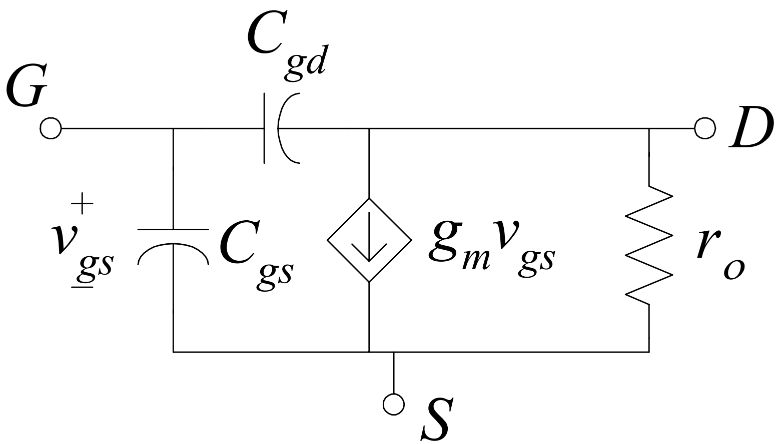
Frequency Response

(Part 2)

Dr. J.E. Rayas Sánchez

The FET High-Frequency Model



$$g_m = 2K(V_{GS} - V_t)$$

$$r_o = \frac{|V_A|}{I_{DS}}$$

$$V_A = 1/\lambda$$

$C_{iss} = C_{gs} + C_{gd}$ (input capacitance)

$C_{oss} = C_{ds} + C_{gd}$ (output capacitance)

$C_{rss} = C_{gd}$ (reverse capacitance)

The FET Unity-Gain Frequency (f_T)

It is the frequency at which the magnitude of the short-circuit current gain of the Common Source configuration becomes unity

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

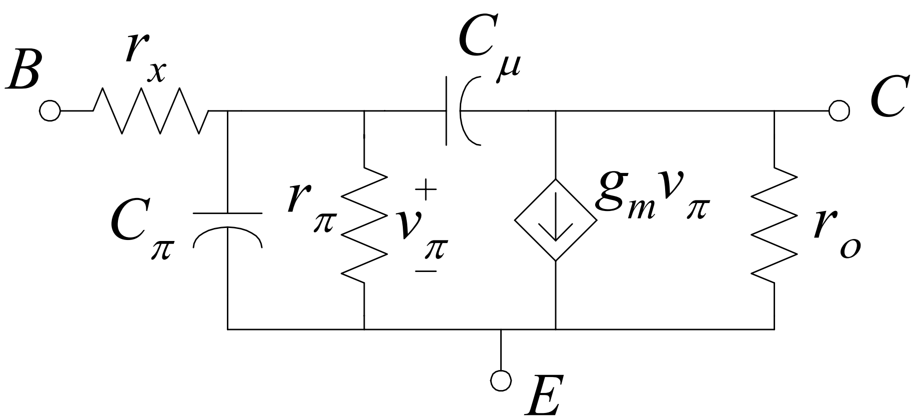
20 MHz $\leq f_T \leq$ 100 MHz for JFETs

100 MHz $\leq f_T \leq$ 2 GHz for MOSFETs

5 GHz $\leq f_T \leq$ 15 GHz for GaAs MESFETs

10 GHz $\leq f_T \leq$ 200 GHz for SiGe MOSFETs

The BJT High-Frequency Hybrid π Model



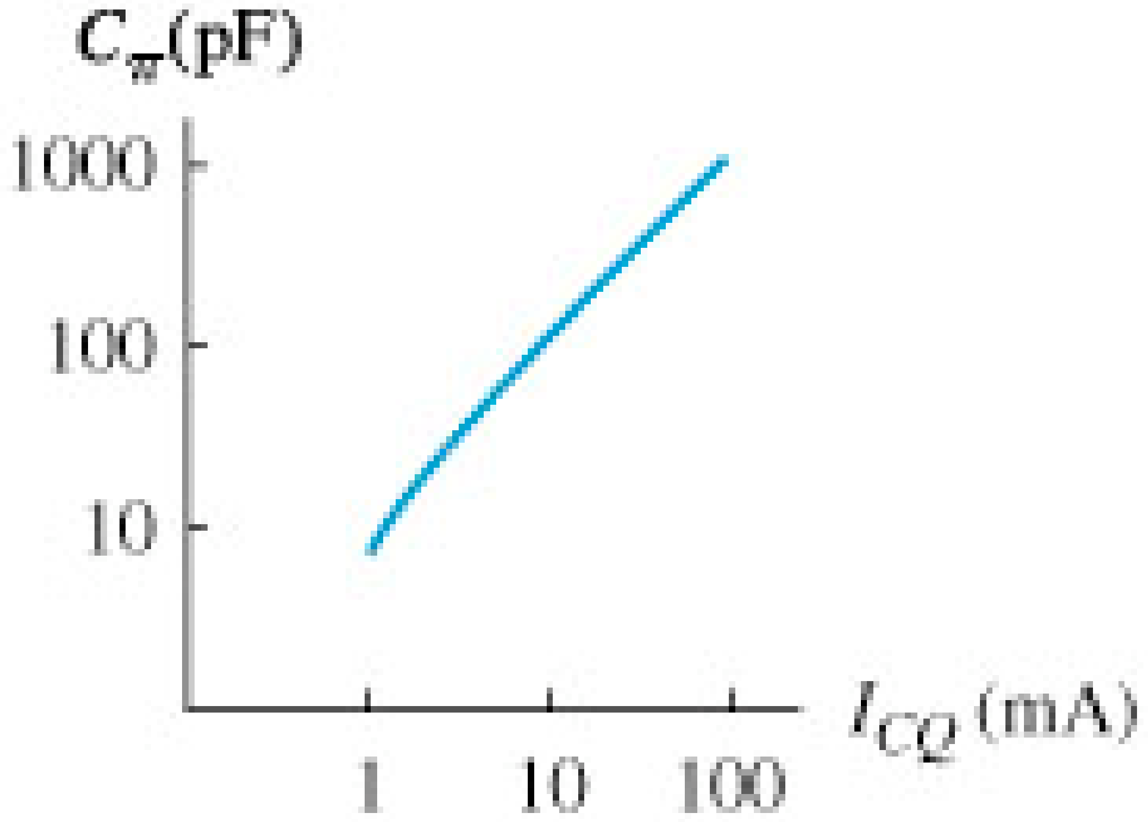
$$g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{\beta}{g_m} \qquad r_o = \frac{V_A}{I_C}$$

$$r_x = h_{ie} - r_\pi \qquad r_\mu = r_\pi / h_{re}$$

$C_\mu = C_{ob} = C_{cb}$ (Collector-Base capacitance)

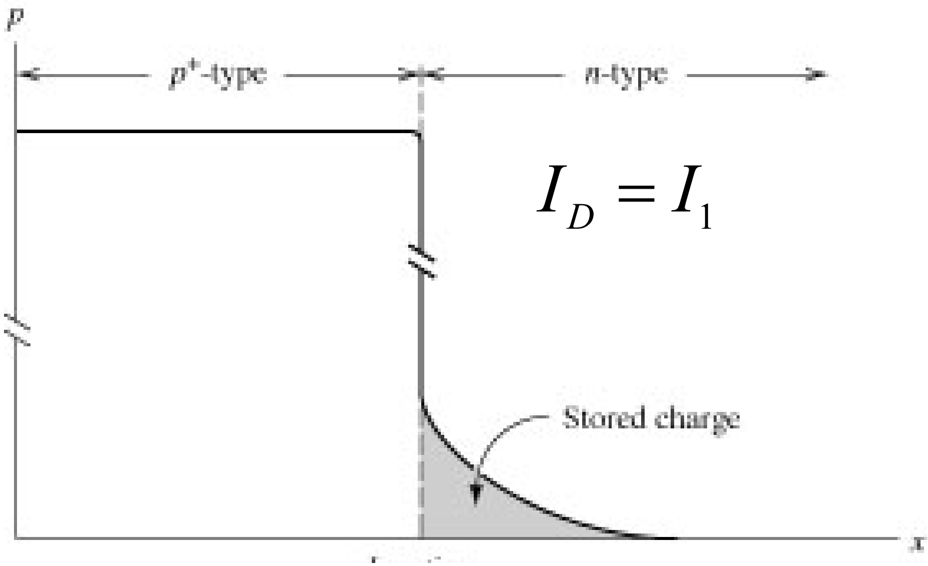
$C_\pi = C_{eb}$ (Base-Emitter capacitance)

Typical Behaviour of C_π

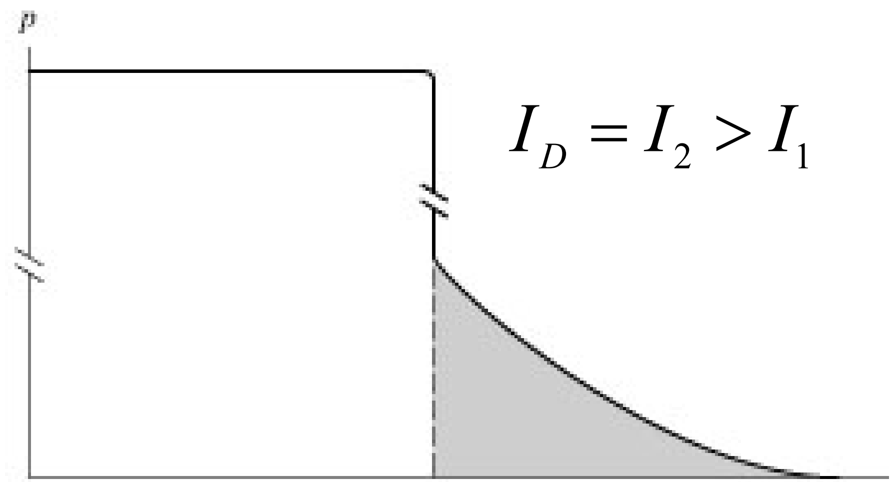


Capacitancia de Difusión (repaso)

$$C_d = \frac{\tau}{V_T} I_D$$



$$I_D = I_1$$



$$I_D = I_2 > I_1$$

$$(C_d >> C_j)$$

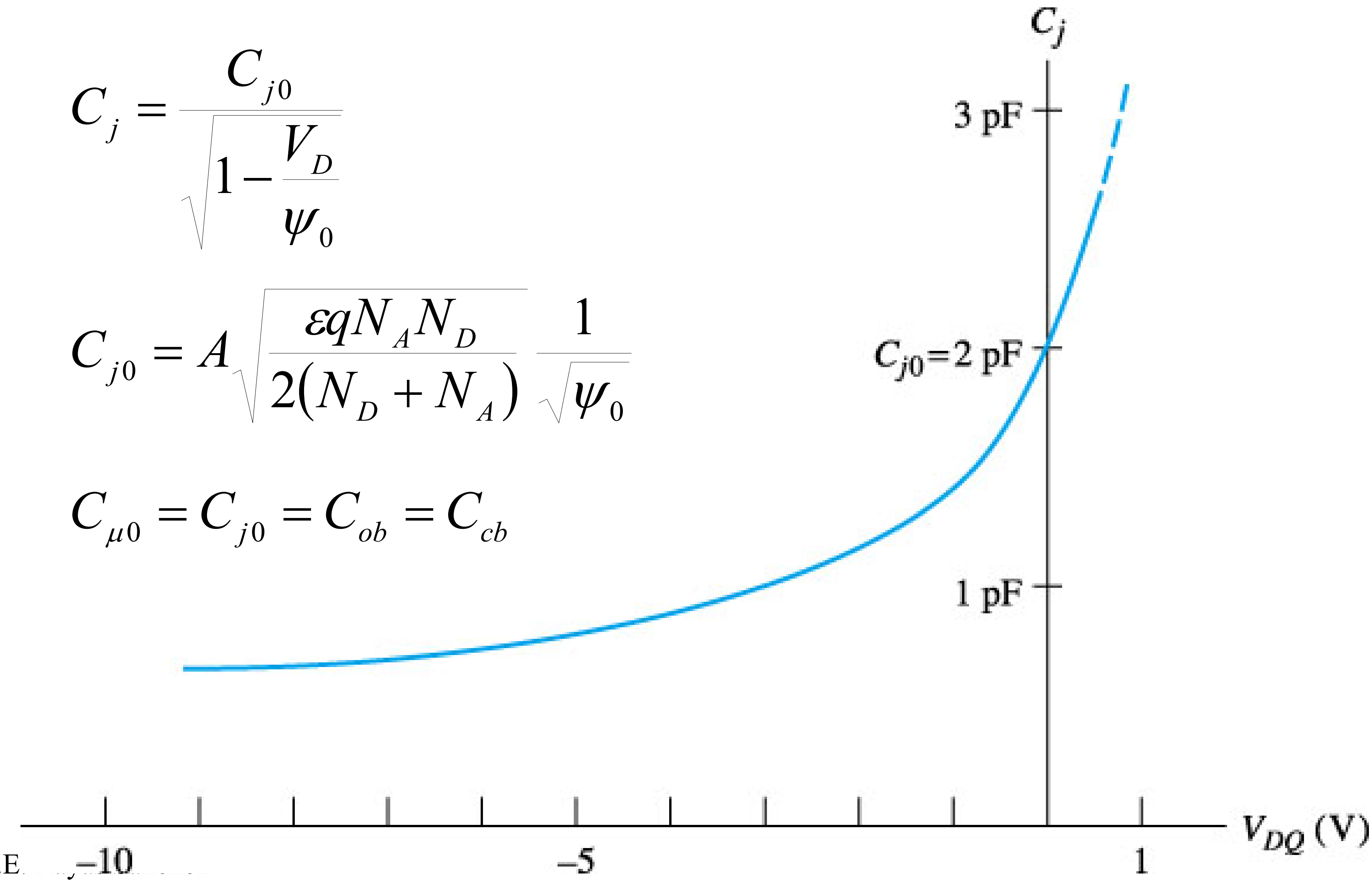
$$C_\pi = C_{eb}$$

Capacitancia de la Región de Desértica (repaso)

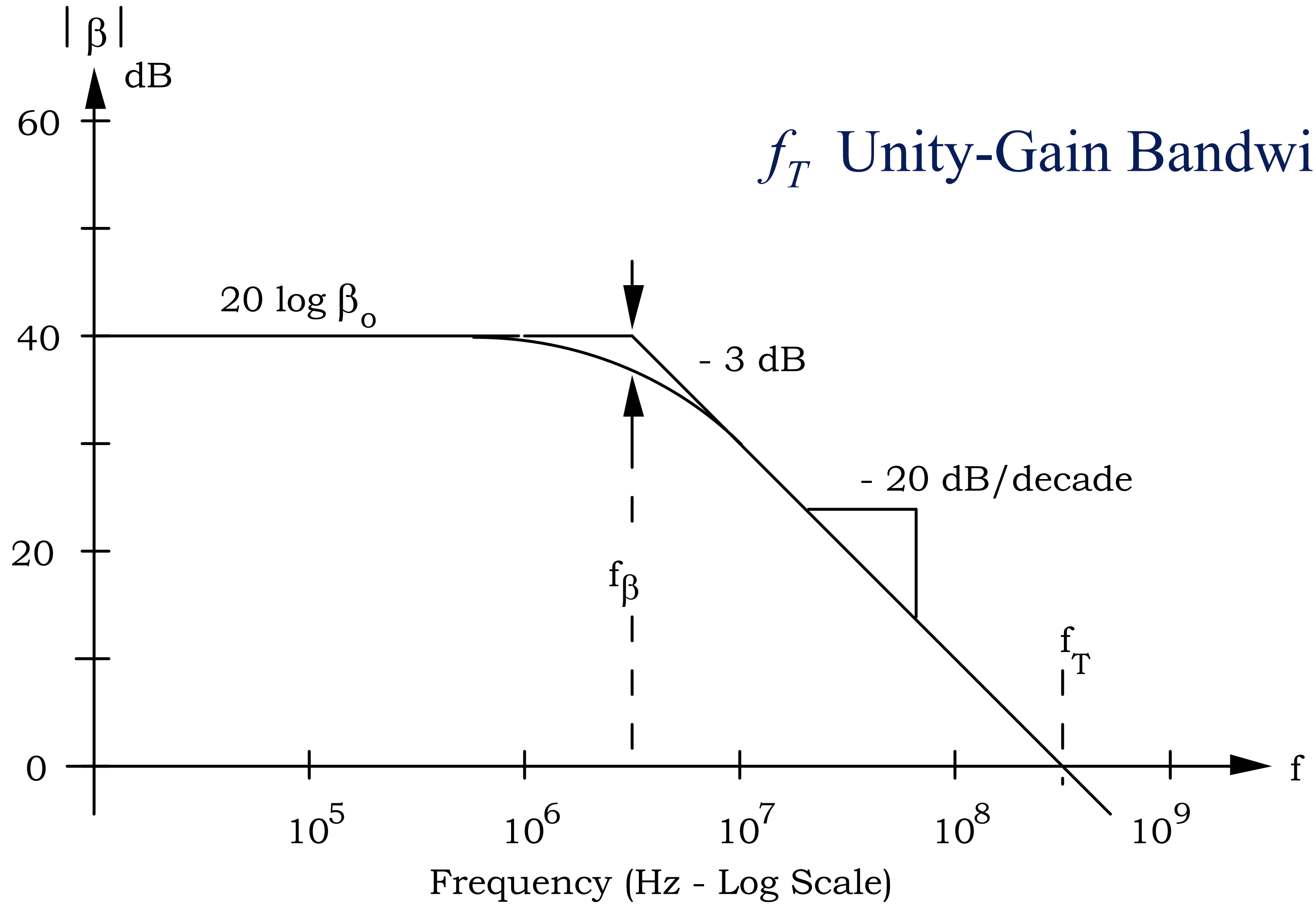
$$C_j = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\psi_0}}}$$

$$C_{j0} = A\sqrt{\frac{\varepsilon q N_A N_D}{2(N_D + N_A)}}\,\frac{1}{\sqrt{\psi_0}}$$

$$C_{\mu 0} = C_{j0} = C_{ob} = C_{cb}$$



Typical Behaviour of β



f_T Unity-Gain Bandwidth

The BJT Unity-Gain Bandwidth (f_T)

It is the frequency at which the magnitude of the short-circuit current gain of the common emitter configuration becomes unity

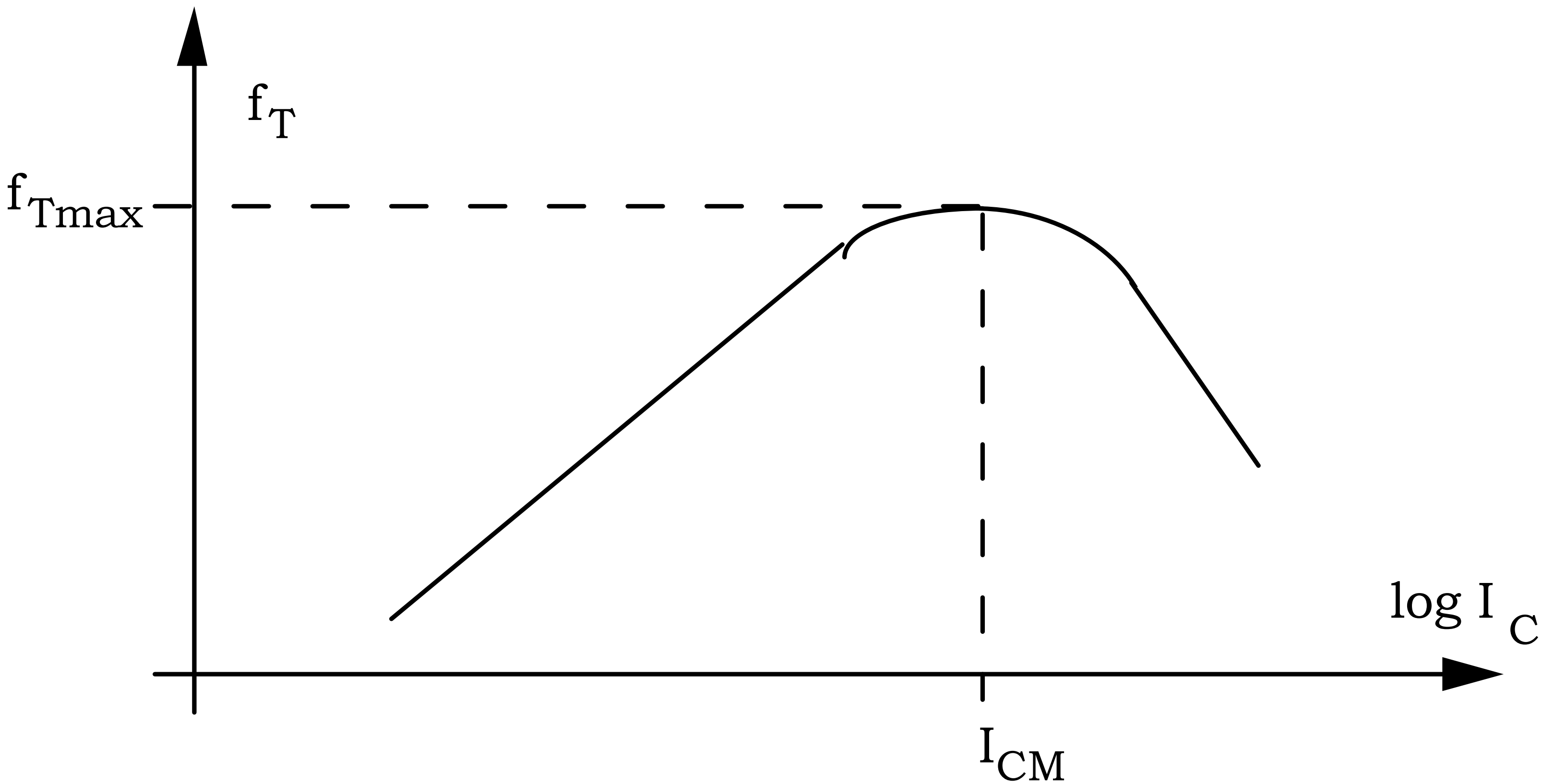
$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

2 MHz $\leq f_T \leq$ 100 MHz for general purpose BJTs

1 GHz $\leq f_T \leq$ 10 GHz for high speed BJTs

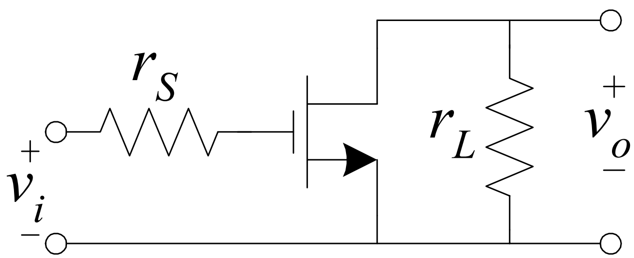
1 GHz $\leq f_T \leq$ 50 GHz for HBTs and HEMTs

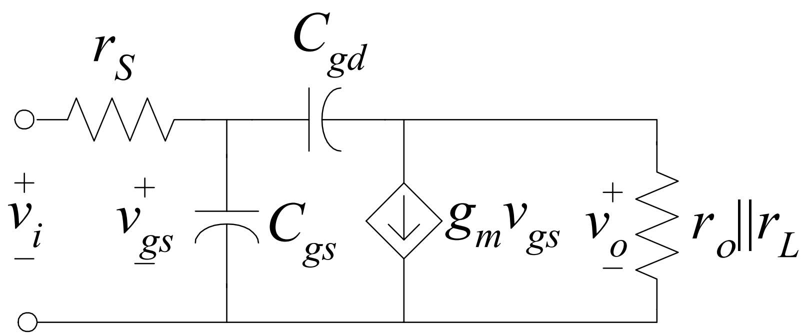
Typical Behaviour of f_T



High-Frequency Response of FET Amplifiers

Common Source





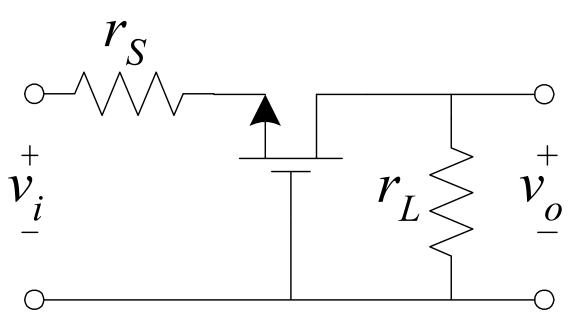
$$R_{gs} = r_S \qquad R_{gd} = \frac{v_{gs} - v_o}{v_{gs}/r_S} = r_S\left(1 - \frac{v_o}{v_{gs}}\right)$$

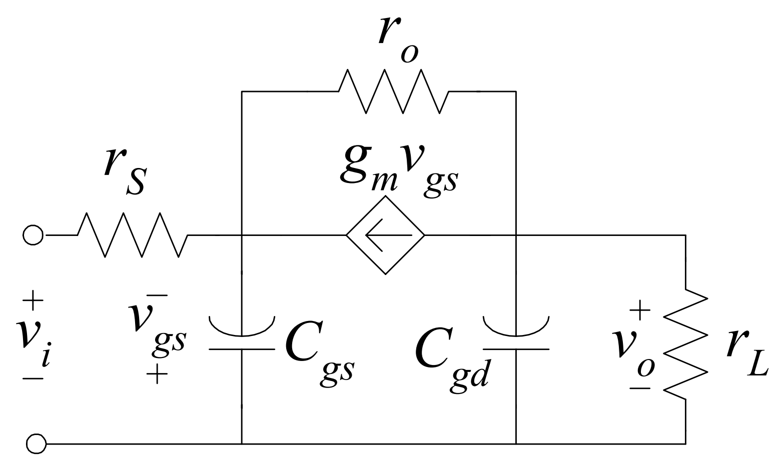
$$v_o = -\left(\frac{v_{gs}}{r_S} + g_m v_{gs}\right)(r_o \| r_L) \qquad R_{gd} = r_S[1 + (r_o \| r_L)(g_m + 1/r_S)]$$

$$\omega_H \approx 1/(R_{gs}C_{gs} + R_{gd}C_{gd})$$

High-Frequency Response of FET Amplifiers

Common Gate





$$R_{gs} = r_S \parallel Z_1 \qquad Z_1 = \frac{-v_{gs}}{v_o / r_L} \qquad v_o = \left[-g_m v_{gs} + \left(\frac{-v_{gs} - v_o}{r_o} \right) \right] r_L$$

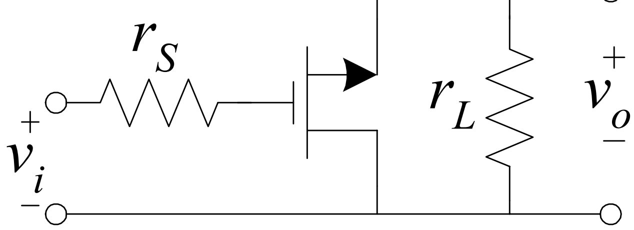
$$Z_1 = \frac{r_L (1 + r_L / r_o)}{g_m r_L + r_L / r_o} \qquad \text{if } r_o \gg r_L \text{ then } Z_1 \approx \frac{1}{g_m}$$

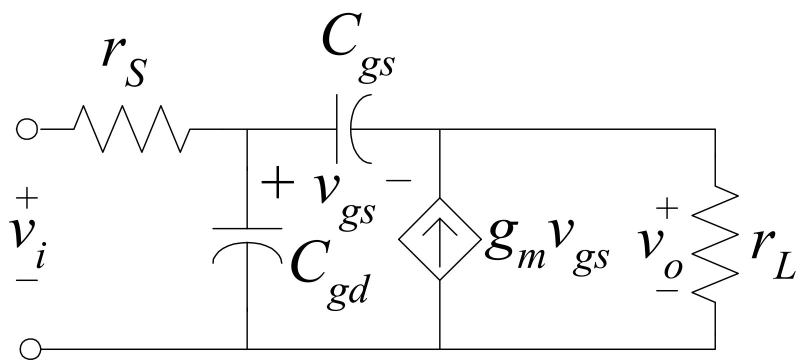
To calculate R_{gd}, assume $r_o \gg r_S$, then $v_{gs} = 0$: $\quad R_{gd} = r_o \parallel r_L$

$$\omega_H \approx 1/(R_{gs} C_{gs} + R_{gd} C_{gd})$$

High-Frequency Response of FET Amplifiers

Common Drain





$$R_{gd} = r_S \qquad R_{gs} = \frac{v_{gs}}{g_m v_{gs} - v_o / r_L} \qquad \frac{v_{gs} + v_o}{r_S} = g_m v_{gs} - \frac{v_o}{r_L}$$

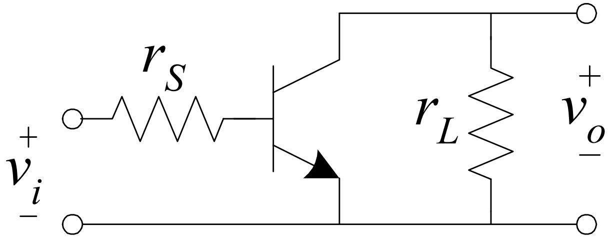
$$v_o = \frac{v_{gs}(g_m r_S - 1)}{1 + r_S / r_L} \approx \frac{-v_{gs}}{1 + r_S / r_L}$$

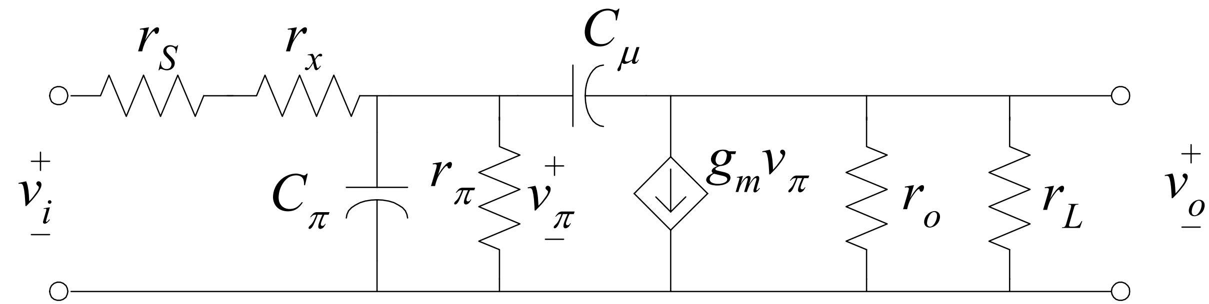
$$R_{gs} = \frac{1}{g_m + 1/(r_L + r_S)} = \frac{r_L + r_S}{1 + g_m (r_L + r_S)}$$

$$\omega_H \approx 1/(R_{gs} C_{gs} + R_{gd} C_{gd})$$

High-Frequency Response of BJT Amplifiers

Common Emitter





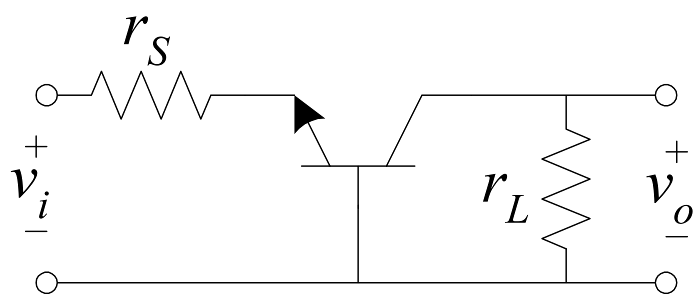
$$R_\pi = (r_S + r_x) \| r_\pi \qquad R_\mu = \frac{v_\pi - v_o}{v_\pi / R_\pi} = R_\pi \left(1 - \frac{v_o}{v_\pi}\right)$$

$$v_o = -\left(\frac{v_\pi}{R_\pi} + g_m v_\pi\right)(r_o \| r_L) \qquad R_\mu = R_\pi [1 + (r_o \| r_L)(g_m + 1/R_\pi)]$$

$$\omega_H \approx 1/(R_\pi C_\pi + R_\mu C_\mu)$$

High-Frequency Response of BJT Amplifiers

Common Base



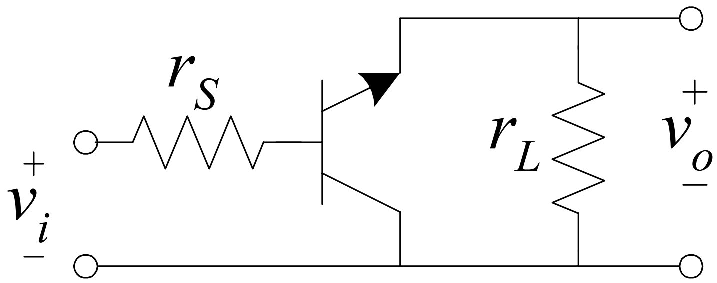
It can be shown that

$$R_\pi = r_\pi \parallel \frac{r_x + r_S}{1 + g_m r_S} \qquad R_\mu = r_L \parallel (r_o + r_x \parallel r_S) \approx r_L$$

$$\omega_H \approx 1/(R_\pi C_\pi + R_\mu C_\mu)$$

High-Frequency Response of BJT Amplifiers

Common Collector



It can be shown that

$$R_\pi = r_\pi \,||\, \frac{r_x + r_S + (r_L \,||\, r_o)}{1 + g_m (r_L \,||\, r_o)} \qquad R_\mu = r_\pi \,||\, \frac{r_L + r_S + r_x}{1 + g_m (r_L + r_S + r_x)}$$

$$\omega_H \approx 1/(R_\pi C_\pi + R_\mu C_\mu)$$

Comparison of Low-Frequency FET Responses

	R_{C_G}	R_{C_S}	R_{C_D}
CS	$R_{SS} + R_{G1} \parallel R_{G2}$	$R_S \parallel \frac{1}{g_m}$	$R_L + R_D$
CG	$R_{G1} \parallel R_{G2}$	$R_{SS} + (R_S \parallel \frac{1}{g_m})$	$R_D + R_L$
CD	$R_{SS} + R_{G1} \parallel R_{G2}$	$(R_S \parallel \frac{1}{g_m}) + R_L$	R_D

$$\omega_L \approx \frac{1}{R_{C_G} C_G} + \frac{1}{R_{C_S} C_S} + \frac{1}{R_{C_D} C_D}$$

Comparison of Low-Frequency BJT Responses

	R_{C_B}	R_{C_E}	R_{C_C}
CE	$R_S + (R_{B1} \parallel R_{B2} \parallel r_\pi)$	$R_E \parallel \dfrac{r_\pi + (R_S \parallel R_{B1} \parallel R_{B2})}{\beta + 1}$	$R_L + (R_C \parallel r_o)$
CB	$R_{B1} \parallel R_{B2} \parallel [r_\pi + (1+\beta)(R_E \parallel R_S)]$	$R_S + R_E \parallel r_\pi \parallel 1/g_m$	$R_L + (R_C \parallel r_o)$
CC	$R_S + R_{B1} \parallel R_{B2} \parallel [r_\pi + (1+\beta)(R_E \parallel R_L)]$	$R_L + R_E \parallel \dfrac{r_\pi + (R_S \parallel R_{B1} \parallel R_{B2})}{\beta + 1}$	$R_C \parallel [r_o + (R_E \parallel R_L)]$

$$\omega_L \approx \frac{1}{R_{C_B} C_B} + \frac{1}{R_{C_E} C_E} + \frac{1}{R_{C_C} C_C}$$

Comparison of High-Frequency FET Responses

	R_{gs}	R_{gd}
CS	r_S	$r_S[1+(r_o \parallel r_L)(g_m+1/r_S)]$
CG	$r_S \parallel \dfrac{1}{g_m}$	$r_o \parallel r_L$
CD	$\dfrac{r_L+r_S}{1+g_m(r_L+r_S)}$	r_S

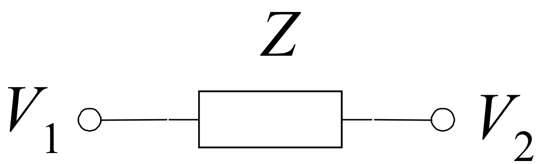
$$\omega_H \approx 1/(R_{gs}C_{gs}+R_{gd}C_{gd})$$

Comparison of High-Frequency BJT Responses

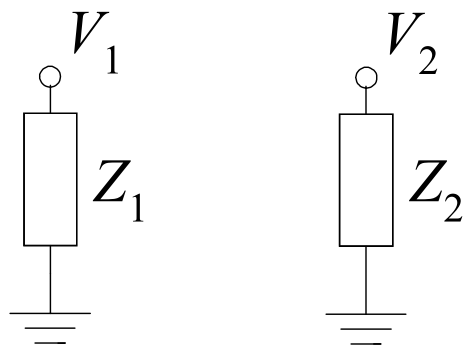
	R_π	R_μ
CE	$(r_S + r_x) \parallel r_\pi$	$R_\pi[1+(r_o \parallel r_L)(g_m + 1/R_\pi)]$
CB	$r_\pi \parallel \dfrac{r_x + r_S}{1+g_m r_S}$	$r_L \parallel (r_o + r_x \parallel r_S)$
CC	$r_\pi \parallel \dfrac{r_x + r_S + (r_L \parallel r_o)}{1+g_m(r_L \parallel r_o)}$	$r_\pi \parallel \dfrac{r_L + r_S + r_x}{1+g_m(r_L + r_S + r_x)}$

$$\omega_H \approx 1/(R_\pi C_\pi + R_\mu C_\mu)$$

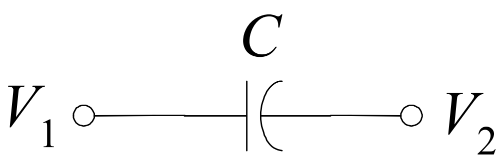
Miller´s Theorem

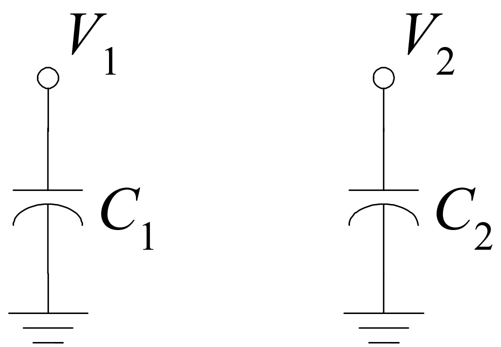


$$A_V = \frac{V_2}{V_1}$$



$$Z_1 = \frac{Z}{1 - A_V} \qquad Z_2 = \frac{Z A_V}{A_V - 1}$$



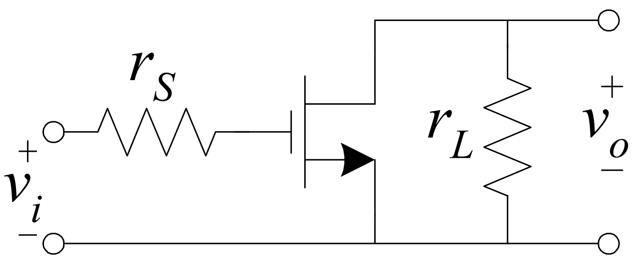


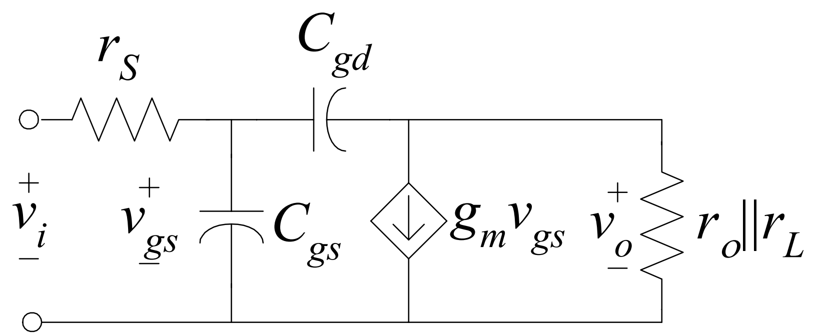
$$C_1 = C(1 - A_V)$$

$$C_2 = C\left(\frac{A_V - 1}{A_V}\right)$$

Miller Effect

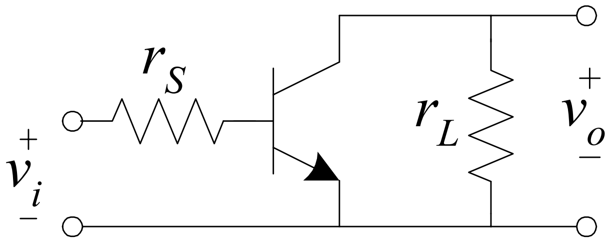
Common Source

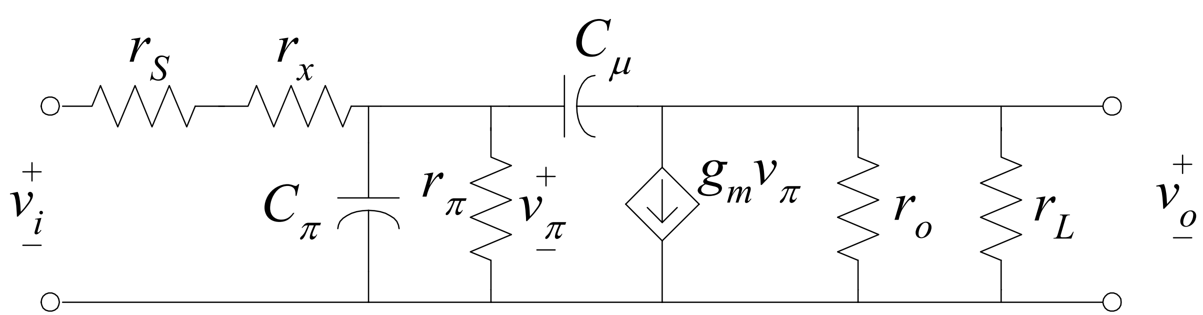




$$R_{gd} = r_S[1 + (r_o \parallel r_L)(g_m + 1/r_S)]$$

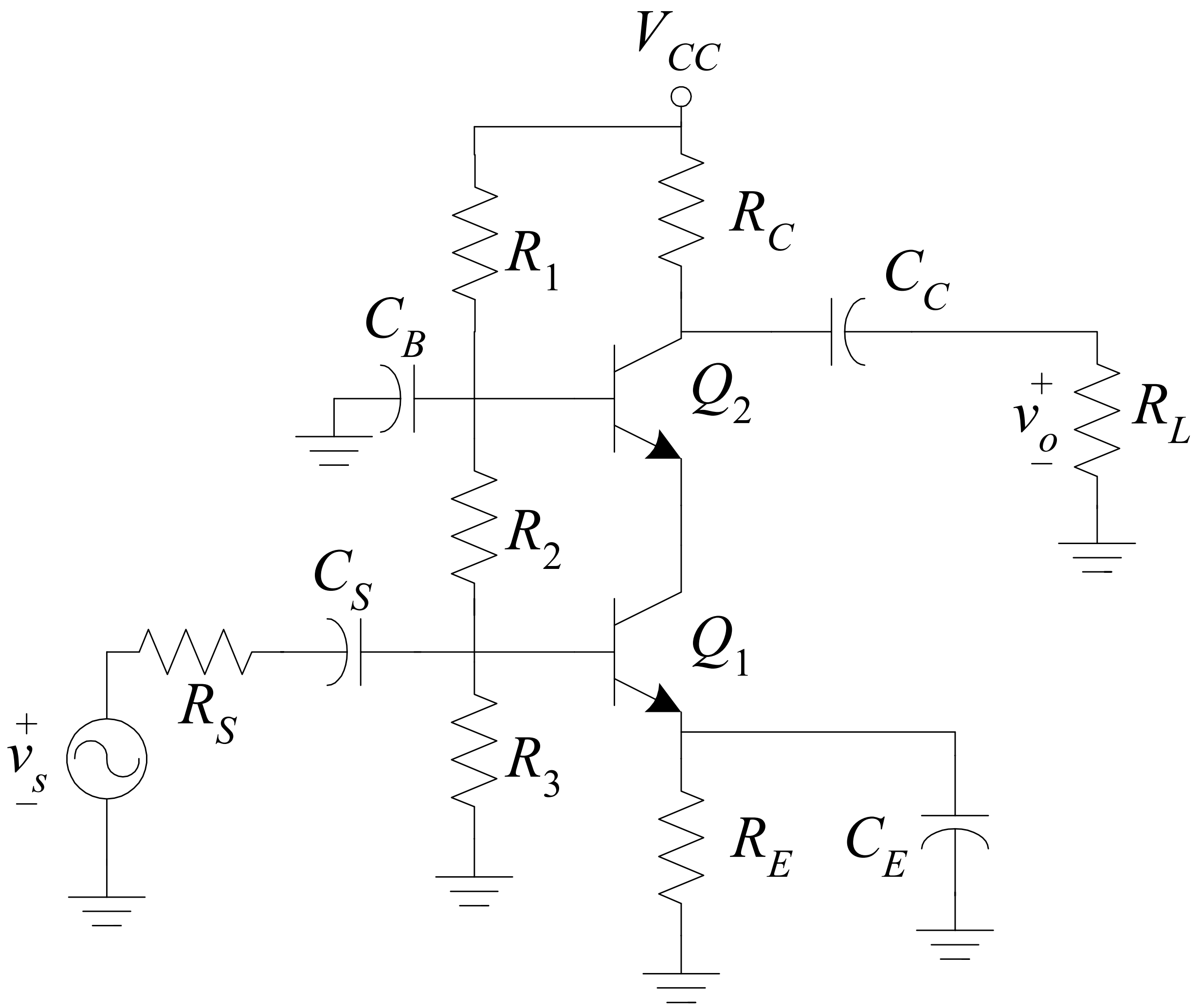
Common Emitter



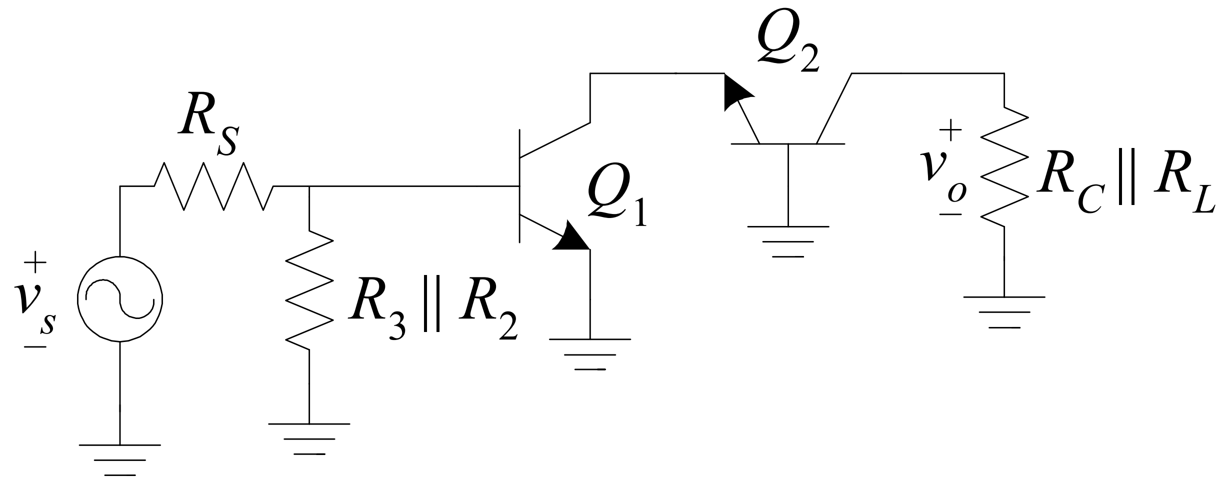


$$R_{\mu} = R_{\pi}[1 + (r_o \parallel r_L)(g_m + 1/R_{\pi})]$$

The Cascode Configuration

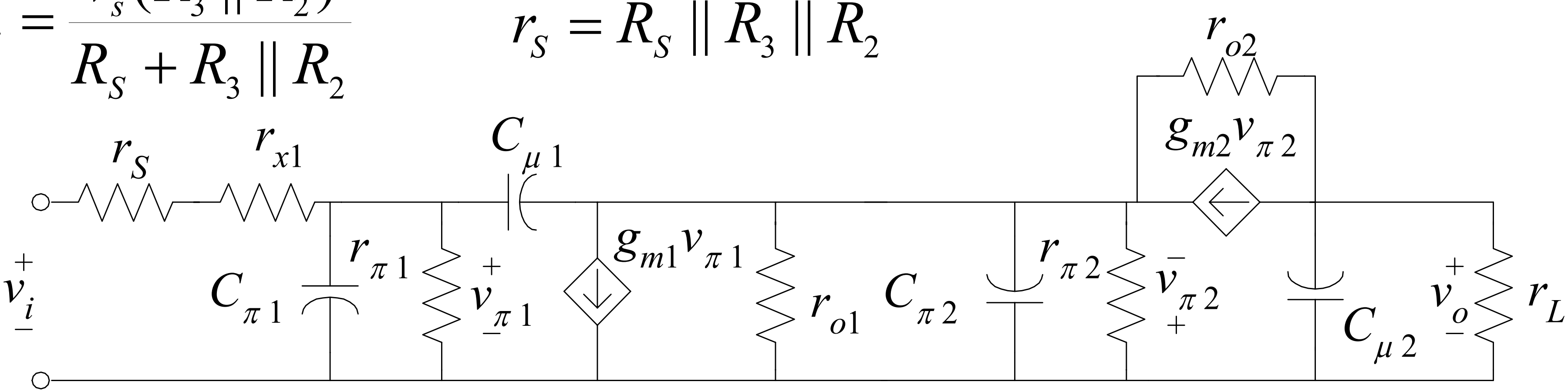


The Cascode Configuration (cont)

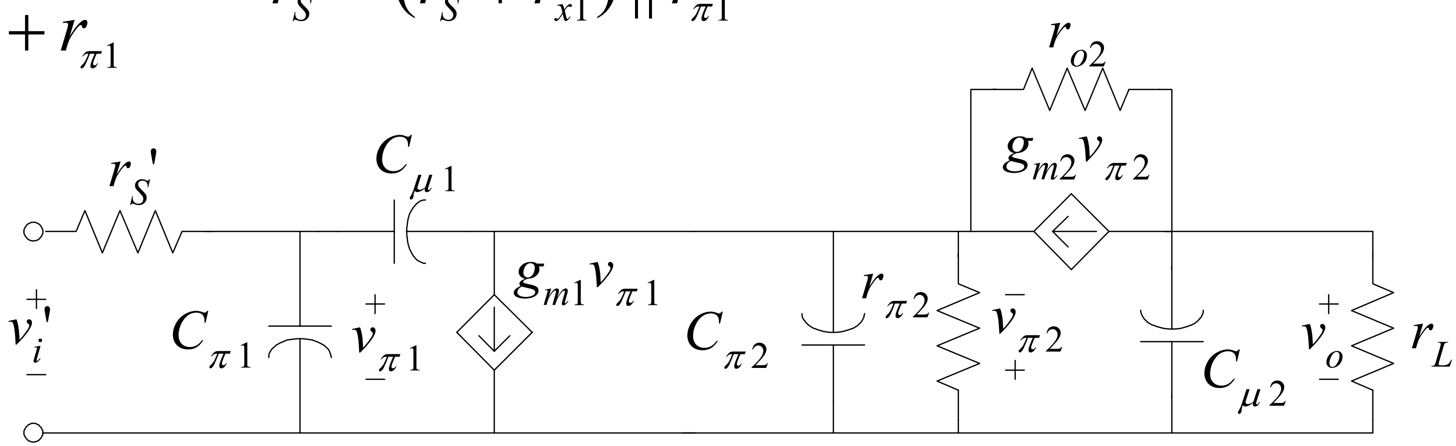


The Cascode Amplifier at High Frequencies

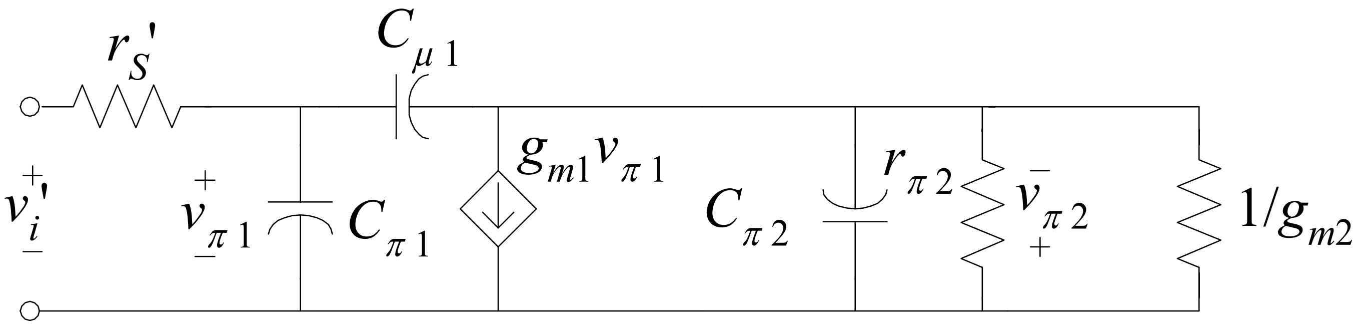
$$v_i = \frac{v_s (R_3 \| R_2)}{R_S + R_3 \| R_2} \qquad r_S = R_S \| R_3 \| R_2$$



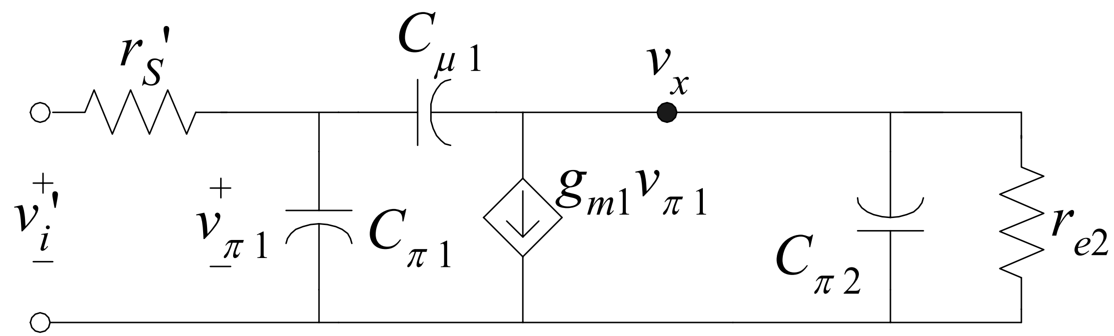
$$v_i' = \frac{v_i r_{\pi 1}}{r_S + r_{x1} + r_{\pi 1}} \qquad r_S' = (r_S + r_{x1}) \| r_{\pi 1}$$



The Cascode Amplifier at High Frequencies



$$r_{\pi 2} \| \frac{1}{g_{m2}} = r_{e2}$$



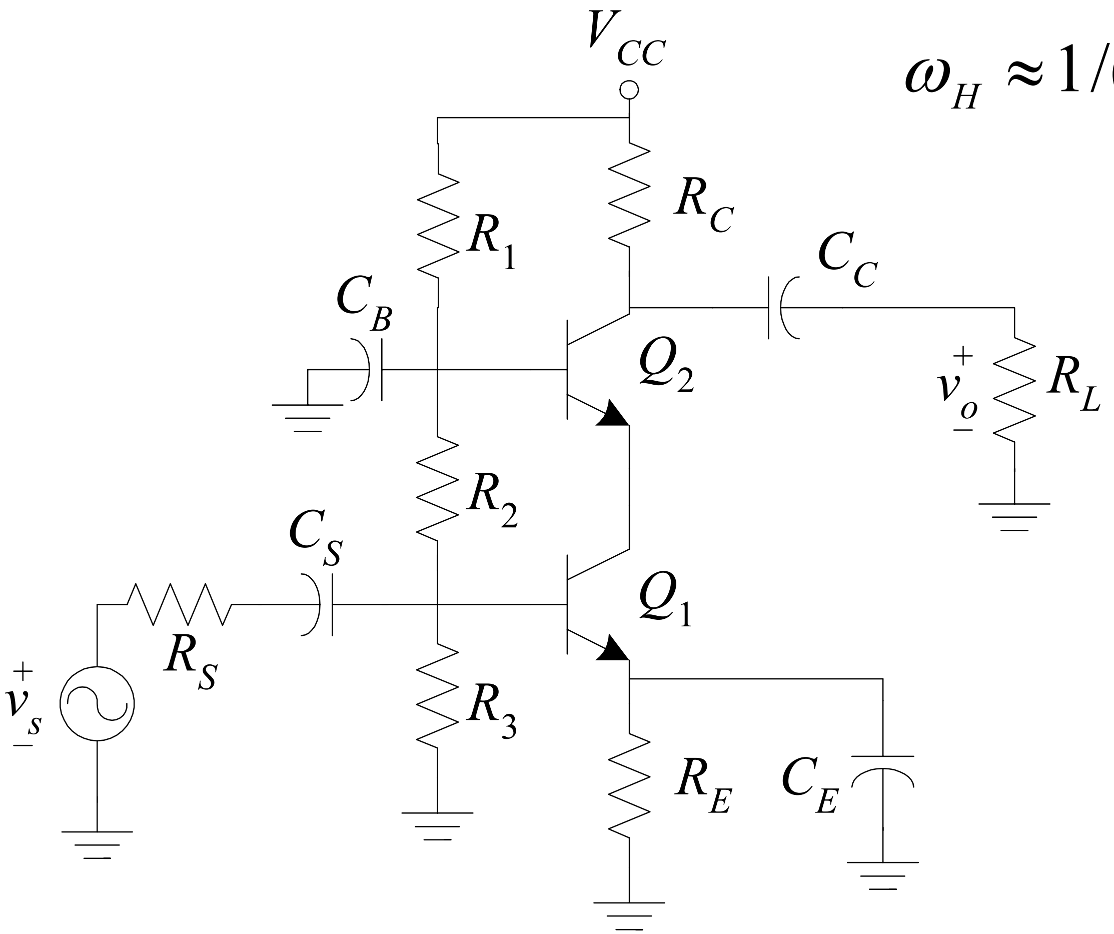
$$R_{\pi 1} = r_S' \qquad R_{\pi 2} = r_{e2}$$

$$R_{\mu 1} = \frac{v_{\pi 1} - v_x}{v_{\pi 1} / r_S'} = r_S'(1 - \frac{v_x}{v_{\pi 1}})$$

$$v_x = -(\frac{v_{\pi 1}}{r_S'} + g_{m1}v_{\pi 1})r_{e2}$$

$$R_{\mu 1} = r_S'(1 + g_m r_{e2} + \frac{r_{e2}}{r_S'}) \approx 2r_S' + r_{e2}$$

The Cascode Configuration



$$\omega_H \approx 1/(R_{\pi 1}C_{\pi 1} + R_{\pi 2}C_{\pi 2} + R_{\mu 1}C_{\mu 1})$$

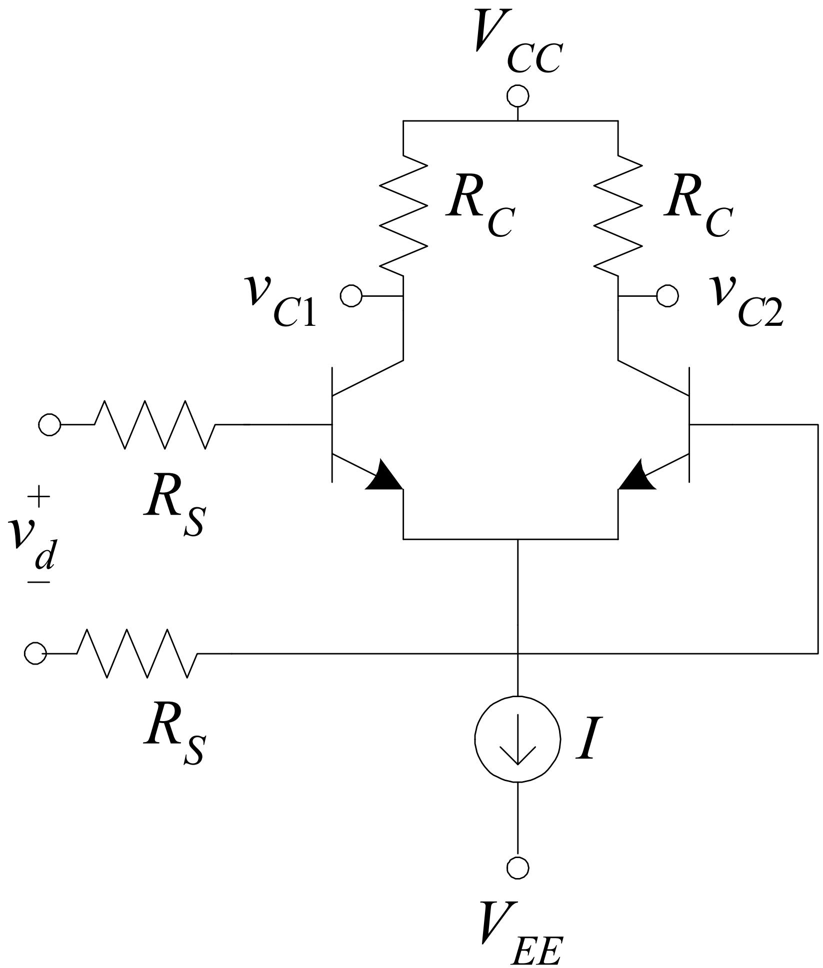
$$R_{\pi 1} = r_S' \qquad R_{\pi 2} = r_{e2}$$

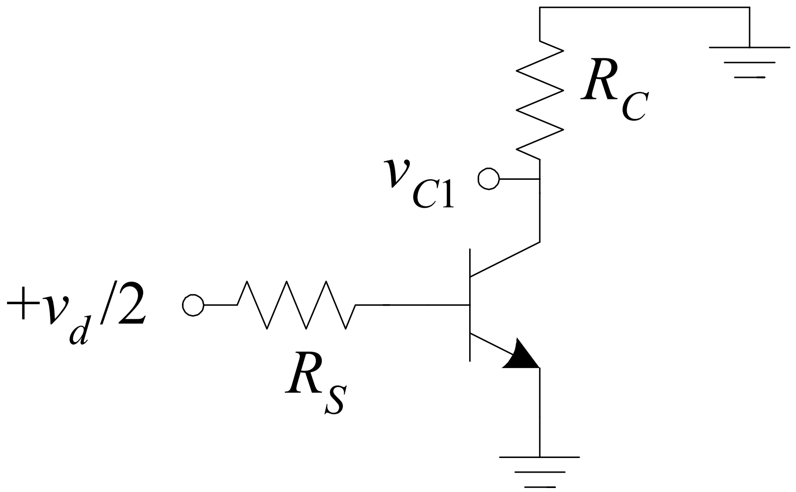
$$R_{\mu 1} = 2r_S' + r_{e2}$$

$$r_S' = (r_S + r_{x1}) \parallel r_{\pi 1}$$

$$r_S = R_S \parallel R_3 \parallel R_2$$

Frequency Response of a Differential Amplifier





$$R_\pi = (R_S + r_x) \parallel r_\pi$$

$$R_\mu = R_\pi[1 + (r_o \parallel R_C)(g_m + 1/R_\pi)]$$

$$\omega_H \approx 1/(R_\pi C_\pi + R_\mu C_\mu)$$

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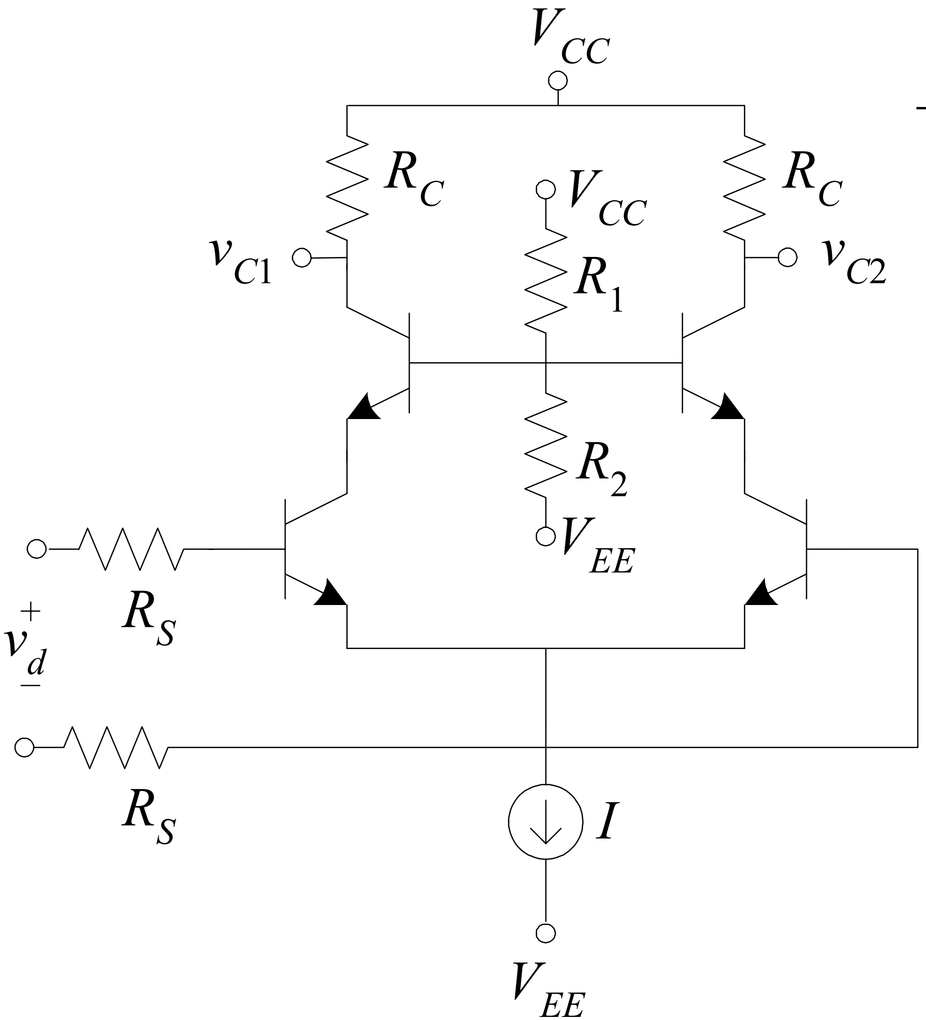
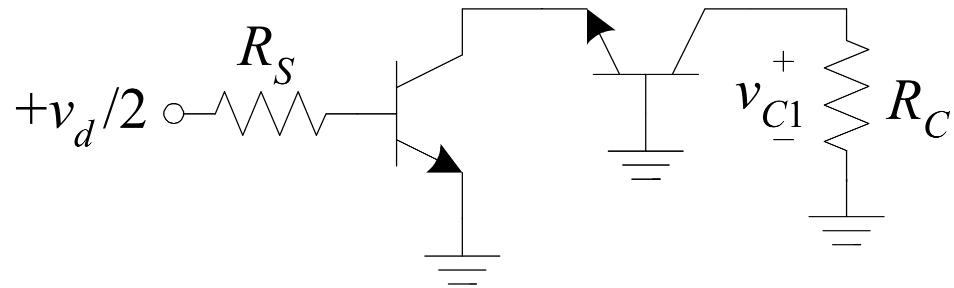
Cascode Differential Amplifier





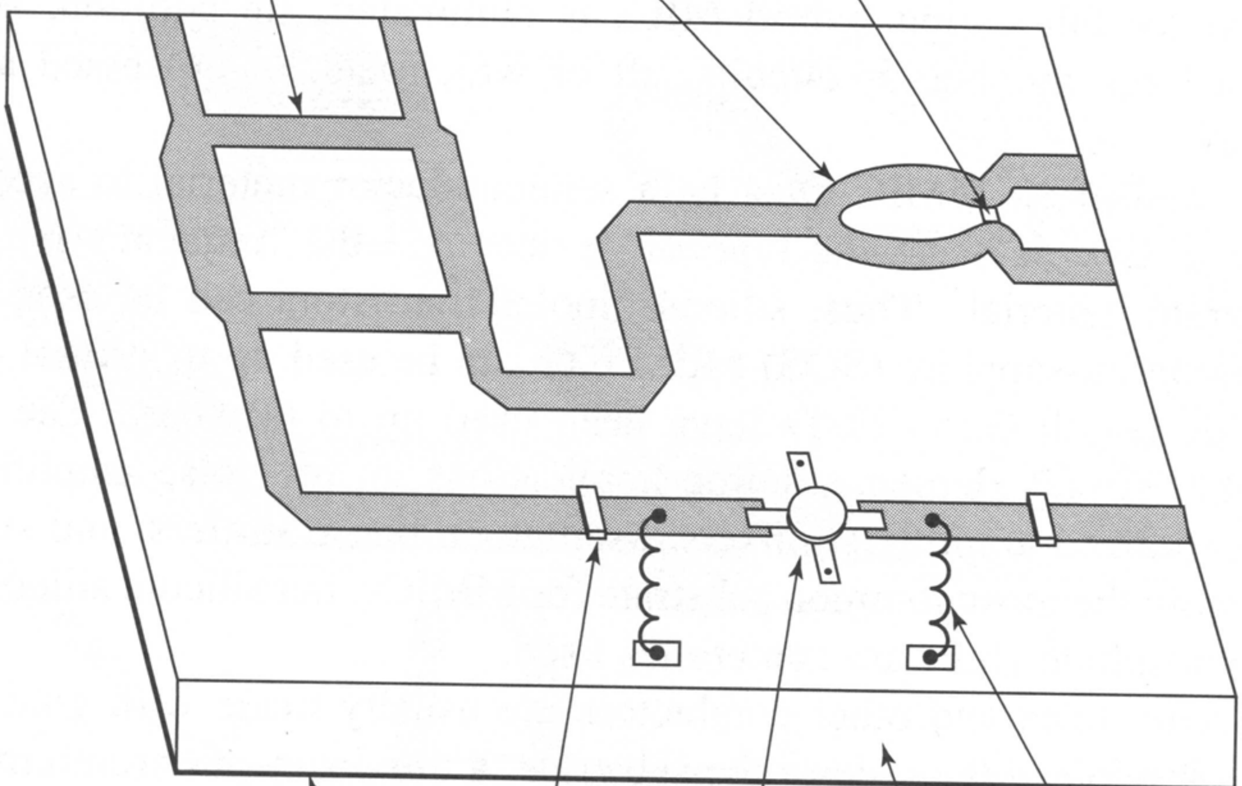
$$\omega_H \approx 1/[(R_{\pi 1} + R_{\pi 2})C_\pi + R_\mu C_\mu]$$

$$R_{\pi 1} = (R_S + r_x) \parallel r_\pi$$

$$R_{\pi 2} = r_e$$

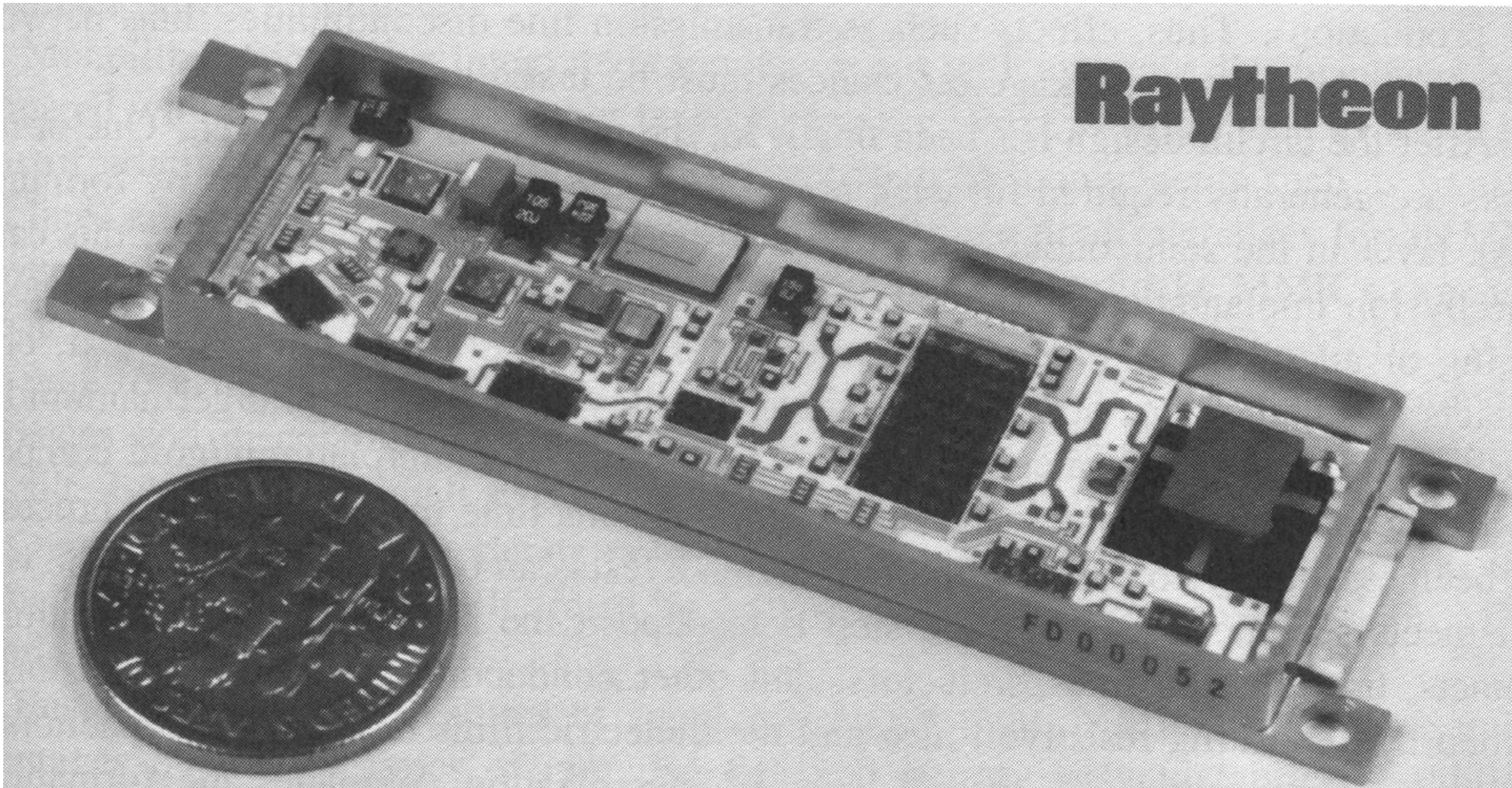
$$R_\mu = 2R_{\pi 1} + r_e$$

Hybrid Microwave Integrated Circuits



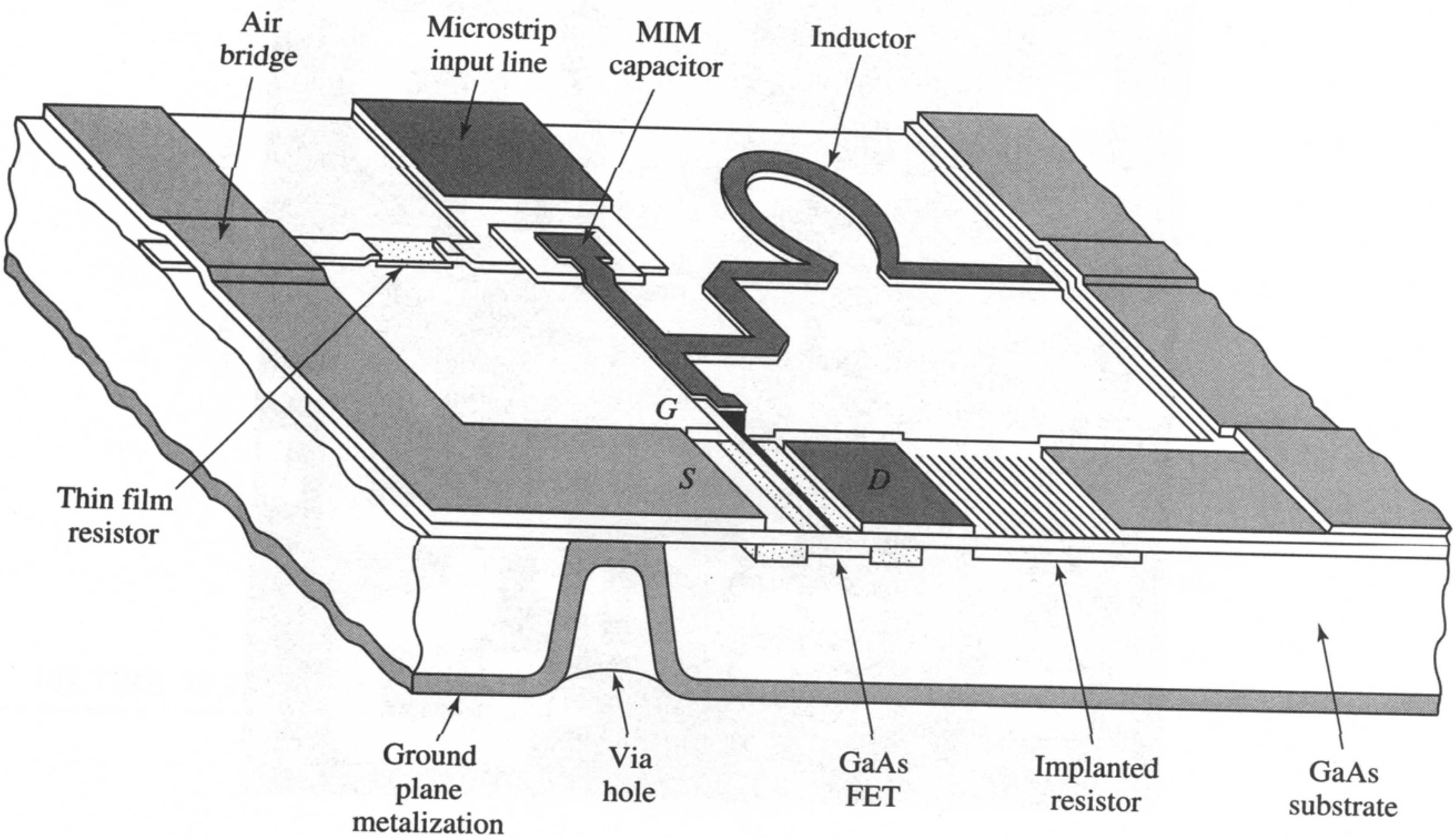
M. Pozar (1998), *Microwave Engineering*. Amherst, MA: John Wiley and Sons.

Hybrid Microwave Integrated Circuits (cont)



M. Pozar (1998), *Microwave Engineering*. Amherst, MA: John Wiley and Sons.

Monolithic Microwave Integrated Circuits



M. Pozar (1998), *Microwave Engineering*. Amherst, MA: John Wiley and Sons.

Active Device Models at Microwave Frequencies