SPACE MAPPING OPTIMIZATION FOR ENGINEERING DESIGN: A TUTORIAL PRESENTATION

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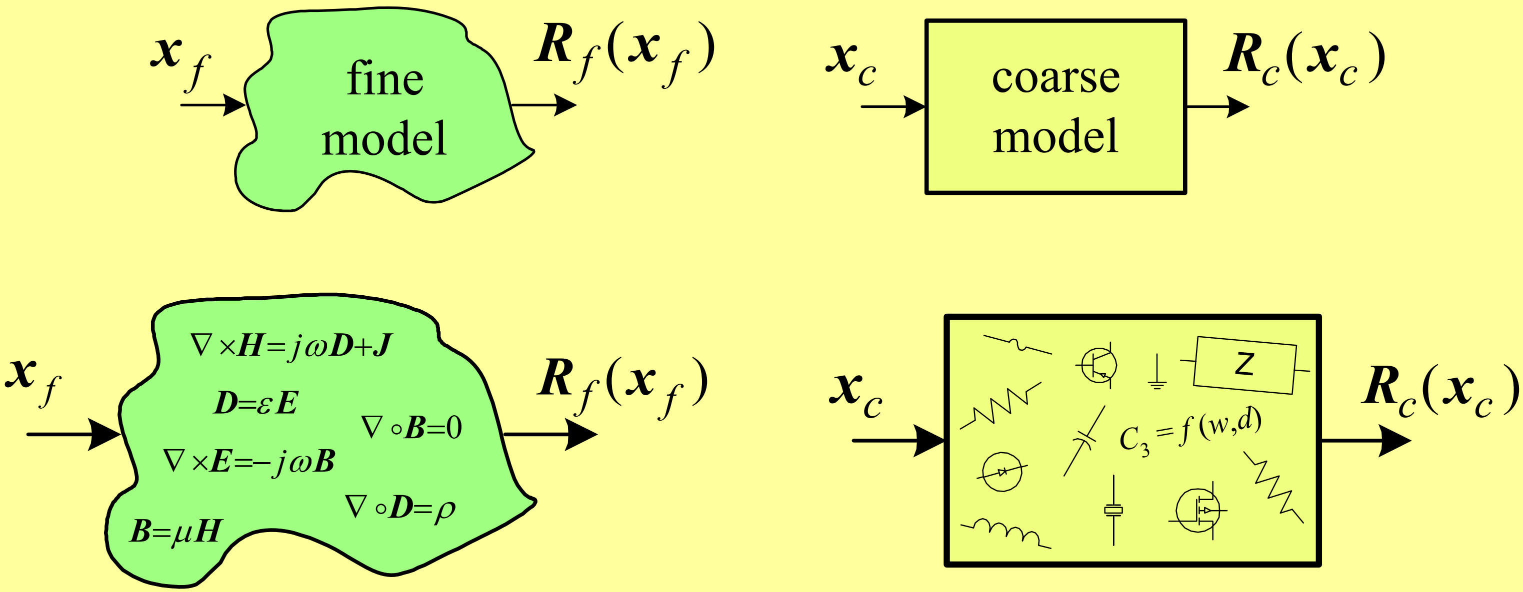
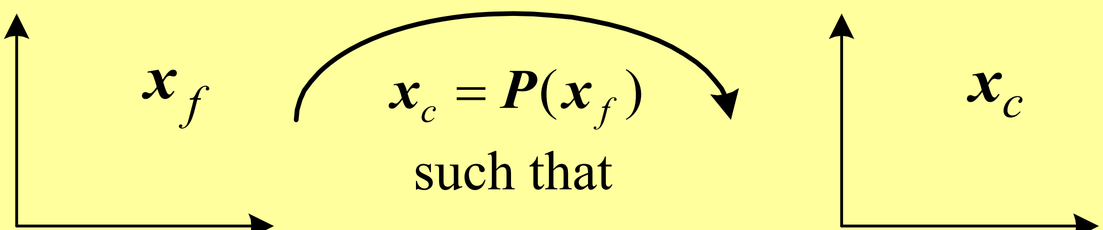
The Aim of Space Mapping
(Bandler et al., 1994-)





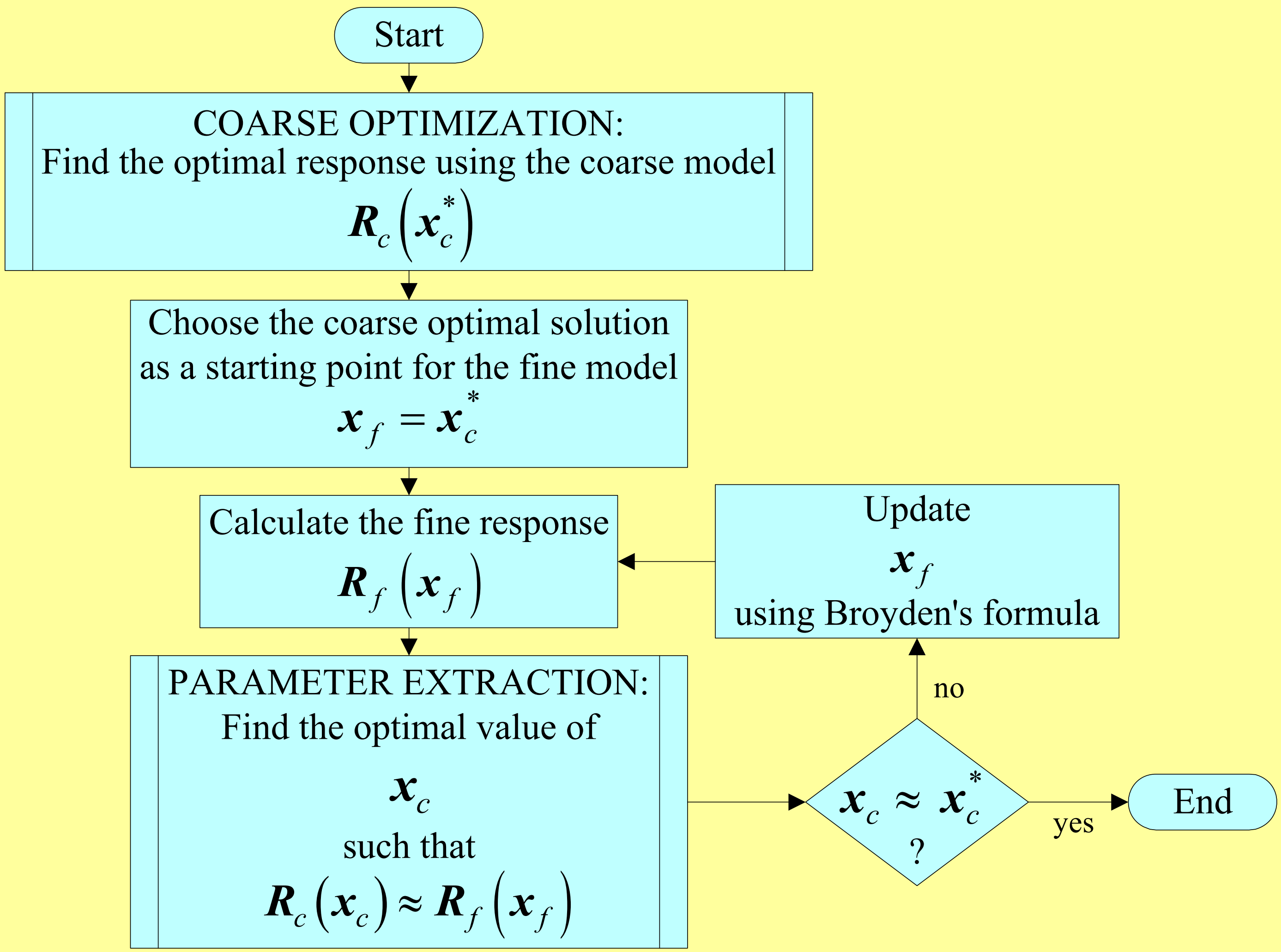
$$x_c = P(x_f)$$

such that

$$R_c(P(x_f)) \approx R_f(x_f)$$

Aggressive Space Mapping (ASM) Algorithm

(Bandler et al., 1995)



ASM Algorithm

0. Initialize $\boldsymbol{x}_f^{(1)} = \boldsymbol{x}_c^*$, $\boldsymbol{B}^{(1)} = \boldsymbol{1}$, $j = 1$

1. Evaluate $\boldsymbol{R}_f(\boldsymbol{x}_f^{(1)})$

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2. Extract $\boldsymbol{x}_c^{(1)}$ such that

$$\boldsymbol{R}_c(\boldsymbol{x}_c^{(1)}) \approx \boldsymbol{R}_f(\boldsymbol{x}_f^{(1)})$$

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3. Evaluate $\boldsymbol{f}^{(1)} = \boldsymbol{x}_c^{(1)} - \boldsymbol{x}_c^*$

   Stop if $\left\| \boldsymbol{f}^{(1)} \right\| \leq \eta$

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4. Solve $\boldsymbol{B}^{(j)} \boldsymbol{h}^{(j)} = -\boldsymbol{f}^{(j)}$ for $\boldsymbol{h}^{(j)}$

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4. Solve $\boldsymbol{B}^{(j)} \boldsymbol{h}^{(j)} = -\boldsymbol{f}^{(j)}$ for $\boldsymbol{h}^{(j)}$

5. Set $\boldsymbol{x}_f^{(j+1)} = \boldsymbol{x}_f^{(j)} + \boldsymbol{h}^{(j)}$

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6. Evaluate $\boldsymbol{R}_f(\boldsymbol{x}_f^{(j+1)})$

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   $\boldsymbol{R}_c(\boldsymbol{x}_c^{(1)}) \approx \boldsymbol{R}_f(\boldsymbol{x}_f^{(1)})$

3. Evaluate $\boldsymbol{f}^{(1)} = \boldsymbol{x}_c^{(1)} - \boldsymbol{x}_c^*$

   Stop if $\|\boldsymbol{f}^{(1)}\| \le \eta$

4. Solve $\boldsymbol{B}^{(j)} \boldsymbol{h}^{(j)} = -\boldsymbol{f}^{(j)}$ for $\boldsymbol{h}^{(j)}$

5. Set $\boldsymbol{x}_f^{(j+1)} = \boldsymbol{x}_f^{(j)} + \boldsymbol{h}^{(j)}$

6. Evaluate $\boldsymbol{R}_f(\boldsymbol{x}_f^{(j+1)})$

7. Extract $\boldsymbol{x}_c^{(j+1)}$ such that

   $\boldsymbol{R}_c(\boldsymbol{x}_c^{(j+1)}) \approx \boldsymbol{R}_f(\boldsymbol{x}_f^{(j+1)})$

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8. Evaluate $\boldsymbol{f}^{(j+1)} = \boldsymbol{x}_c^{(j+1)} - \boldsymbol{x}_c^*$

   Stop if $\left\| \boldsymbol{f}^{(j+1)} \right\| \leq \eta$

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0. Initialize $\boldsymbol{x}_f^{(1)}=\boldsymbol{x}_c^*$, $\boldsymbol{B}^{(1)}=\boldsymbol{1}$, $j=1$

1. Evaluate $\boldsymbol{R}_f(\boldsymbol{x}_f^{(1)})$

2. Extract $\boldsymbol{x}_c^{(1)}$ such that

   $\boldsymbol{R}_c(\boldsymbol{x}_c^{(1)})\approx\boldsymbol{R}_f(\boldsymbol{x}_f^{(1)})$

3. Evaluate $\boldsymbol{f}^{(1)}=\boldsymbol{x}_c^{(1)}-\boldsymbol{x}_c^*$

   Stop if $\left\|\boldsymbol{f}^{(1)}\right\|\leq\eta$

4. Solve $\boldsymbol{B}^{(j)}\boldsymbol{h}^{(j)}=-\boldsymbol{f}^{(j)}$ for $\boldsymbol{h}^{(j)}$

5. Set $\boldsymbol{x}_f^{(j+1)}=\boldsymbol{x}_f^{(j)}+\boldsymbol{h}^{(j)}$

6. Evaluate $\boldsymbol{R}_f(\boldsymbol{x}_f^{(j+1)})$

7. Extract $\boldsymbol{x}_c^{(j+1)}$ such that

   $\boldsymbol{R}_c(\boldsymbol{x}_c^{(j+1)})\approx\boldsymbol{R}_f(\boldsymbol{x}_f^{(j+1)})$

8. Evaluate $\boldsymbol{f}^{(j+1)}=\boldsymbol{x}_c^{(j+1)}-\boldsymbol{x}_c^*$

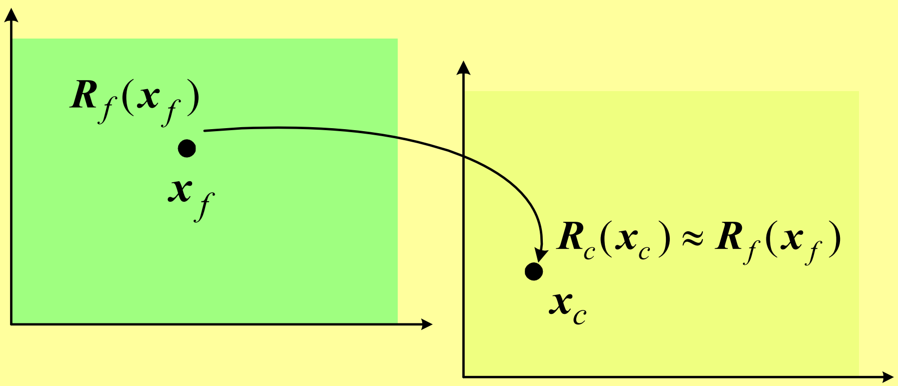
   Stop if $\left\|\boldsymbol{f}^{(j+1)}\right\|\leq\eta$

9. Update $\boldsymbol{B}^{(j+1)}=\boldsymbol{B}^{(j)}+\dfrac{\boldsymbol{f}^{(j+1)}\boldsymbol{h}^{(j)^T}}{\boldsymbol{h}^{(j)^T}\boldsymbol{h}^{(j)}}$

10. Set $j=j+1$; go to Step 4

Parameter Extraction

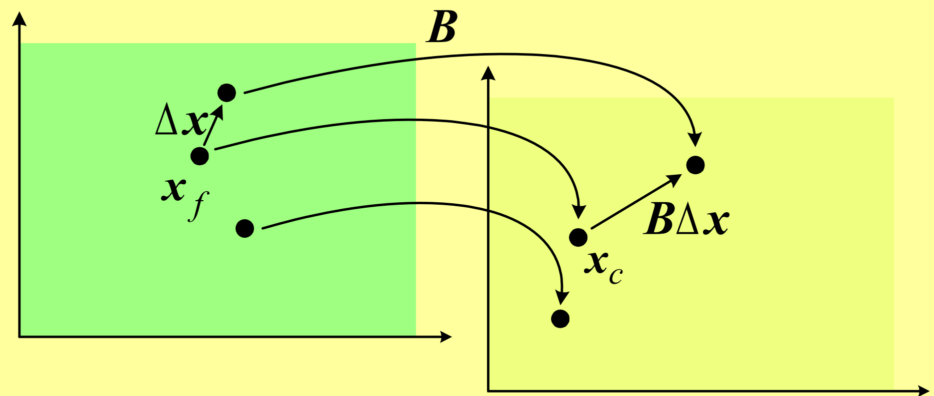
single point parameter extraction matches the responses of both models at a single point



it is formulated as

$$\min_{x_c} \left\| R_f(x_f) - R_c(x_c) \right\|$$

multi-point parameter extraction simultaneously matches the responses at a number of corresponding points



it is formulated as

$$\min_{x_c} \left\| \begin{bmatrix} e_1^T & \cdots & e_i^T & \cdots \end{bmatrix}^T \right\|$$

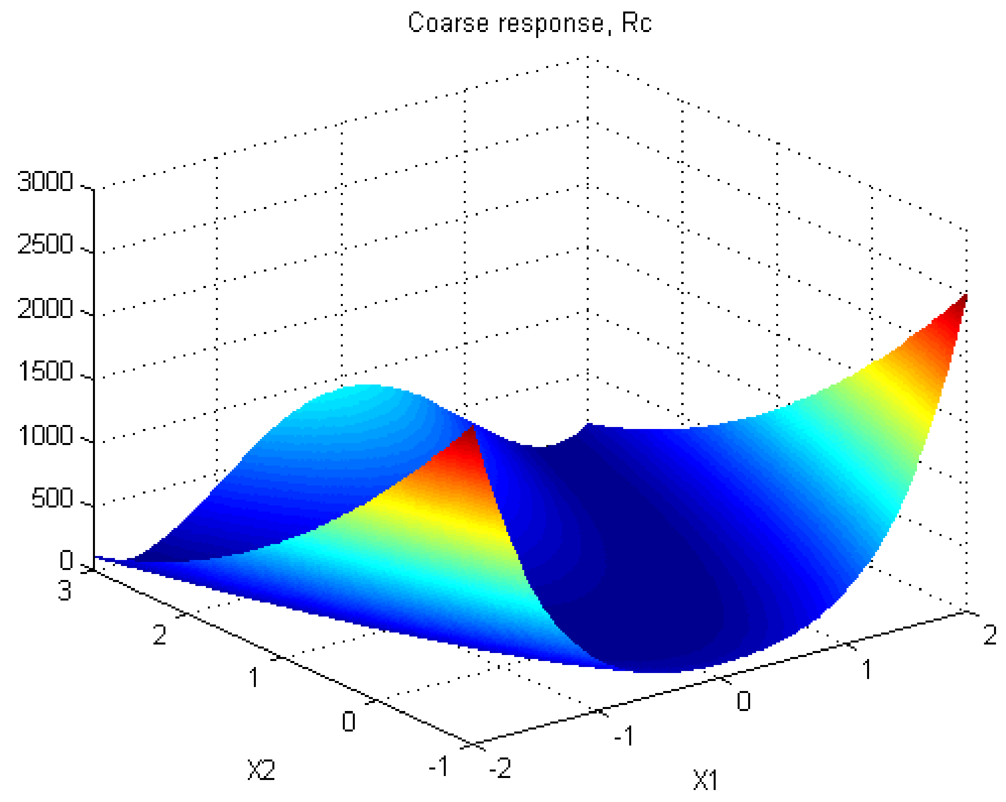
where

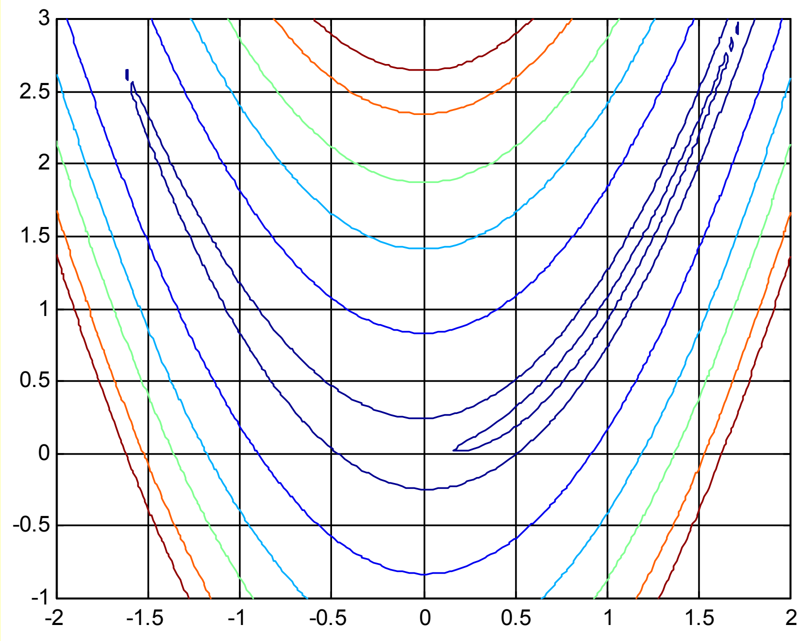
$$e_i = R_f(x_f + \Delta x_i) - R_c(x_c + B\Delta x_i)$$

Illustration With Two Rosenbrock Functions

Coarse Model: Rosenbrock Function

$$R_c(\boldsymbol{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2, \text{ where } \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



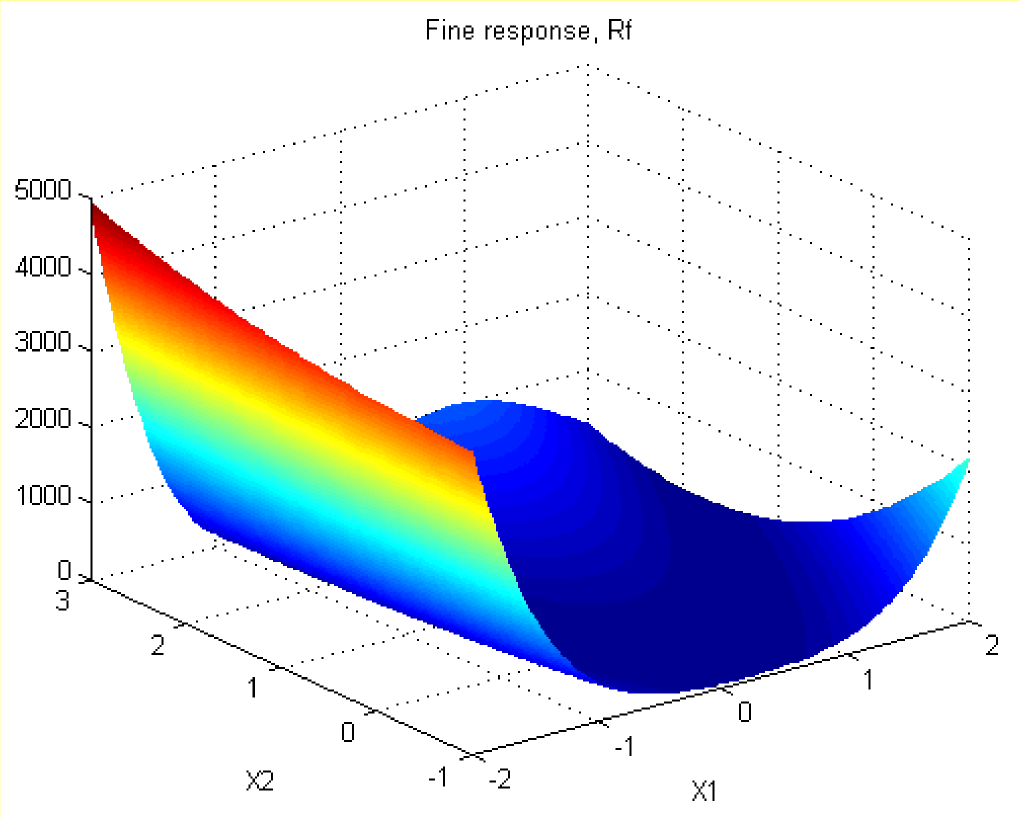


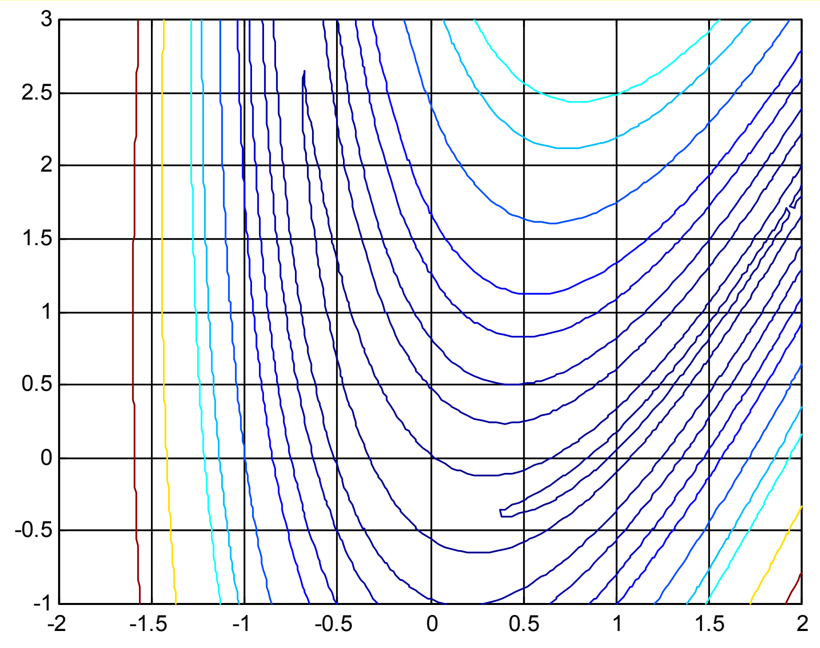
$$\boldsymbol{x}_c^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad R_c^* = R_c(\boldsymbol{x}_c^*) = 0$$

Illustration With Two Rosenbrock Functions (continued)

Fine Model Example: Transformed Rosenbrock Function

$$R_f(\boldsymbol{x}) = 100(u_2 - u_1^2)^2 + (1-u_1)^2, \text{ where } \quad \boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} -0.3 \\ 0.3 \end{bmatrix}$$

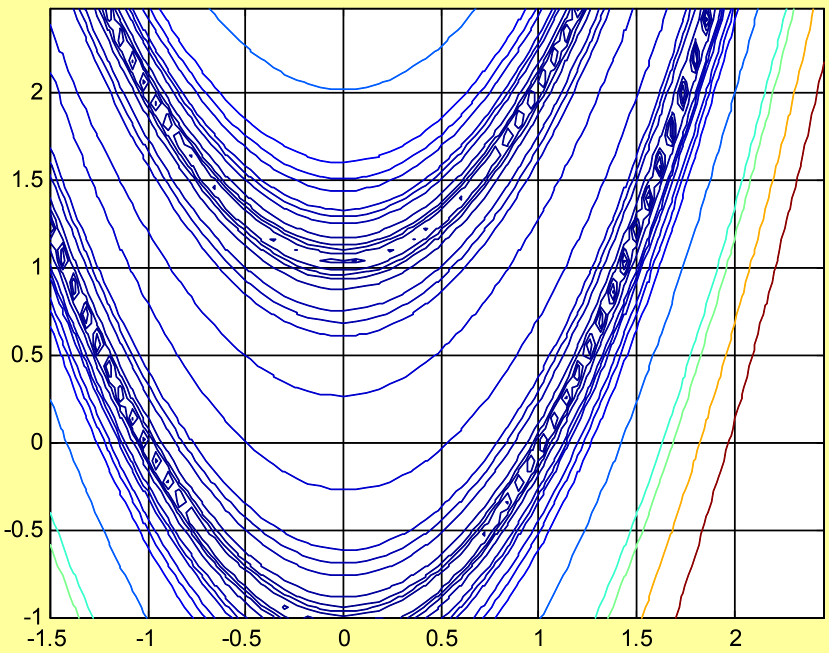




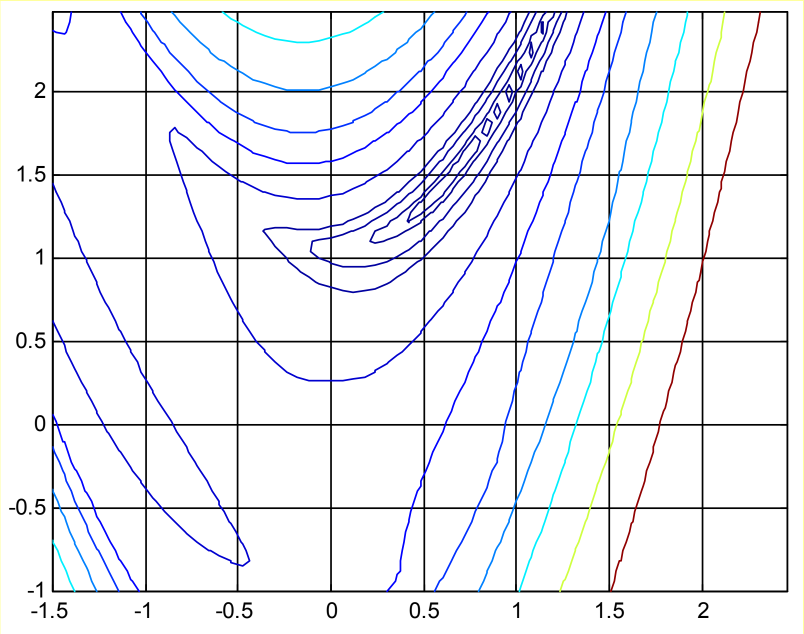
$$R_f(\boldsymbol{x}_c^*) = 108.32$$

First l_1 Parameter Extraction

single point parameter extraction



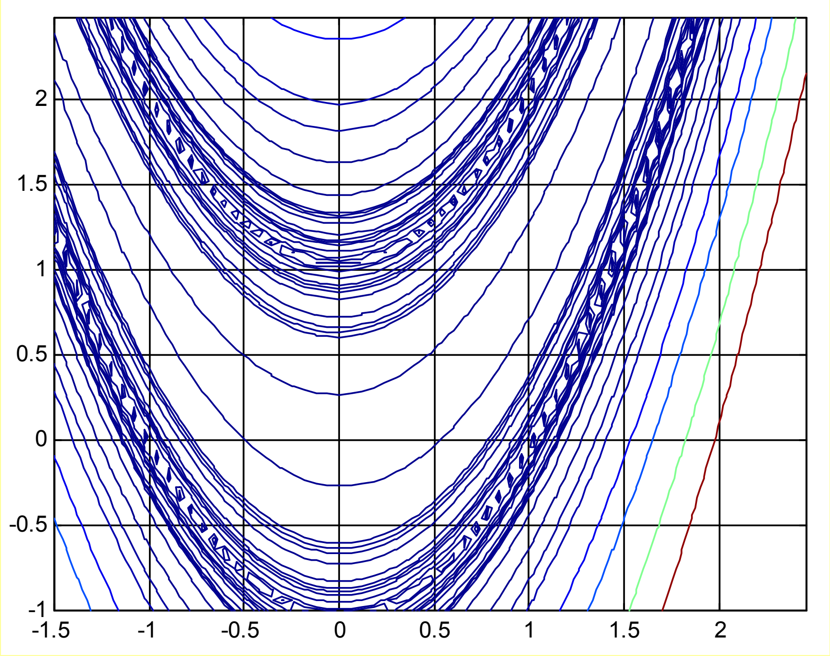
multi-point parameter extraction (with 2 additional points)



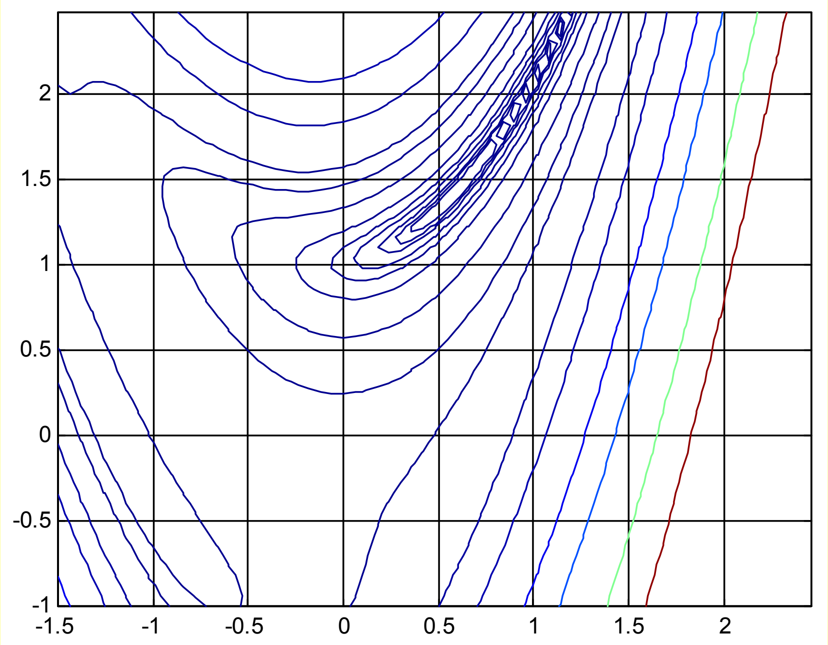
$$\boldsymbol{x}_c^{(1)} = \begin{bmatrix} 0.6 \\ 1.4 \end{bmatrix}$$

First l_2 Parameter Extraction

single point parameter extraction



multi-point parameter extraction (with 2 additional points)



$$\boldsymbol{x}_c^{(1)} = \begin{bmatrix} 0.6 \\ 1.4 \end{bmatrix}$$

Space Mapping Solution Process

0. $\boldsymbol{x}_f^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\boldsymbol{B}^{(1)} = \boldsymbol{1}$, $j = 1$

1. $R_f(\boldsymbol{x}_f^{(1)}) = 108.32$

2. When $\boldsymbol{x}_c^{(1)} = \begin{bmatrix} 0.6 \\ 1.4 \end{bmatrix}$, $R_c(\boldsymbol{x}_c^{(1)}) = R_f(\boldsymbol{x}_f^{(1)})$

3. $\boldsymbol{f}^{(1)} = \begin{bmatrix} 0.6 \\ 1.4 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.4 \\ 0.4 \end{bmatrix}$

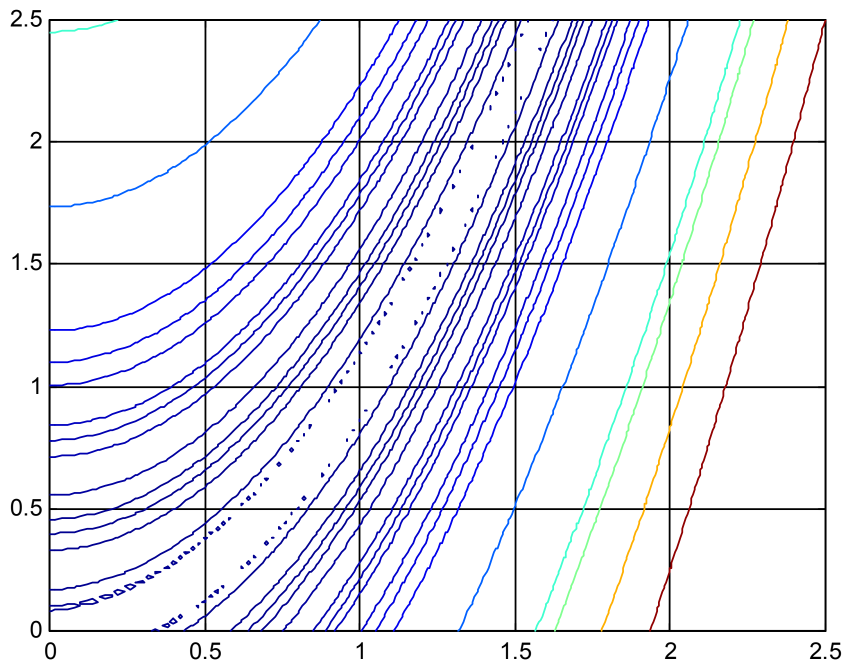
4. Since $\boldsymbol{B}^{(1)} = \boldsymbol{1}$, $\boldsymbol{h}^{(1)} = \begin{bmatrix} 0.4 \\ -0.4 \end{bmatrix}$

5. Set $\boldsymbol{x}_f^{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.4 \\ -0.4 \end{bmatrix} = \begin{bmatrix} 1.4 \\ 0.6 \end{bmatrix}$

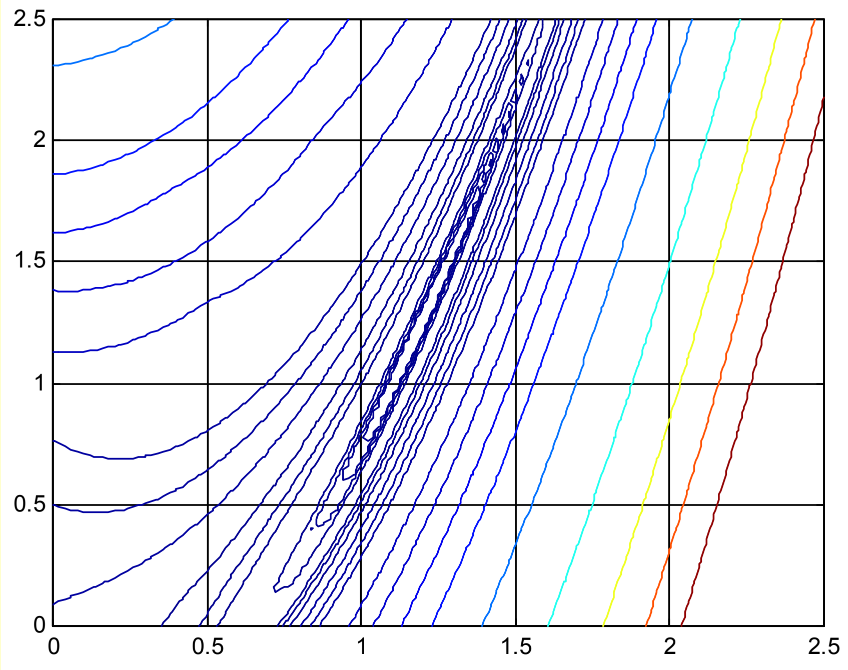
6. $R_f(\boldsymbol{x}_f^{(2)}) = 1.8207$

Second l_1 Parameter Extraction

single point parameter extraction



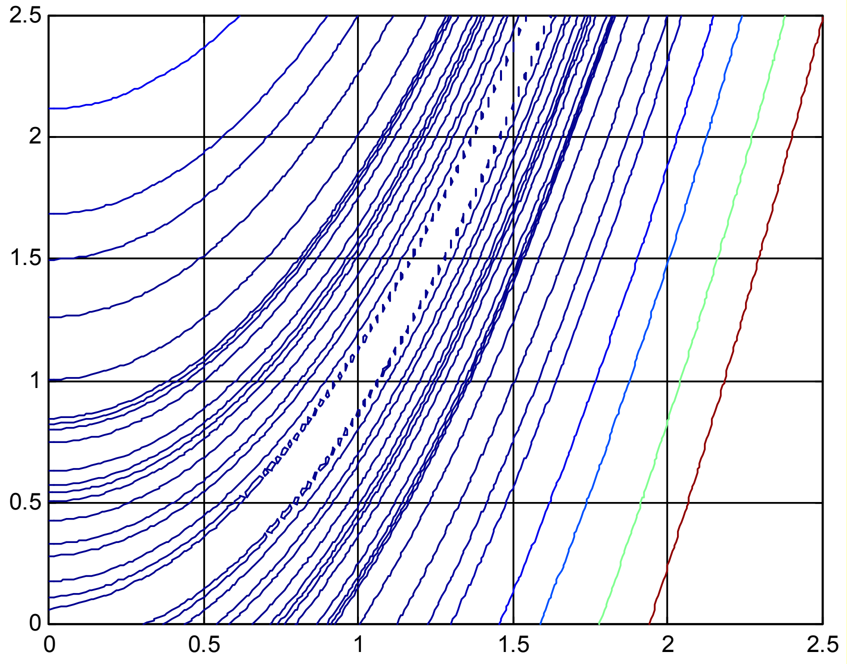
multi-point parameter extraction (with 4 additional points)



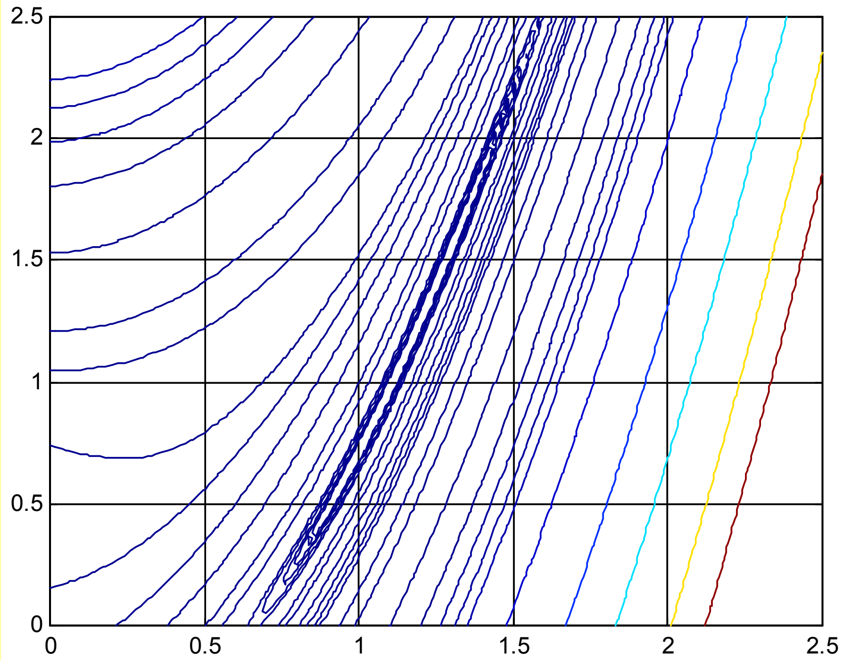
$$\boldsymbol{x}_c^{(2)} = \begin{bmatrix} 1.12 \\ 1.12 \end{bmatrix}$$

Second l_2 Parameter Extraction

single point parameter extraction



multi-point parameter extraction (with 4 additional points)



$$\boldsymbol{x}_c^{(2)} = \begin{bmatrix} 1.12 \\ 1.12 \end{bmatrix}$$

Space Mapping Solution Process (continued)

7. When $\boldsymbol{x}_c^{(2)} = \begin{bmatrix} 1.12 \\ 1.12 \end{bmatrix}$, $R_c(\boldsymbol{x}_c^{(2)}) = R_f(\boldsymbol{x}_f^{(2)})$

8. $\boldsymbol{f}^{(2)} = \begin{bmatrix} 1.12 \\ 1.12 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.12 \\ 0.12 \end{bmatrix}$

9. $\boldsymbol{B}^{(2)} = \boldsymbol{B}^{(1)} + \dfrac{\boldsymbol{f}^{(2)} \boldsymbol{h}^{(1)^T}}{\boldsymbol{h}^{(1)^T} \boldsymbol{h}^{(1)}} = \begin{bmatrix} 1.15 & -0.15 \\ 0.15 & 0.85 \end{bmatrix}$

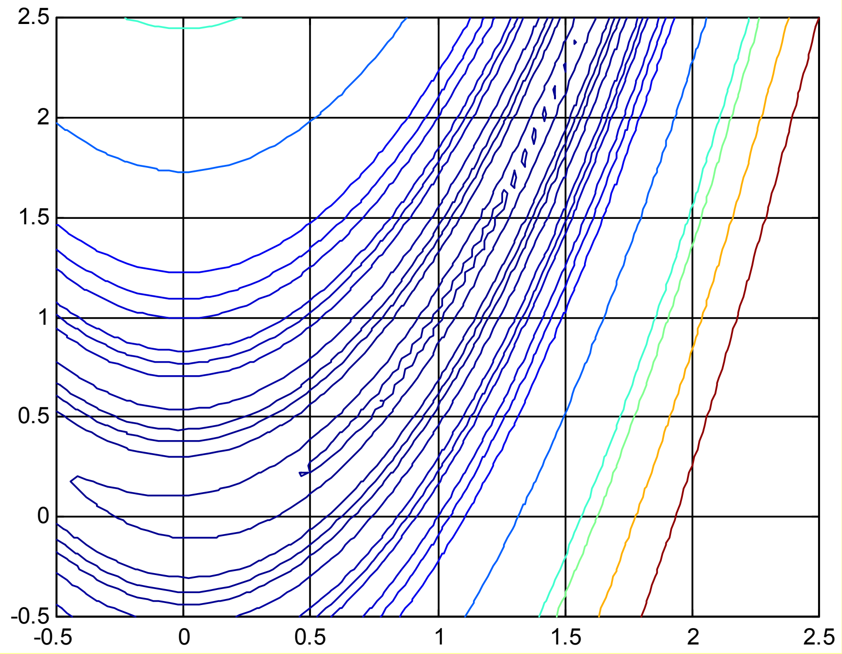
4b. $\boldsymbol{h}^{(2)} = -\boldsymbol{B}^{(2)^{-1}} \boldsymbol{f}^{(2)} = \begin{bmatrix} -0.12 \\ -0.12 \end{bmatrix}$

5b. Set $\boldsymbol{x}_f^{(3)} = \begin{bmatrix} 1.4 \\ 0.6 \end{bmatrix} + \begin{bmatrix} -0.12 \\ -0.12 \end{bmatrix} = \begin{bmatrix} 1.28 \\ 0.48 \end{bmatrix}$

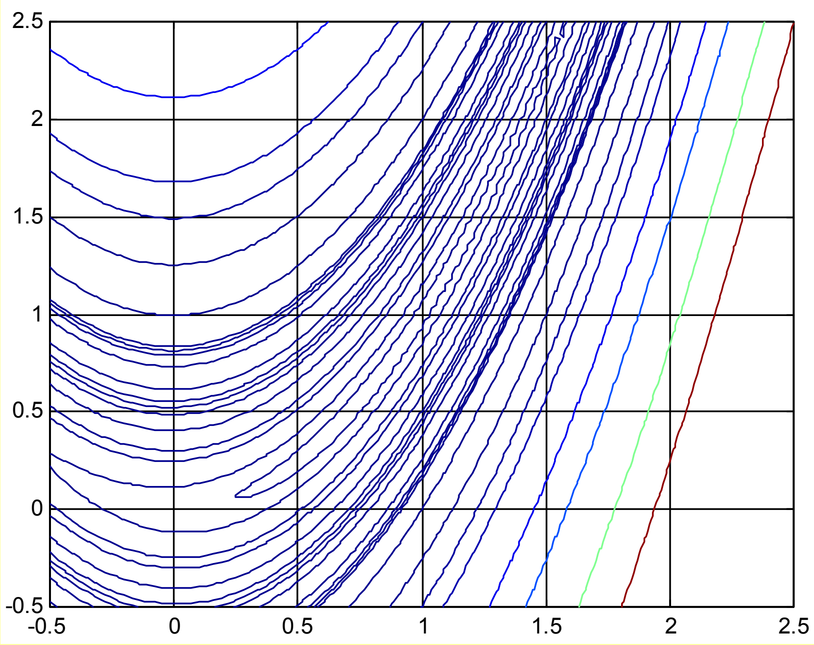
6b. $R_f(\boldsymbol{x}_f^{(3)}) = 0.1308$

Third Parameter Extraction

single l_1 point parameter extraction



single l_2 point parameter extraction



$$\boldsymbol{x}_c^{(3)} = \begin{bmatrix} 1.012 \\ 0.988 \end{bmatrix}$$

Space Mapping Solution Process (continued)

7b. When $\boldsymbol{x}_c^{(3)} = \begin{bmatrix} 1.012 \\ 0.988 \end{bmatrix}$, $R_c(\boldsymbol{x}_c^{(3)}) = R_f(\boldsymbol{x}_f^{(3)})$

8b. $\boldsymbol{f}^{(3)} = \begin{bmatrix} 1.012 \\ 0.988 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.012 \\ -0.012 \end{bmatrix}$

9b. $\boldsymbol{B}^{(3)} = \boldsymbol{B}^{(2)} + \dfrac{\boldsymbol{f}^{(3)} \boldsymbol{h}^{(2)^T}}{\boldsymbol{h}^{(2)^T} \boldsymbol{h}^{(2)}} = \begin{bmatrix} 1.1 & -0.2 \\ 0.2 & 0.9 \end{bmatrix}$

4c. $\boldsymbol{h}^{(3)} = -\boldsymbol{B}^{(3)^{-1}} \boldsymbol{f}^{(3)} = \begin{bmatrix} -0.0082 \\ 0.0151 \end{bmatrix}$

5c. $\boldsymbol{x}_f^{(4)} = \begin{bmatrix} 1.28 \\ 0.48 \end{bmatrix} + \begin{bmatrix} -0.0082 \\ 0.0151 \end{bmatrix} = \begin{bmatrix} 1.2718 \\ 0.4951 \end{bmatrix}$

6c. $R_f(\boldsymbol{x}_f^{(4)}) = 9.2 \times 10^{-8}$, then $\bar{\boldsymbol{x}}_f = \boldsymbol{x}_f^{(4)}$
and we can end the algorithm

Conclusions

the Aggressive Space Mapping (ASM) algorithm with multiple point parameter extraction was illustrated making use of two analytical Rosenbrock functions

the improvement in the uniqueness of the parameter extraction due to multiple point matching was graphically illustrated

the ASM algorithm was executed in a step by step fashion, and the corresponding space mapped solution was accomplished after three SM iterations for the case of shifting, scaling and rotation of the input parameters