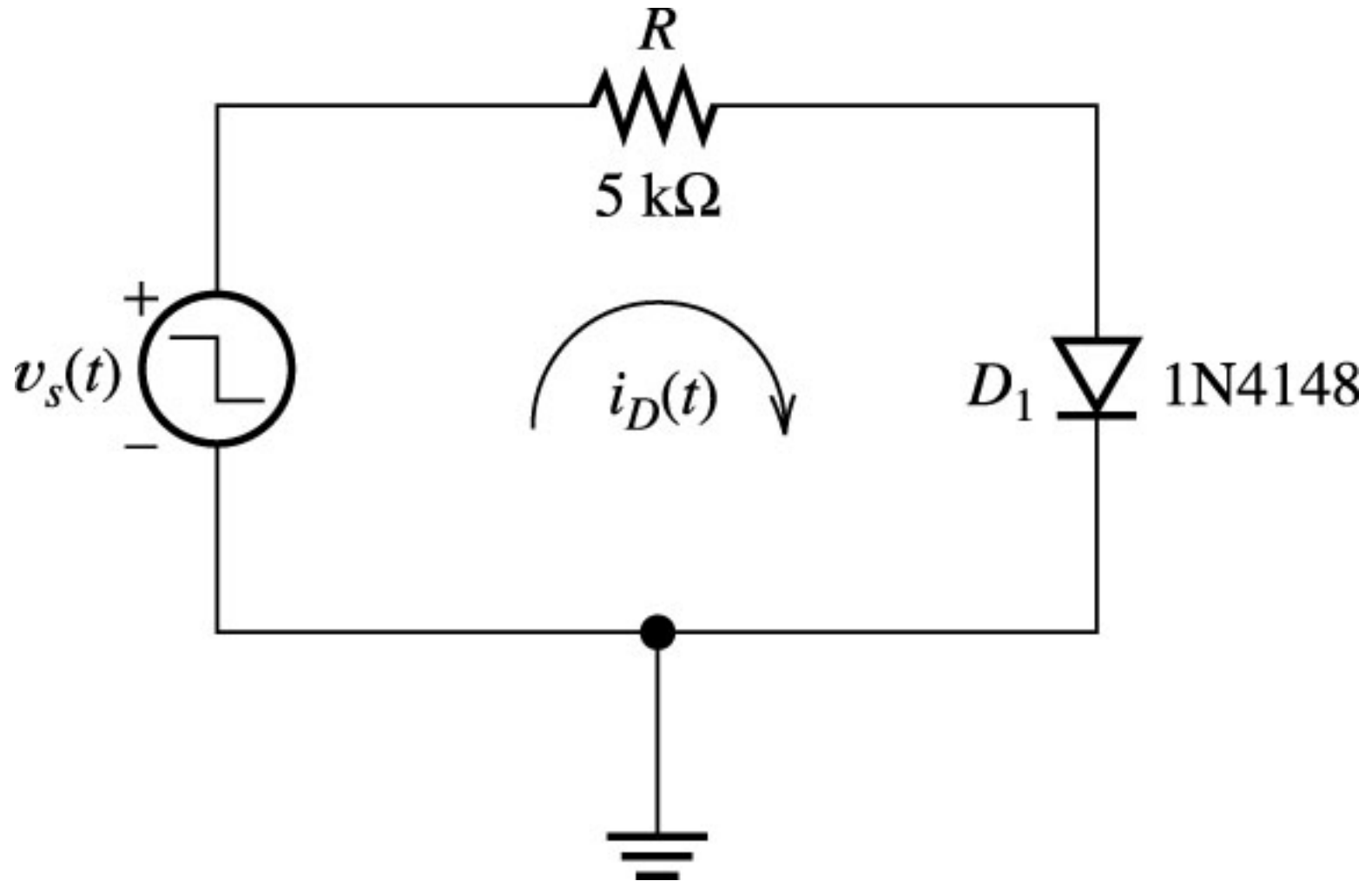
Capacitancia de Difusión (cont.)

$$C_d = \frac{\tau}{V_T} I_D$$

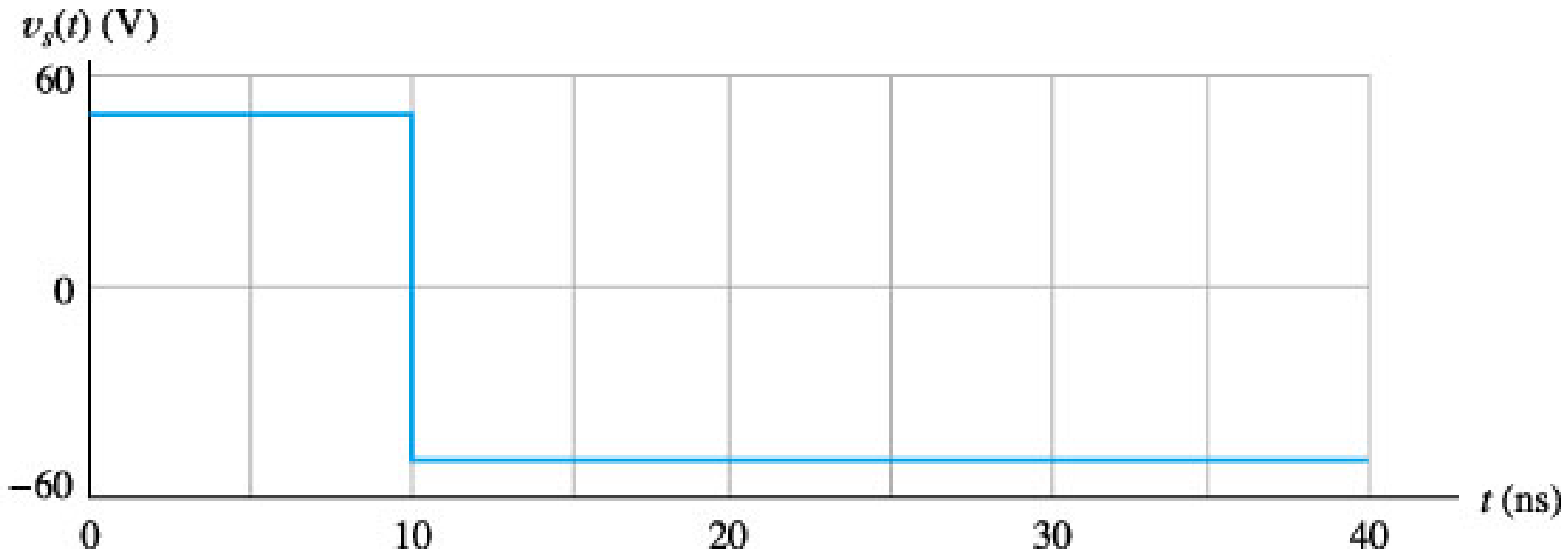


$$(C_d \gg C_j)$$

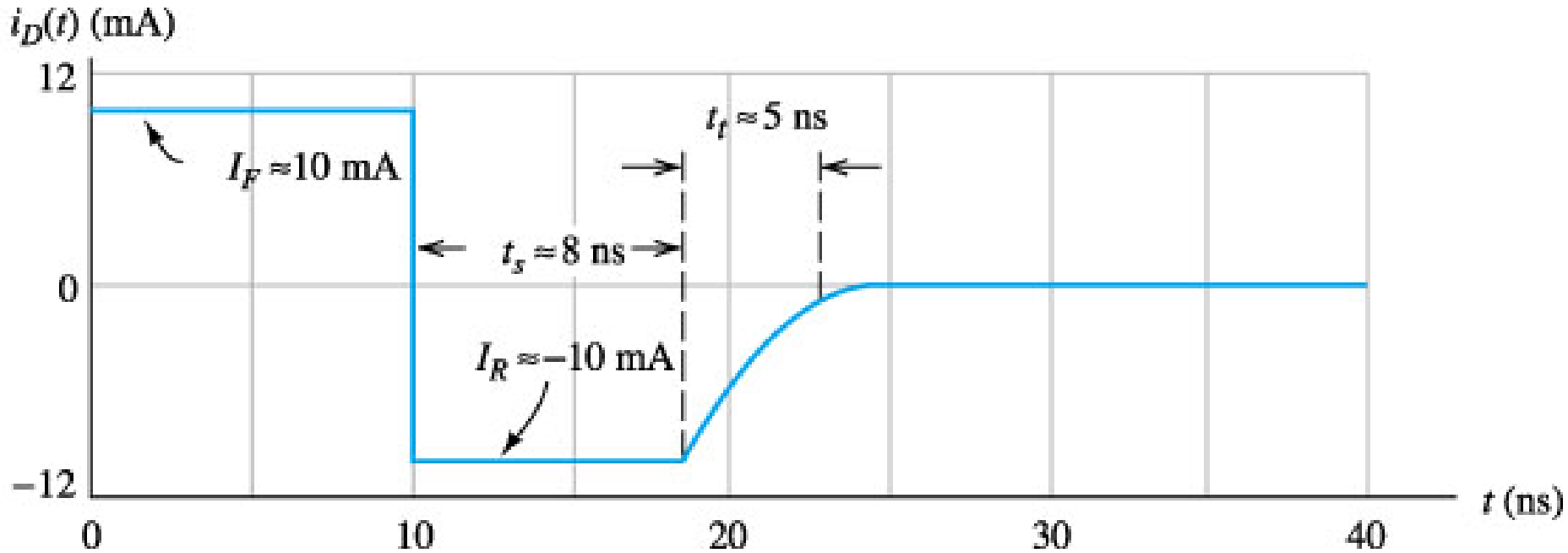
Efectos de C_j y C_d en la Conmutación



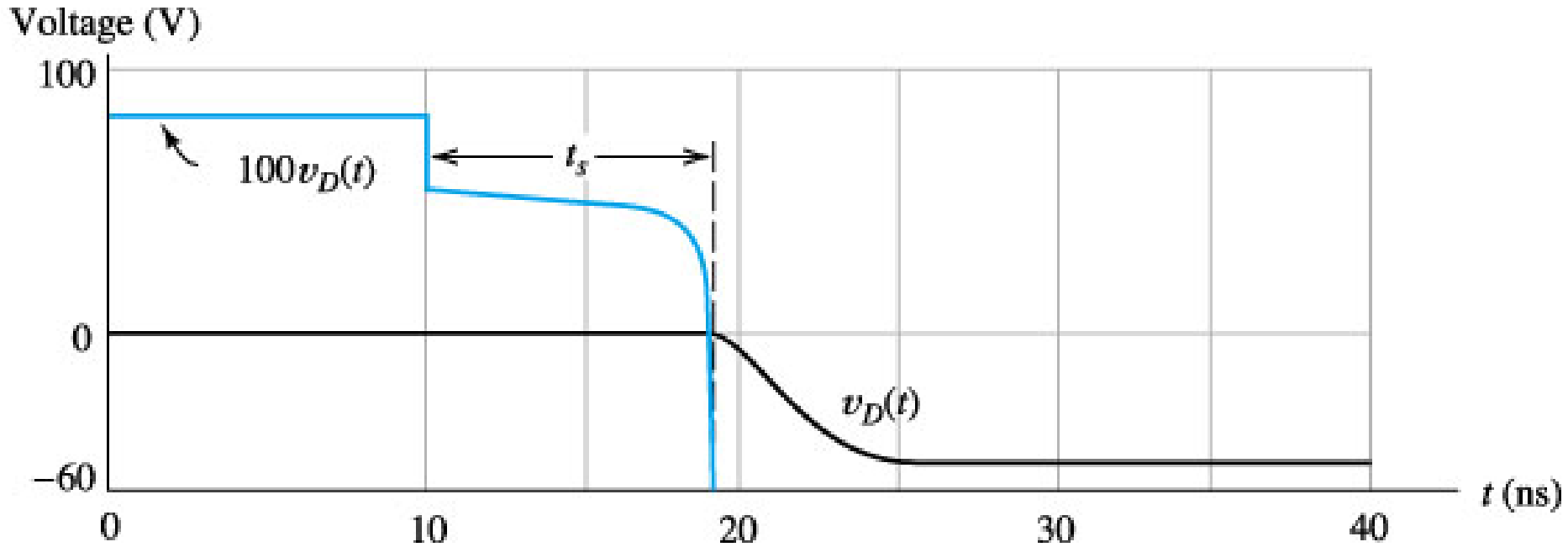
Efectos de C_j y C_d en la Conmutación (cont.)



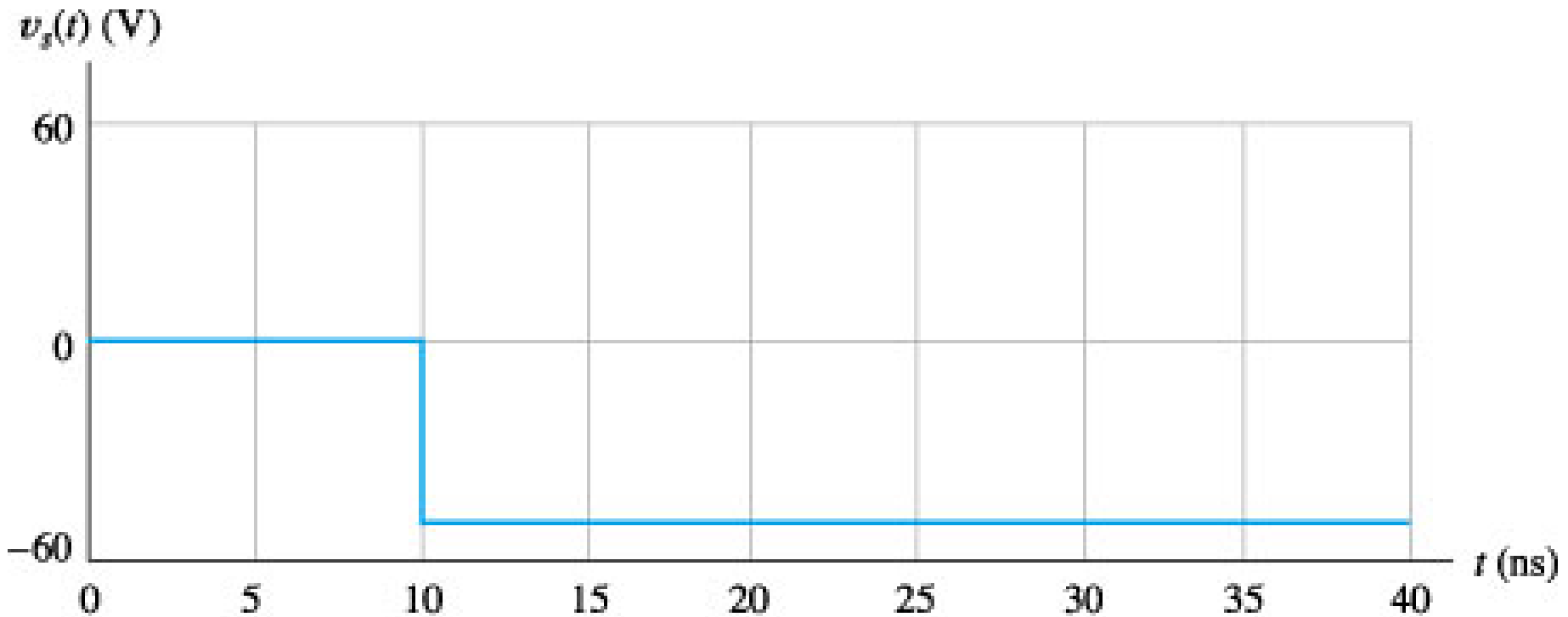
Efectos de C_j y C_d en la Conmutación (cont.)



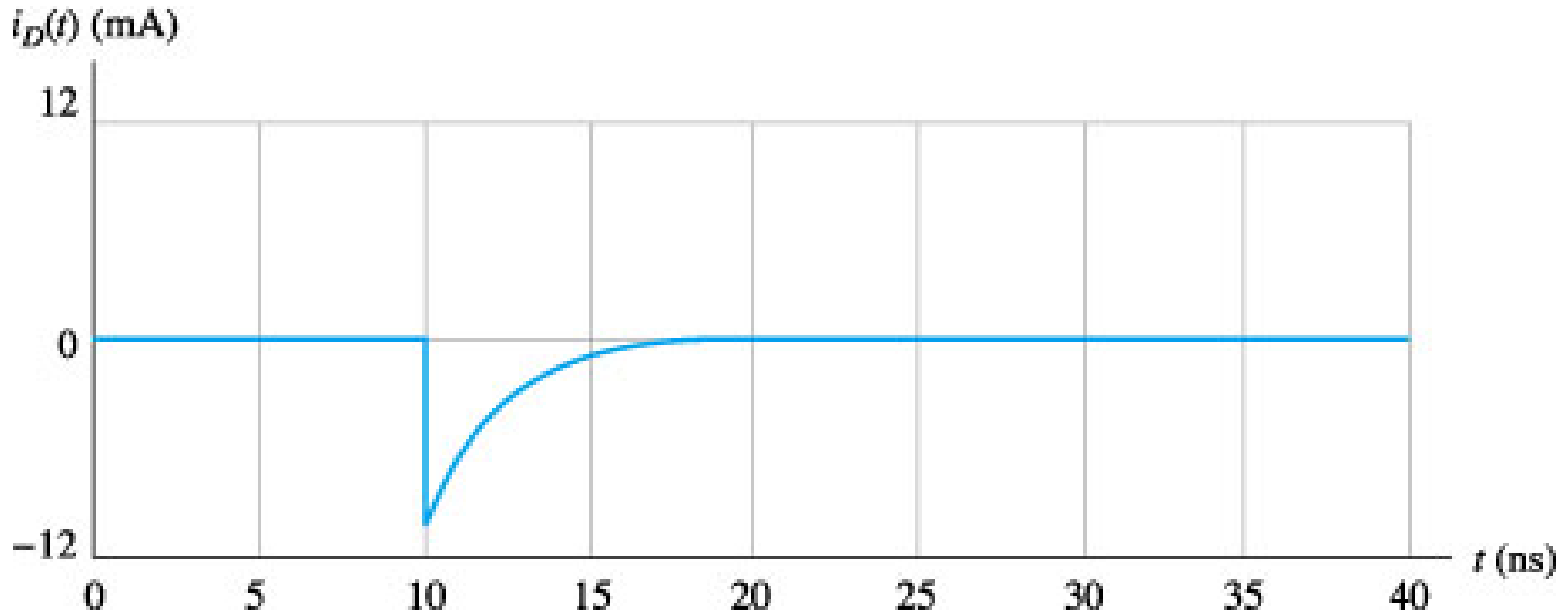
Efectos de C_j y C_d en la Conmutación (cont.)



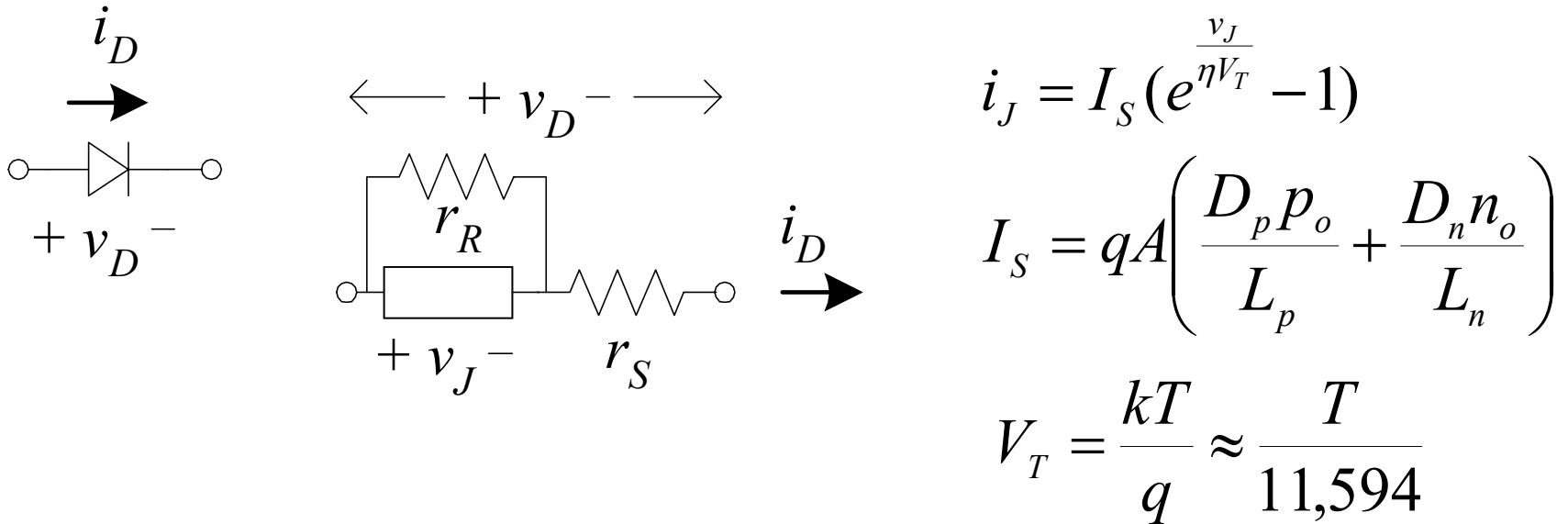
Efectos de C_j y C_d en la Conmutación (cont.)



Efectos de C_j y C_d en la Conmutación (cont.)



Modelo del Diodo para Señal Grande



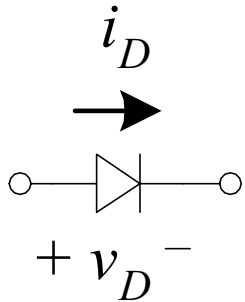
r_R Resistencia de reversa ($I_R \gg I_S$)

r_S Resistencia de volumen y de los contactos metálicos

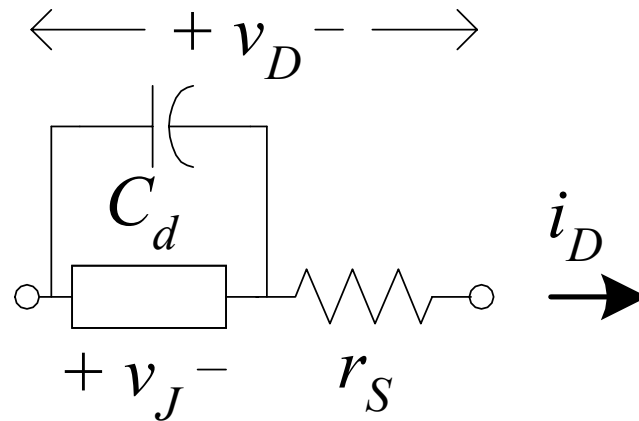
$$r_R \approx \frac{PIV}{I_{R_{\max}}}$$

$$r_S \approx \frac{V_{F_{\max}} - V_{F_{\text{nom}}}}{I_{F_{\max}} - I_{F_{\text{nom}}}}$$

Modelo del Diodo para Señal Grande (2)



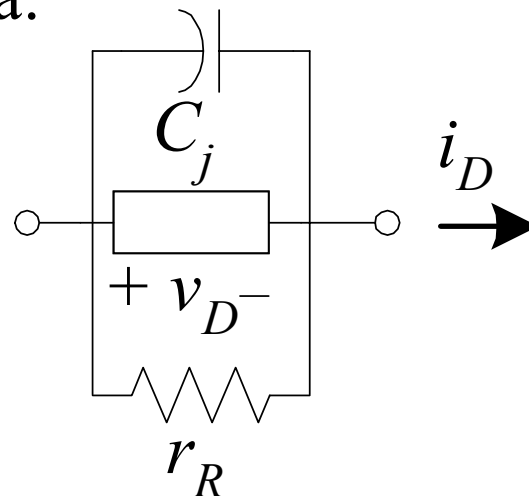
Polarización Directa:



$$i_J = I_S (e^{\frac{v_J}{\eta V_T}} - 1)$$

$$C_d = \frac{\tau}{V_T} i_J$$

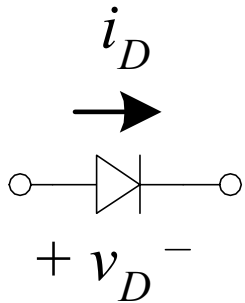
Polarización Inversa:



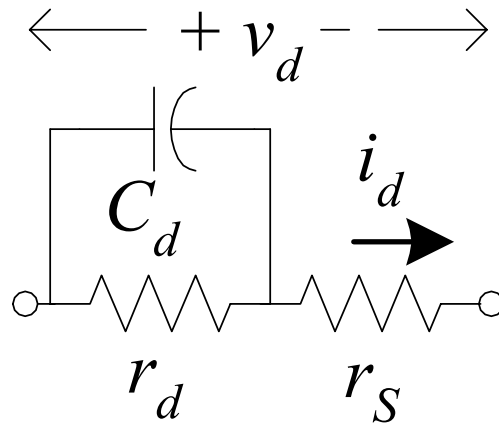
$$C_j = \frac{C_{j0}}{\sqrt{\psi_0 - V_D}}$$

$$C_{j0} = A \sqrt{\frac{\epsilon q N_A N_D}{2(N_D + N_A)}}$$

Modelo del Diodo para Señal Pequeña



Polarización Directa:



$$i_J = I_S (e^{\frac{v_J}{\eta V_T}} - 1) \approx I_S e^{\frac{v_J}{\eta V_T}}$$

$$\frac{di_J}{dv_J} = \frac{I_S}{\eta V_T} e^{\frac{v_J}{\eta V_T}} = \frac{I_D}{\eta V_T}$$

$$r_d = \frac{\eta V_T}{I_D}$$

Polarización Inversa:

