Frequency Response

(Part 1)

A diagram of this presentation was taken from the web site of the author of the book:

A.R. Hambley, *Electronics: A Top-Down Approach to Computer-Aided Circuit Design*. Englewood Cliffs, NJ: Prentice Hall, 2000.

s-Domain Analysis

Voltage gain as a transfer function of the complex frequency $s = j\omega$

$$T(s) \equiv \frac{v_o(s)}{v_i(s)}$$

$$T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \cdots + a_1 s + a_0}{s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}$$

n is the order of the circuit $(m < n)$

for a stable circuit, all the roots of the denominator polynomial must have negative real parts

s-Domain Analysis (cont.)

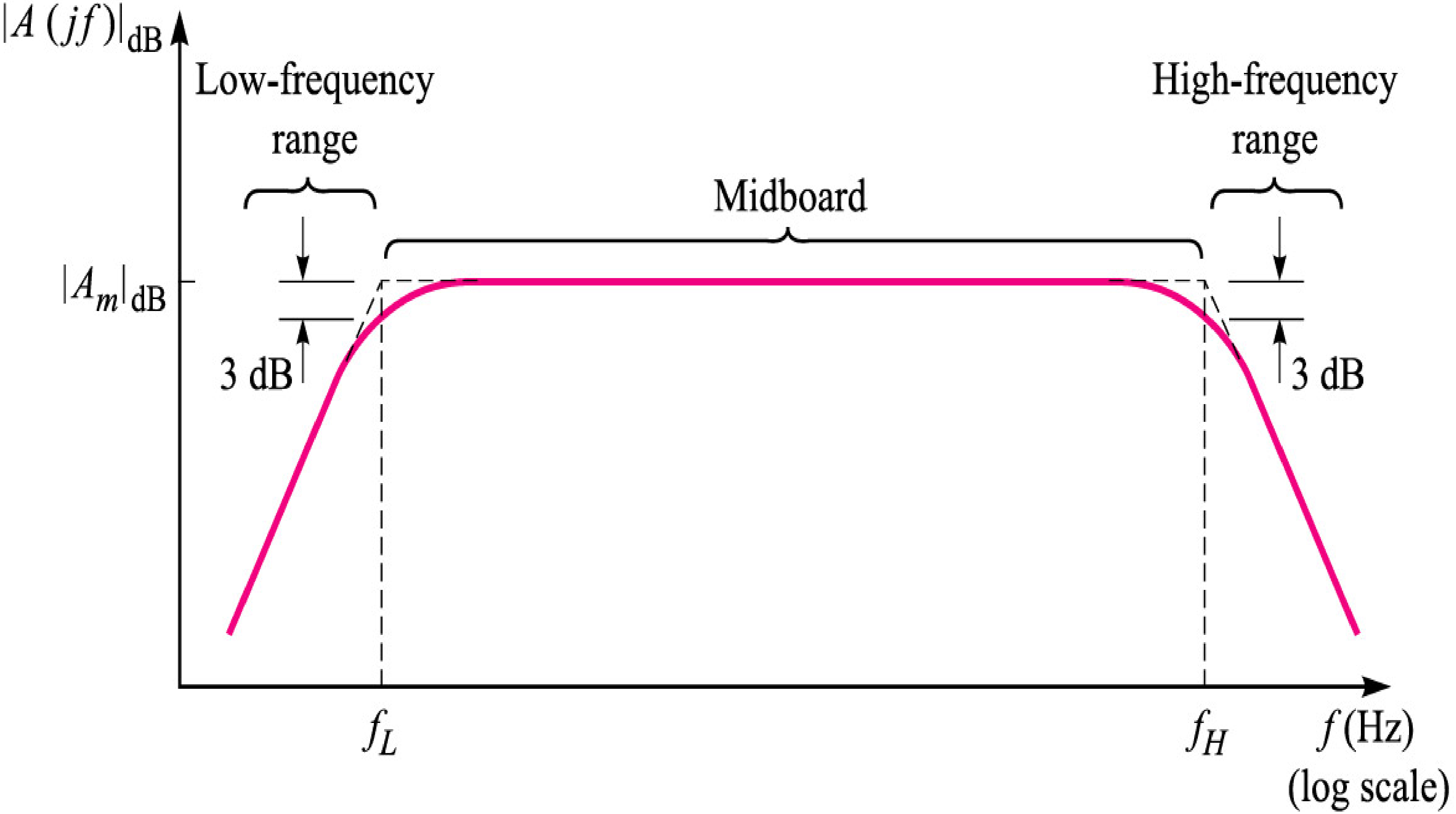
Poles and Zeros

$$T(s) = a_m \frac{(s - Z_1)(s - Z_2)\cdots(s - Z_m)}{(s - P_1)(s - P_2)\cdots(s - P_n)}$$

$Z_1, \ldots, Z_m$ are the transfer function zeros, or transmission zeros

$P_1, \ldots, P_m$ are the transfer function poles, transmission poles or natural modes

The Amplifier Transfer Function



The Amplifier Transfer Function (continue)

A_M midband gain

ω_L cutoff low frequency, 3-dB low frequency

ω_H cutoff high frequency, 3-dB high frequency

Bandwidth (BW)

$BW = f_H - f_L$ (Hz) $BW = \omega_H - \omega_L$ (rad/sec)

usually $\omega_H >> \omega_L$, $BW \approx \omega_H$

Gain-Bandwith Product (GB)

$GB = A_M \, \omega_H$

The Gain Function

$$A(s) = \frac{v_o(s)}{v_i(s)} = A_M F_L(s) F_H(s)$$

A_M midband gain

$F_L(s)$ low frequency response

$F_H(s)$ high frequency response

When $\omega_H >> \omega >> \omega_L$, $A(s) \approx A_M$

When $\omega >> \omega_L$, $F_L(s) \approx 1$

When $\omega << \omega_H$, $F_H(s) \approx 1$

Low Frequency Response

$$F_L(s) = \frac{(s+\omega_{Z1})(s+\omega_{Z2})\cdots(s+\omega_{Zn_L})}{(s+\omega_{P1})(s+\omega_{P2})\cdots(s+\omega_{Pn_L})}$$

n_L number of poles (or zeros) of $F_L(s)$

$$|F_L(j\omega)|^2 = \frac{(\omega^2+\omega_{Z1}^2)(\omega^2+\omega_{Z2}^2)\cdots(\omega^2+\omega_{Zn_L}^2)}{(\omega^2+\omega_{P1}^2)(\omega^2+\omega_{P2}^2)\cdots(\omega^2+\omega_{Pn_L}^2)}$$

$$\lim_{\omega\to\infty} |F_L(j\omega)| = 1$$

$$\lim_{\omega\to 0} |F_L(j\omega)| = \sqrt{\frac{\omega_{Z1}^2\omega_{Z2}^2\cdots\omega_{Zn_L}^2}{\omega_{P1}^2\omega_{P2}^2\cdots\omega_{Pn_L}^2}}$$

$$\exists i \ni \omega_{Zi} = 0, \lim_{\omega\to 0} |F_L(j\omega)| = 0$$

Low Frequency Response (cont)

$$F_L(s) = \frac{(s+\omega_{Z1})(s+\omega_{Z2})\cdots(s+\omega_{Zn_L})}{(s+\omega_{P1})(s+\omega_{P2})\cdots(s+\omega_{Pn_L})}$$

$$\omega_{P1},\ldots,\omega_{Pn_L} > 0$$

if $\quad \omega_{P1} >> \omega_{P2},\ldots,\omega_{Pn_L},\omega_{Z1},\ldots,\omega_{Zn_L} \quad$ then

$F_L(s) \approx \dfrac{s}{(s+\omega_{P1})}$ and $\omega_L \approx \omega_{P1}$ (ω_{P1} is a dominant pole)

What if there is no dominant pole?

Low Frequency Response - No Dominant Pole

$$F_L(s) = \frac{(s+\omega_{Z1})(s+\omega_{Z2})}{(s+\omega_{P1})(s+\omega_{P2})} \qquad |F_L(j\omega)|^2 = \frac{(\omega^2+\omega_{Z1}^2)(\omega^2+\omega_{Z2}^2)}{(\omega^2+\omega_{P1}^2)(\omega^2+\omega_{P2}^2)}$$

$$\text{Como } |F_L(j\omega_L)|^2 = \frac{1}{2} \qquad \frac{(\omega_L^2+\omega_{Z1}^2)(\omega_L^2+\omega_{Z2}^2)}{(\omega_L^2+\omega_{P1}^2)(\omega_L^2+\omega_{P2}^2)} = \frac{1}{2}$$

$$\frac{\omega_L^4+\omega_L^2(\omega_{Z1}^2+\omega_{Z2}^2)+\omega_{Z1}^2\omega_{Z2}^2}{\omega_L^4+\omega_L^2(\omega_{P1}^2+\omega_{P2}^2)+\omega_{P1}^2\omega_{P2}^2} = \frac{1}{2}$$

$$\frac{1+(1/\omega_L^2)(\omega_{Z1}^2+\omega_{Z2}^2)+(1/\omega_L^4)\omega_{Z1}^2\omega_{Z2}^2}{1+(1/\omega_L^2)(\omega_{P1}^2+\omega_{P2}^2)+(1/\omega_L^4)\omega_{P1}^2\omega_{P2}^2} = \frac{1}{2} \approx \frac{1+(1/\omega_L^2)(\omega_{Z1}^2+\omega_{Z2}^2)}{1+(1/\omega_L^2)(\omega_{P1}^2+\omega_{P2}^2)}$$

$$(1/\omega_L^2)[\omega_{P1}^2+\omega_{P2}^2-2(\omega_{Z1}^2+\omega_{Z2}^2)] = 1$$

$$\omega_L \approx \sqrt{\omega_{P1}^2+\omega_{P2}^2-2(\omega_{Z1}^2+\omega_{Z2}^2)}$$

Low Frequency Response - No Dominant Pole

$$F_L(s) = \frac{(s+\omega_{Z1})(s+\omega_{Z2})\cdots(s+\omega_{Zn_L})}{(s+\omega_{P1})(s+\omega_{P2})\cdots(s+\omega_{Pn_L})}$$

If there is no dominant pole

$$\omega_L \approx \sqrt{\omega_{P1}^2 + \omega_{P2}^2 + \ldots + \omega_{Pn_L}^2 - 2(\omega_{Z1}^2 + \omega_{Z2}^2 + \ldots + \omega_{Zn_L}^2)}$$

High Frequency Response

$$F_H(s) = \frac{(1+s/\omega_{Z1})(1+s/\omega_{Z2})\cdots(1+s/\omega_{Zn_H})}{(1+s/\omega_{P1})(1+s/\omega_{P2})\cdots(1+s/\omega_{Pn_H})}$$

n_H number of poles (or zeros) of $F_H(s)$

$$|F_H(j\omega)|^2 = \frac{[1+(\omega/\omega_{Z1})^2][1+(\omega/\omega_{Z2})^2]\cdots[1+(\omega/\omega_{Zn_H})^2]}{[1+(\omega/\omega_{P1})^2][1+(\omega/\omega_{P2})^2]\cdots[1+(\omega/\omega_{Pn_H})^2]}$$

$$\lim_{\omega\to 0} |F_H(j\omega)| = 1$$

$$\lim_{\omega\to\infty} |F_H(j\omega)| = \sqrt{\frac{\omega_{P1}^2\omega_{P2}^2\cdots\omega_{Pn_L}^2}{\omega_{Z1}^2\omega_{Z2}^2\cdots\omega_{Zn_L}^2}} \qquad \exists i \ni \omega_{Zi} = \infty, \lim_{\omega\to\infty} |F_H(j\omega)| = 0$$

High Frequency Response (cont)

$$F_H(s) = \frac{(1+s/\omega_{Z1})(1+s/\omega_{Z2})\cdots(1+s/\omega_{Zn_H})}{(1+s/\omega_{P1})(1+s/\omega_{P2})\cdots(1+s/\omega_{Pn_H})}$$

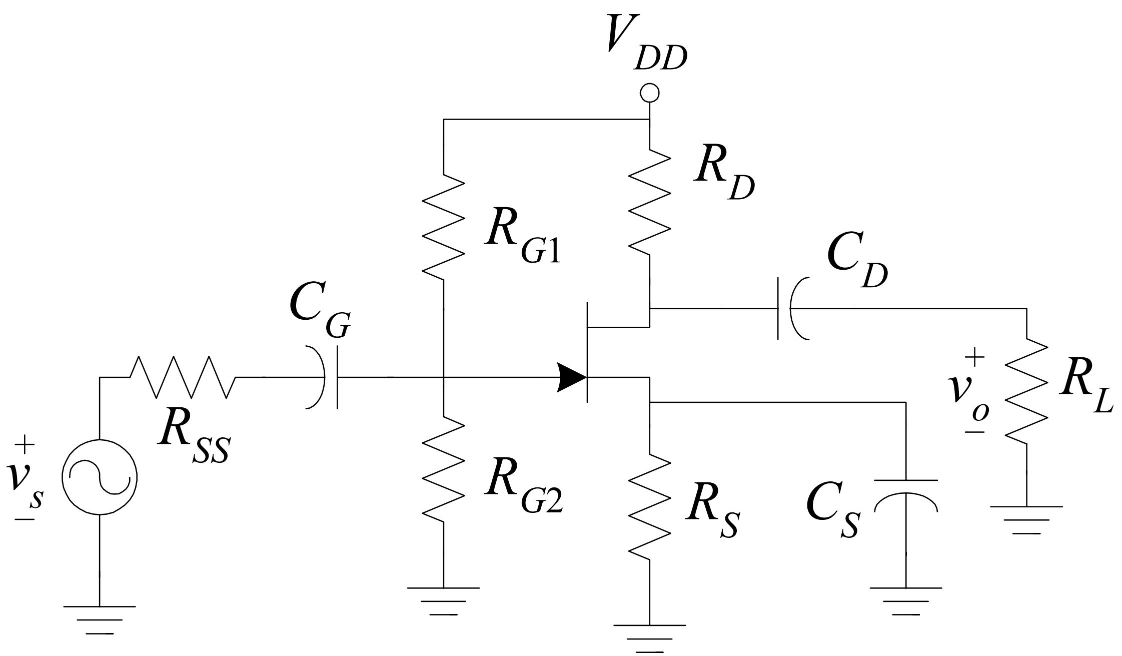
if $\omega_{P1} << \omega_{P2},\ldots,\omega_{Pn_L},\omega_{Z1},\ldots,\omega_{Zn_L}$ then

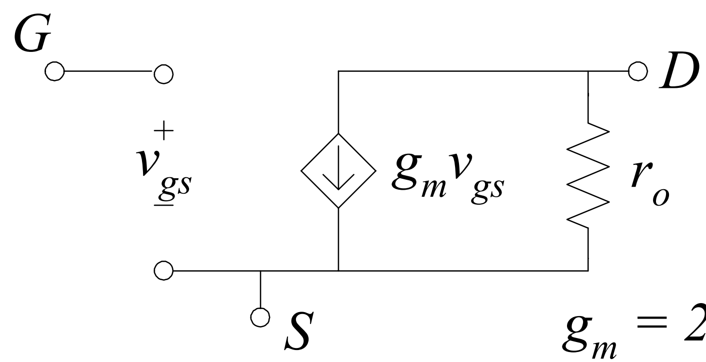
$F_H(s) \approx \dfrac{1}{(1+s/\omega_{P1})}$ and $\omega_H \approx \omega_{P1}$ (ω_{P1} is a dominant pole)

If there is no dominant pole

$$\omega_H \approx \frac{1}{\sqrt{1/\omega_{P1}^2 + 1/\omega_{P2}^2 + \ldots + 1/\omega_{Pn_H}^2 - 2(1/\omega_{Z1}^2 + 1/\omega_{Z2}^2 + \ldots + 1/\omega_{Zn_H}^2)}}$$

Example: Common Source Low Frequency

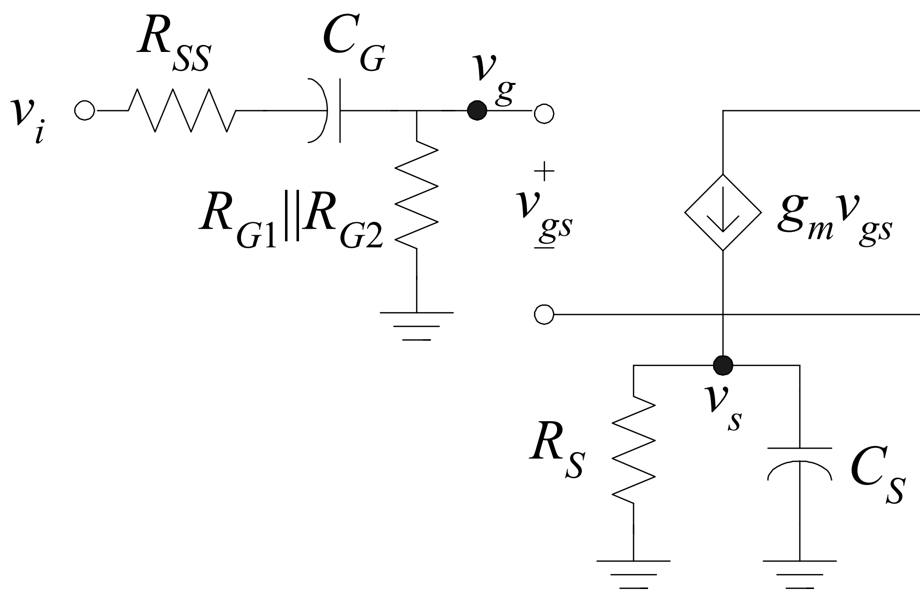




$$g_m = 2K(V_{GS} - V_t)$$

$$r_o = V_A/I_D = 1/\lambda I_D$$

Example (cont)

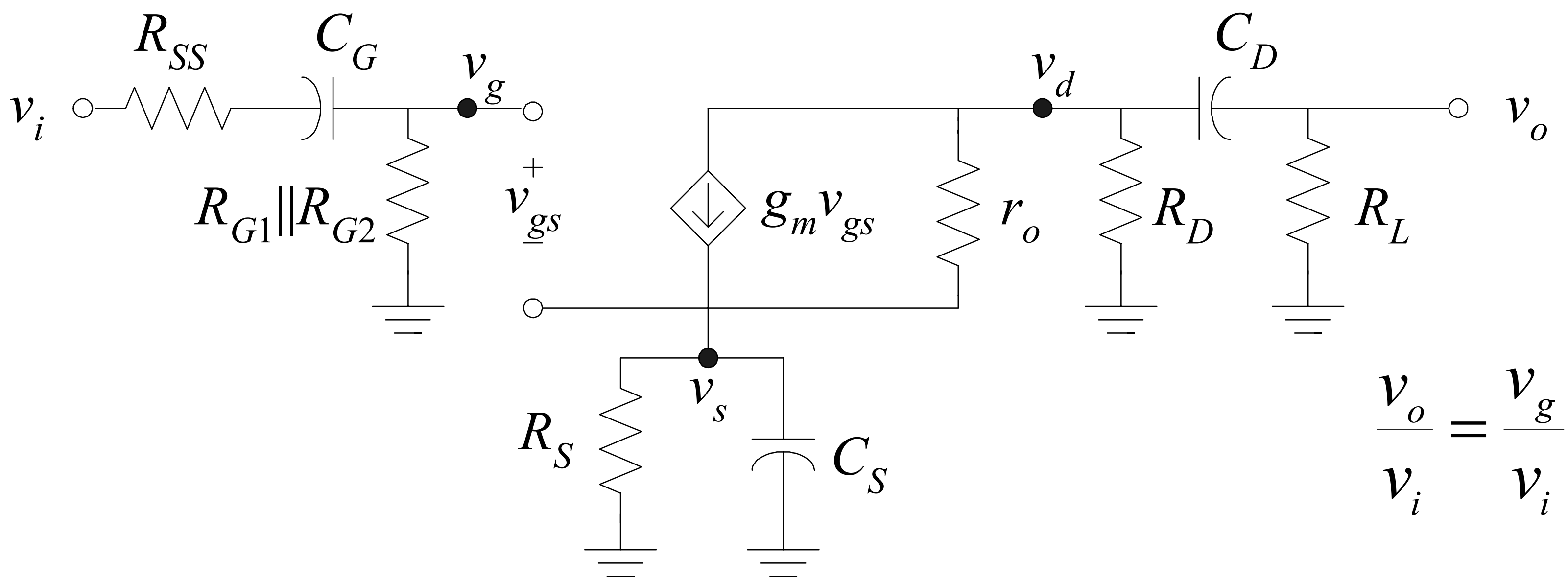


$$\frac{v_o}{v_i} = \frac{v_g}{v_i}\frac{v_s}{v_g}\frac{v_d}{v_s}\frac{v_o}{v_d}$$

$$v_g = \frac{(R_{G1} \parallel R_{G2})v_i}{R_{SS} + (R_{G1} \parallel R_{G2}) + 1/sC_G}$$

$$\frac{v_g}{v_i} = \frac{(R_{G1} \parallel R_{G2})C_G s}{1 + [R_{SS} + (R_{G1} \parallel R_{G2})]C_G s}$$

Example (cont)

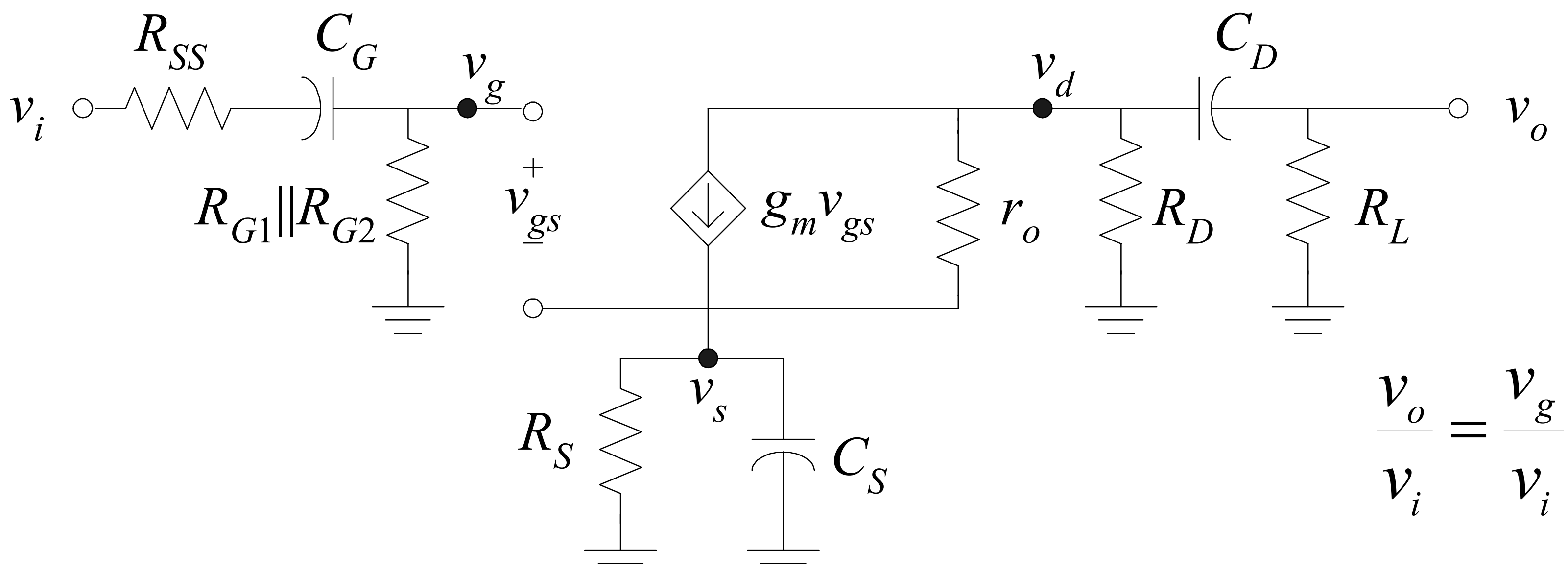


$$\frac{v_o}{v_i} = \frac{v_g}{v_i}\frac{v_s}{v_g}\frac{v_d}{v_s}\frac{v_o}{v_d}$$

$$v_o = \frac{R_L v_d}{R_L + 1/sC_D}$$

$$\frac{v_o}{v_d} = \frac{R_L C_D s}{1 + R_L C_D s}$$

Example (cont)



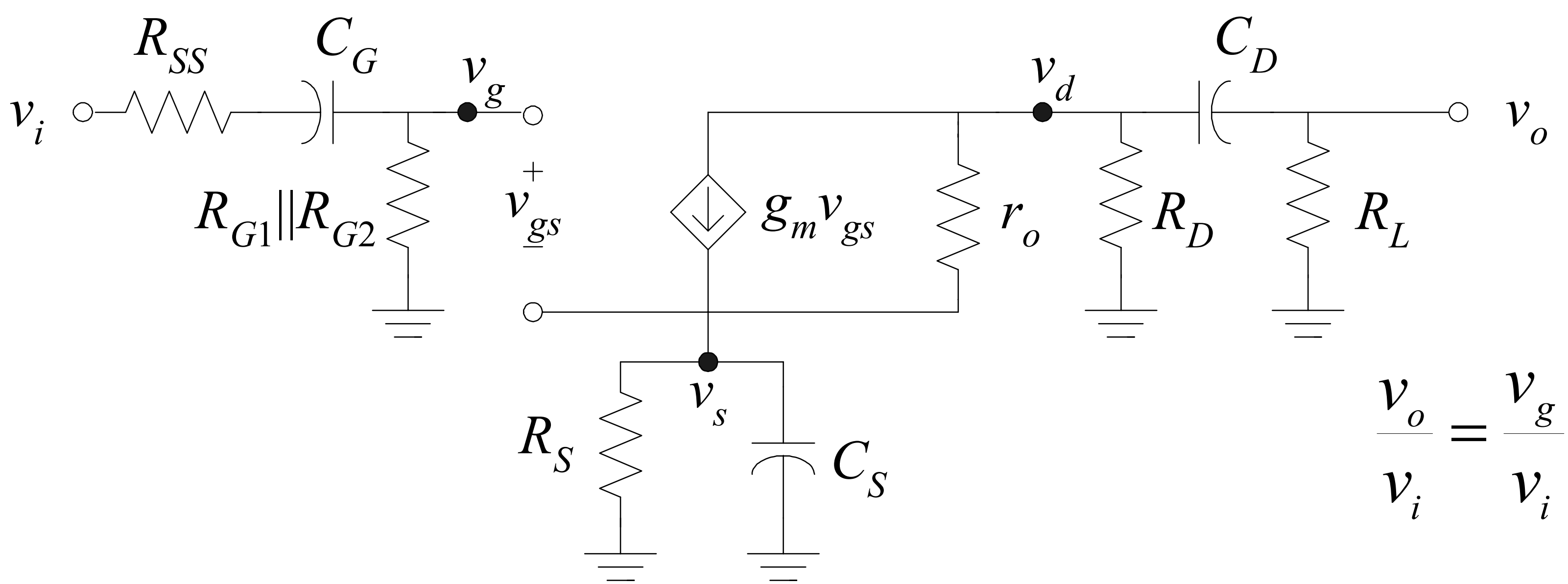
$$\frac{v_o}{v_i} = \frac{v_g}{v_i}\frac{v_s}{v_g}\frac{v_d}{v_s}\frac{v_o}{v_d}$$

$$v_s = i_s Z_s \qquad Z_s = R_S \,||\, \frac{1}{sC_S} = \frac{(R_S)(1/sC_S)}{R_S + 1/sC_S} \qquad Z_s = \frac{R_S}{1 + R_S C_S s}$$

$$v_d = -i_s Z_d$$

$$Z_d = R_D \,||\, (\frac{1}{sC_D} + R_L) \qquad Z_d = \frac{R_D(1 + R_L C_D s)}{1 + (R_D + R_L)C_D s} \qquad \frac{v_d}{v_s} = \frac{-Z_d}{Z_s}$$

Example (cont)

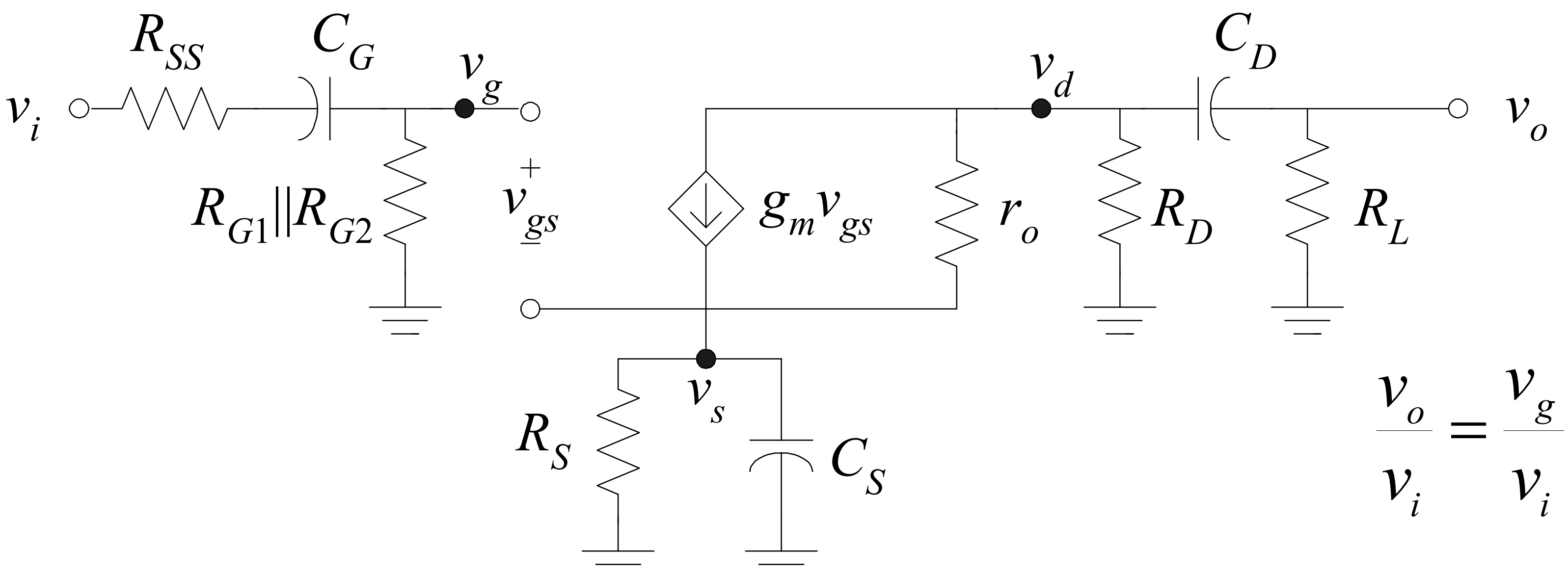


$$\frac{v_o}{v_i} = \frac{v_g}{v_i}\frac{v_s}{v_g}\frac{v_d}{v_s}\frac{v_o}{v_d}$$

$$\frac{v_d}{v_s} = \frac{-Z_d}{Z_s} \qquad Z_s = \frac{R_S}{1+R_S C_S s} \qquad Z_d = \frac{R_D(1+R_L C_D s)}{1+(R_D+R_L)C_D s}$$

$$\frac{v_d}{v_s} = \frac{-R_D}{R_S}\frac{(1+R_S C_S s)(1+R_L C_D s)}{1+(R_D+R_L)C_D s}$$

Example (cont)



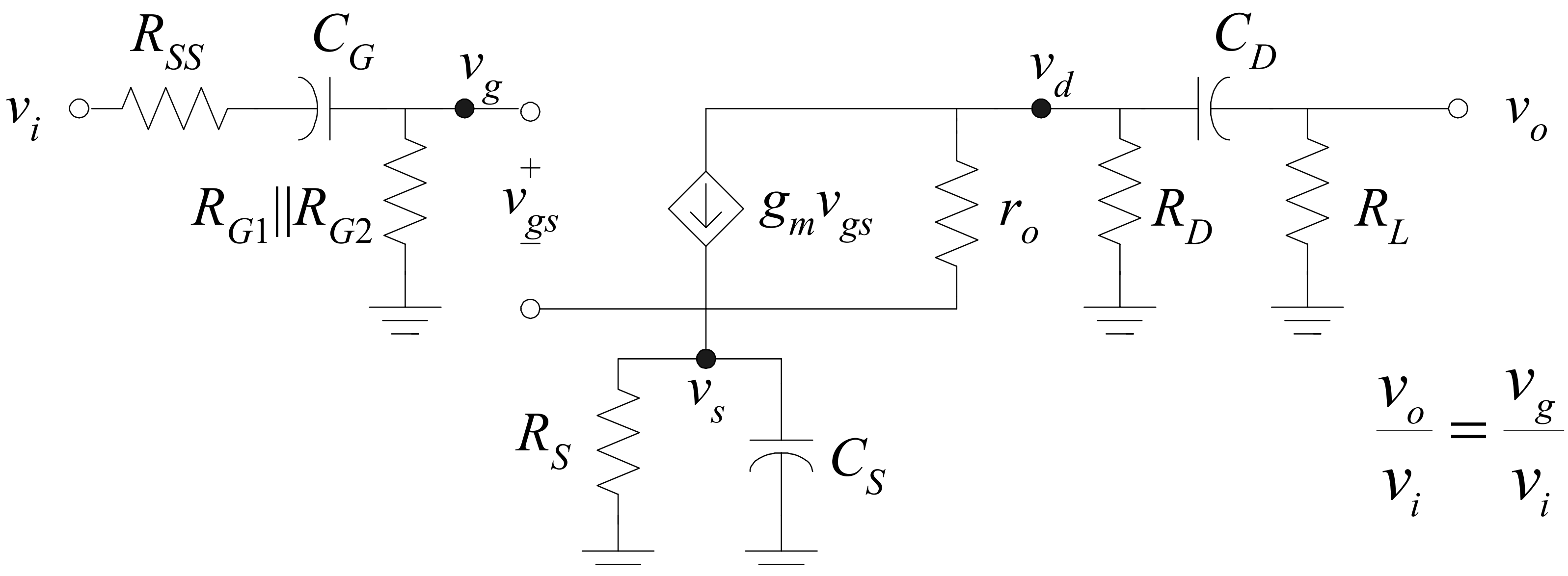
$$\frac{v_o}{v_i} = \frac{v_g}{v_i}\frac{v_s}{v_g}\frac{v_d}{v_s}\frac{v_o}{v_d}$$

$$v_s = i_s Z_s = [g_m v_{gs} + (v_d - v_s)/r_o]Z_s = [g_m v_g + v_d/r_o - v_s(g_m + 1/r_o)]Z_s$$

Como $r_o >> \frac{1}{g_m}$, $\quad v_s \approx [g_m v_g + v_d/r_o - v_s g_m]Z_s$

Como $\frac{v_d}{v_s} = \frac{-Z_d}{Z_s}$, $\quad v_s(1 + g_m Z_s + Z_d/r_o) \approx g_m Z_s v_g$

Example (cont)



$$\frac{v_o}{v_i} = \frac{v_g}{v_i}\frac{v_s}{v_g}\frac{v_d}{v_s}\frac{v_o}{v_d}$$

$$v_s(1 + g_m Z_s + Z_d / r_o) \approx g_m Z_s v_g$$

$$\frac{v_s}{v_g} \approx \frac{g_m Z_s}{1 + g_m Z_s + Z_d / r_o}$$

Si $g_m Z_s >> Z_d / r_o$,

$$\frac{v_s}{v_g} \approx \frac{g_m Z_s}{1 + g_m Z_s} = \frac{g_m R_S}{1 + g_m R_S + R_S C_S s} = \frac{\dfrac{g_m R_S}{1 + g_m R_S}}{1 + \dfrac{R_S C_S}{1 + g_m R_S} s} = \frac{\dfrac{g_m R_S}{1 + g_m R_S}}{1 + (R_S \| 1 / g_m) C_S s}$$

Example (cont)

$$\frac{v_o}{v_i} = \frac{v_g}{v_i}\frac{v_s}{v_g}\frac{v_d}{v_s}\frac{v_o}{v_d}$$

$$\frac{v_g}{v_i} = \frac{(R_{G1} \| R_{G2})C_G s}{1+[R_{SS}+(R_{G1} \| R_{G2})]C_G s}$$

$$\frac{v_s}{v_g} = \frac{g_m R_S/(1+g_m R_S)}{1+(R_S \| 1/g_m)C_S s}$$

$$\frac{v_d}{v_s} = \frac{-R_D}{R_S}\frac{(1+R_S C_S s)(1+R_L C_D s)}{1+(R_D+R_L)C_D s}$$

$$\frac{v_o}{v_d} = \frac{R_L C_D s}{1+R_L C_D s}$$

$$\omega_{Z1} = \frac{1}{R_S C_S}$$

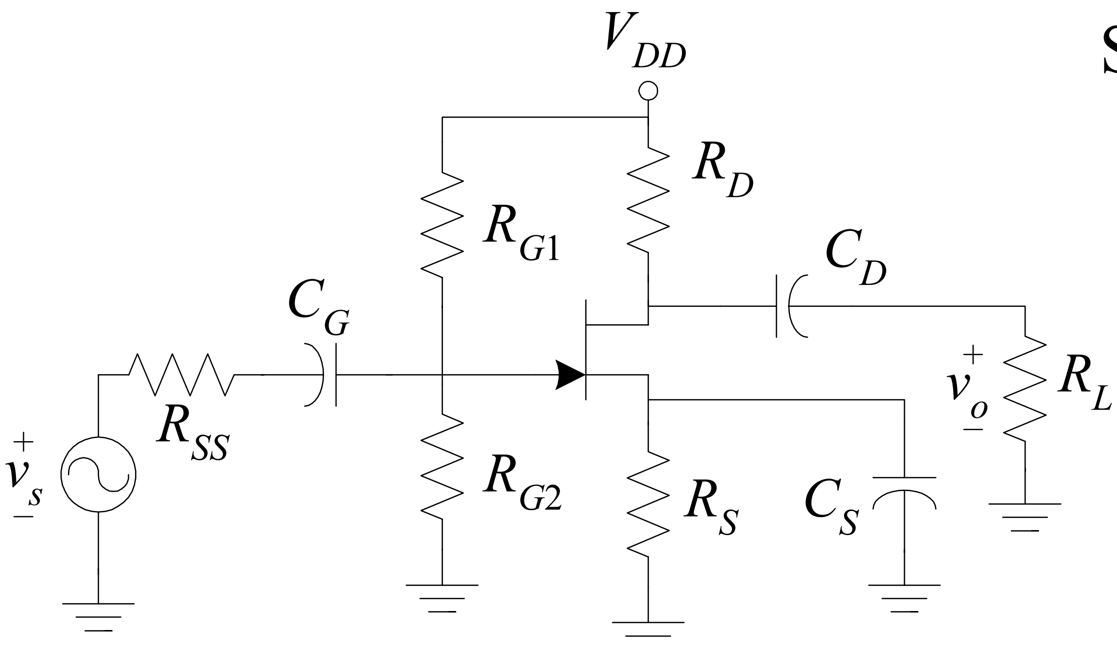
$$\omega_{P1} = \frac{1}{[R_{SS}+(R_{G1} \| R_{G2})]C_G}$$

$$\omega_{P2} = \frac{1}{(R_S \| 1/g_m)C_S}$$

$$\omega_{P3} = \frac{1}{(R_D+R_L)C_D}$$

$$\omega_L \approx \sqrt{\omega_{P1}^2+\omega_{P2}^2+\omega_{P3}^2-2\omega_{Z1}^2}$$

Example (cont)



$$\text{Si } r_o >> \frac{1}{g_m}, \;\; g_m Z_s >> Z_d / r_o$$

$$Z_s = R_S \,||\, \frac{1}{sC_S}$$

$$Z_d = R_D \,||\, (\frac{1}{sC_D} + R_L)$$

$$\omega_{Z1} = \frac{1}{R_S C_S}$$

$$\omega_{P1} = \frac{1}{[R_{SS} + (R_{G1} \,||\, R_{G2})]C_G}$$

$$\omega_{P2} = \frac{1}{(R_S || 1/g_m)C_S}$$

$$\omega_{P3} = \frac{1}{(R_D + R_L)C_D}$$

$$\omega_L \approx \sqrt{\omega_{P1}^2 + \omega_{P2}^2 + \omega_{P3}^2 - 2\omega_{Z1}^2}$$

The Time Constant Method

- It is an approximate method to calculate ω_L and ω_H
- Works well for most amplifiers
- Assumes that all the poles are real, that there is a dominant pole and no dominant zero
- No poles and zeros calculations required
- Very suitable for deriving design formulas

The Short-Circuit Time Constant Method (LF)

1) Replace the amplifier by its low frequency model
2) Calculate the resistance R_i in parallel with the capacitor C_i, considering all the remaining capacitors as short circuits
3) Repeat step 2) for each capacitor ($i = 1, 2, ..., n_L$)
4) Calculate ω_L using

$$\omega_L \approx \sum_{i=1}^{n_L} \frac{1}{R_i C_i}$$

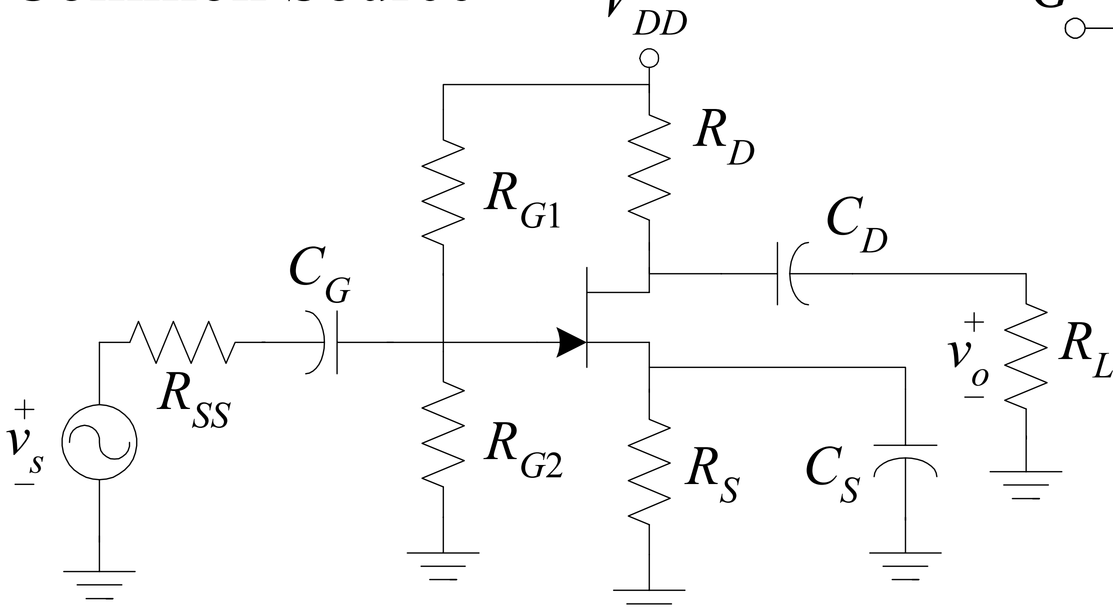
The Open-Circuit Time Constant Method (HF)

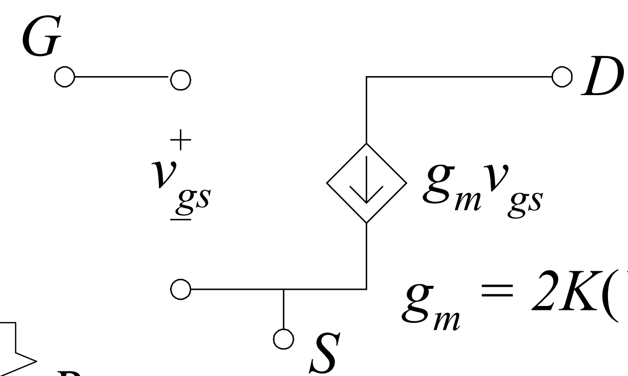
1) Replace the amplifier by its high frequency model
2) Calculate the resistance R_i in parallel with the capacitor C_i, considering all the remaining capacitors as open circuits
3) Repeat step 2) for each capacitor (i = 1, 2, ..., n_H)
4) Calculate ω_H using

$$\omega_H \approx \frac{1}{\sum_{i=1}^{n_H} R_i C_i}$$

Low-Frequency Response of FET Amplifiers

Common Source





$$g_m = 2K(V_{GS} - V_t)$$

$$R_{C_G} = R_{SS} + R_{G1} \| R_{G2}$$

$$R_{C_S} = R_S \| \frac{1}{g_m}$$

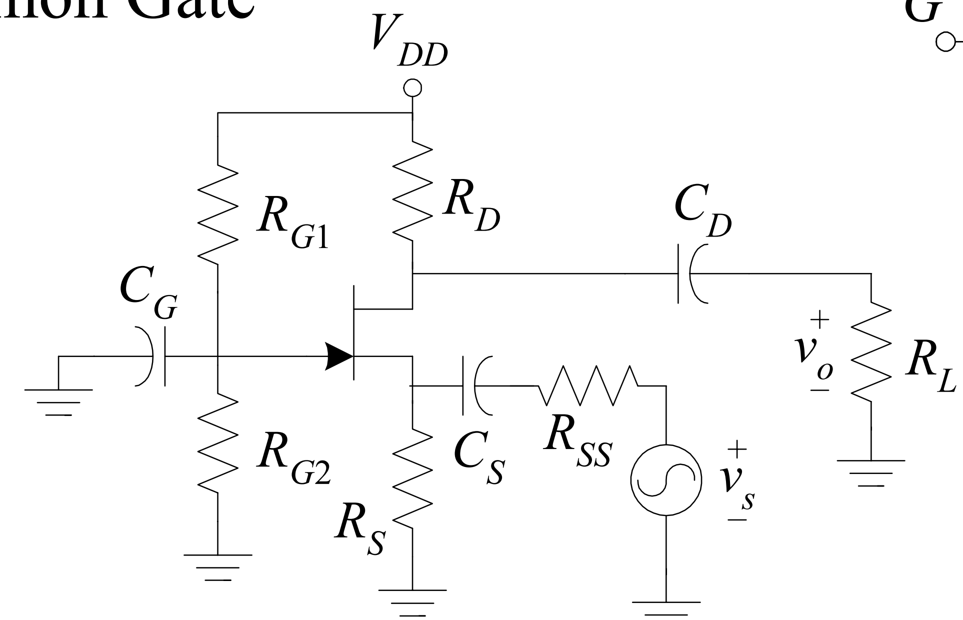
$$R_{C_D} = R_L + R_D$$

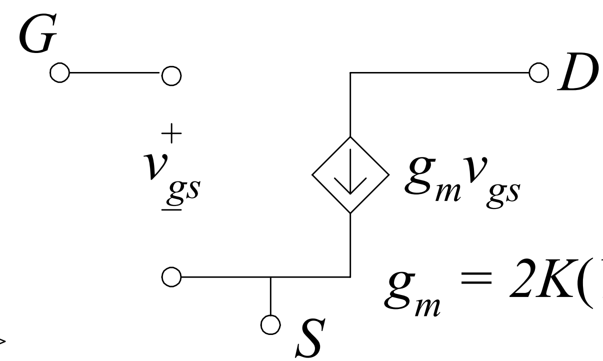
$$\omega_L \approx \frac{1}{R_{C_G} C_G} + \frac{1}{R_{C_S} C_S} + \frac{1}{R_{C_D} C_D}$$

usually $1/(R_{C_S} C_S)$ is the dominant pole

Low-Frequency Response of FET Amplifiers

Common Gate





$$g_m = 2K(V_{GS} - V_t)$$

$$R_{C_G} = R_{G1} \parallel R_{G2}$$

$$R_{C_S} = R_{SS} + \left(R_S \parallel \frac{1}{g_m}\right)$$

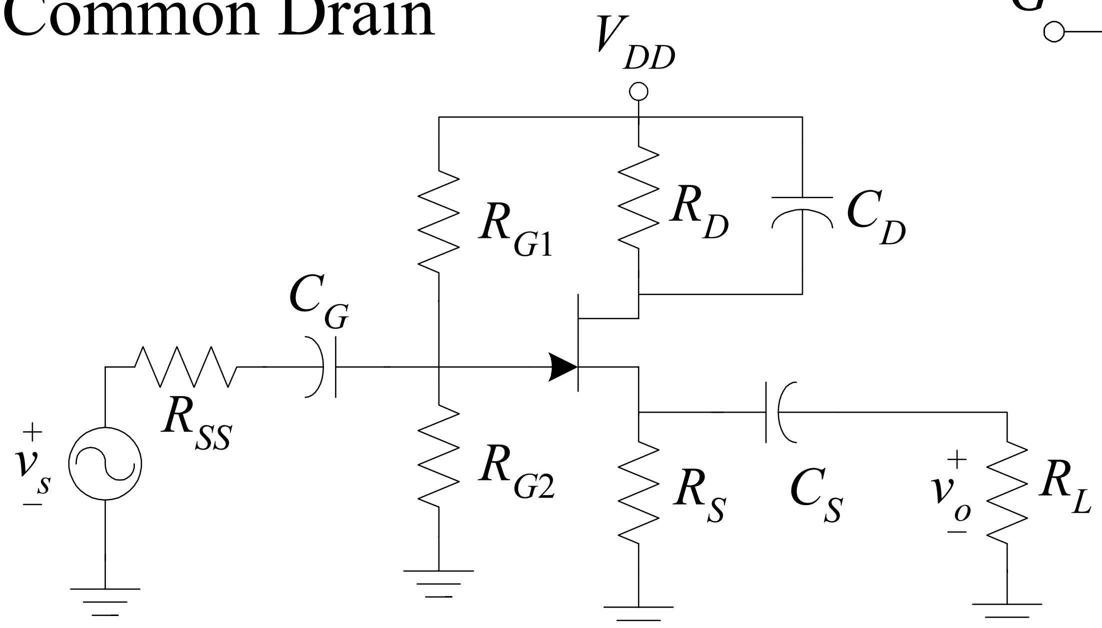
$$R_{C_D} = R_D + R_L$$

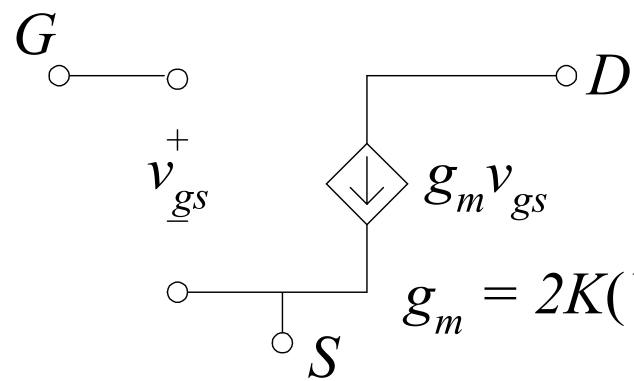
$$\omega_L \approx \frac{1}{R_{C_G} C_G} + \frac{1}{R_{C_S} C_S} + \frac{1}{R_{C_D} C_D}$$

usually $1/(R_{C_S} C_S)$ is the dominant pole

Low-Frequency Response of FET Amplifiers

Common Drain





$$g_m = 2K(V_{GS} - V_t)$$

$$R_{C_G} = R_{SS} + R_{G1} \parallel R_{G2}$$

$$R_{C_S} = (R_S \parallel \frac{1}{g_m}) + R_L$$

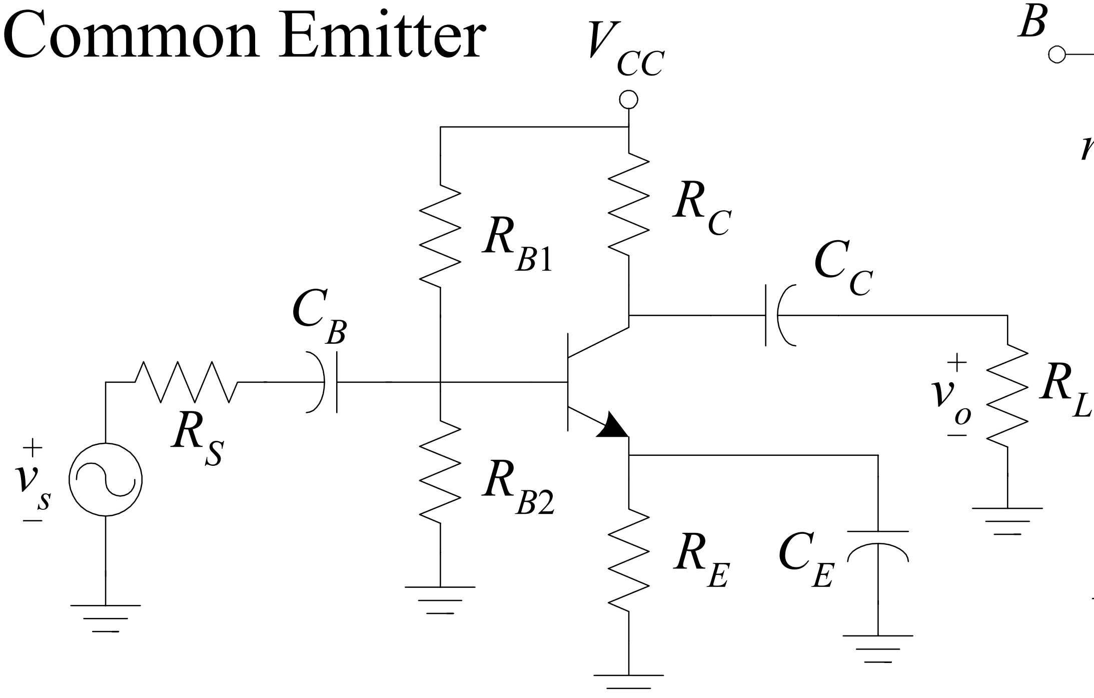
$$R_{C_D} = R_D$$

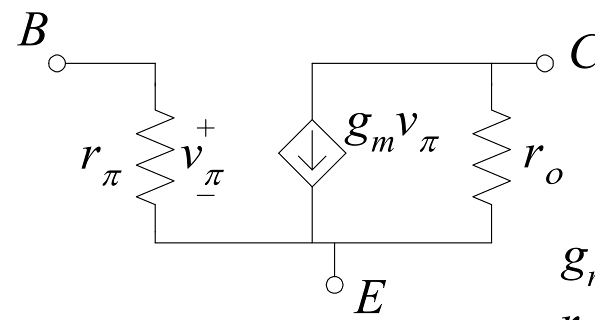
$$\omega_L \approx \frac{1}{R_{C_G} C_G} + \frac{1}{R_{C_S} C_S} + \frac{1}{R_{C_D} C_D}$$

usually $1/(R_{C_S} C_S)$ is the dominant pole

Low-Frequency Response of BJT Amplifiers

Common Emitter





$$g_m = I_C/V_T$$
$$r_\pi = \beta/g_m$$
$$r_o = V_A/I_C$$

$$R_{C_B} = R_S + (R_{B1} \,||\, R_{B2} \,||\, r_\pi)$$

$$R_{C_E} = R_E \,||\, \frac{r_\pi + (R_S \,||\, R_{B1} \,||\, R_{B2})}{\beta + 1}$$

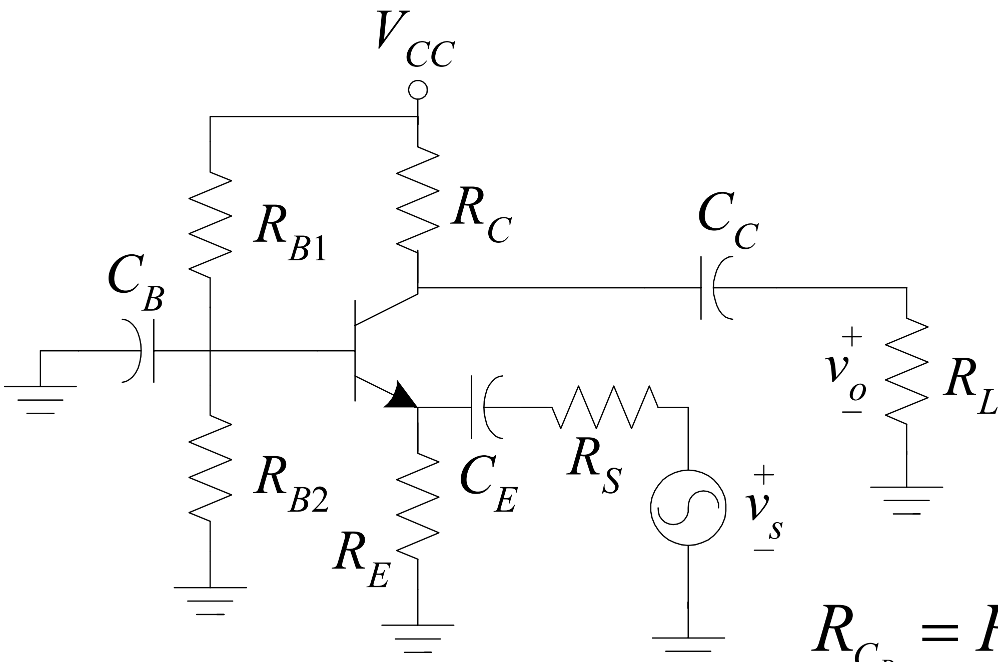
$$R_{C_C} = R_L + (R_C \,||\, r_o)$$

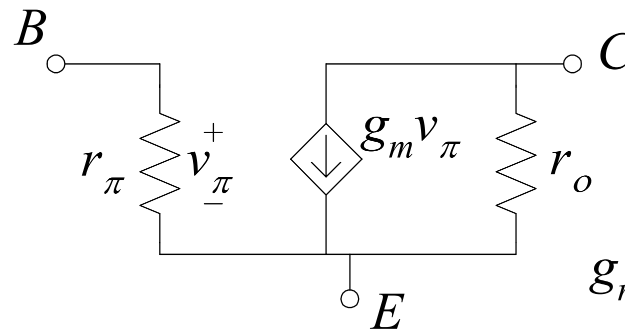
$$\omega_L \approx \frac{1}{R_{C_B} C_B} + \frac{1}{R_{C_E} C_E} + \frac{1}{R_{C_C} C_C}$$

usually $1/(R_{C_E} C_E)$ is the dominant pole

Low-Frequency Response of BJT Amplifiers

Common Base





$$g_m = I_C/V_T$$
$$r_\pi = \beta/g_m$$
$$r_o = V_A/I_C$$

$$R_{C_E} = R_S + R_E \,||\, r_\pi \,||\, 1/g_m$$

$$R_{C_B} = R_{B1} \,||\, R_{B2} \,||\, [r_\pi + (1+\beta)(R_E \,||\, R_S)]$$

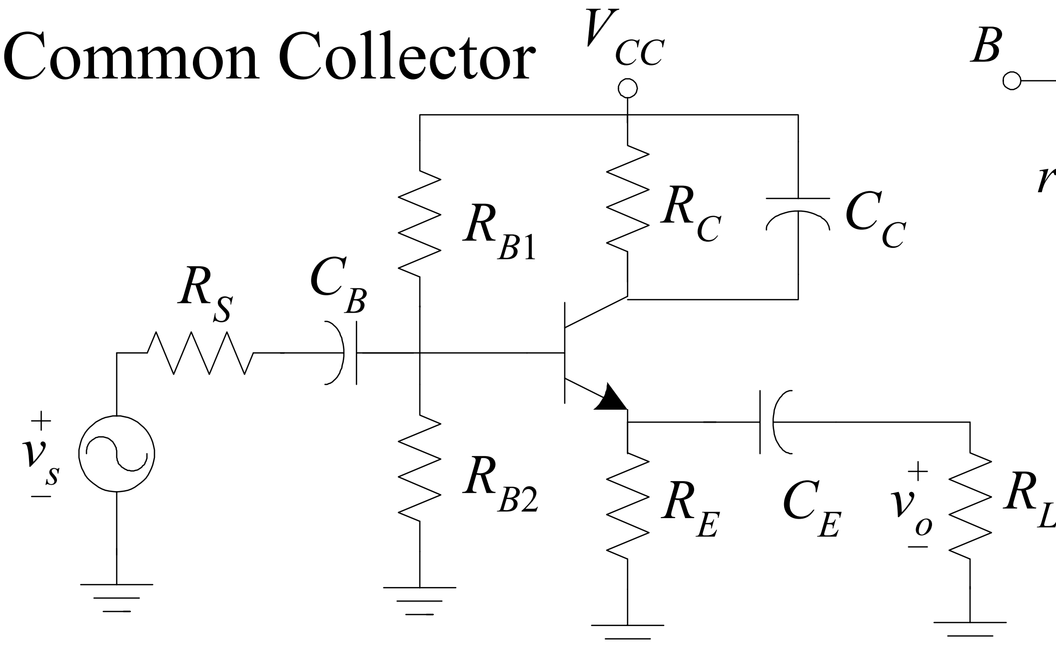
$$R_{C_C} = R_L + (R_C \,||\, r_o)$$

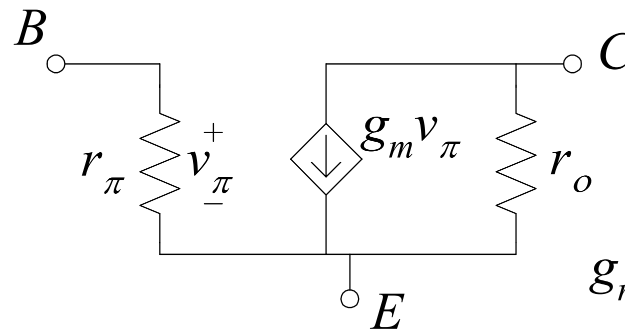
$$\omega_L \approx \frac{1}{R_{C_B} C_B} + \frac{1}{R_{C_E} C_E} + \frac{1}{R_{C_C} C_C}$$

usually $1/(R_{C_E} C_E)$ is the dominant pole

Low-Frequency Response of BJT Amplifiers

Common Collector





$$g_m = I_C/V_T$$
$$r_\pi = \beta/g_m$$
$$r_o = V_A/I_C$$

$$R_{C_C} = R_C \,\|\, [r_o + (R_E \,\|\, R_L)]$$

$$\omega_L \approx \frac{1}{R_{C_B} C_B} + \frac{1}{R_{C_E} C_E} + \frac{1}{R_{C_C} C_C} \qquad R_{C_E} = R_L + R_E \,\|\, \frac{r_\pi + (R_S \,\|\, R_{B1} \,\|\, R_{B2})}{\beta + 1}$$

$$R_{C_B} = R_S + R_{B1} \,\|\, R_{B2} \,\|\, [r_\pi + (1+\beta)(R_E \,\|\, R_L)]$$

usually $1/(R_{C_E} C_E)$ is the dominant pole