An Introduction to Space Mapping

Dr. José Ernesto Rayas-Sánchez

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Outline

- Space mapping origins
- Principal SM-based optimization methods
- Space mapping terminology
- The general aim of space mapping
- Designing with a fine and a coarse model
- Input space mapping approach to design
- An algorithm for constrained Broyden-based input SM
- The space mapped solution versus the optimal solution
- Examples

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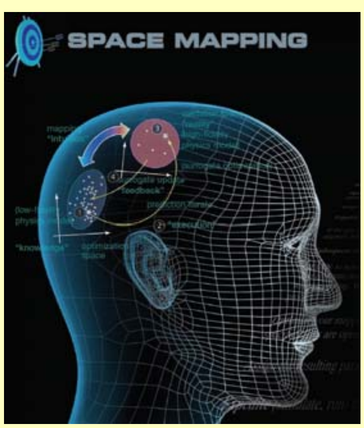
Space Mapping Origins

- Space Mapping (SM) was originally created by Dr. John W. Bandler et al. in 1994 at McMaster University
- SM was invented as an extremely efficient design optimization procedure for optimizing computationally expensive functions
- SM proved to be very useful also for developing highly accurate and efficient models
- SM has experienced an impressive evolution; many advanced SM techniques are now available

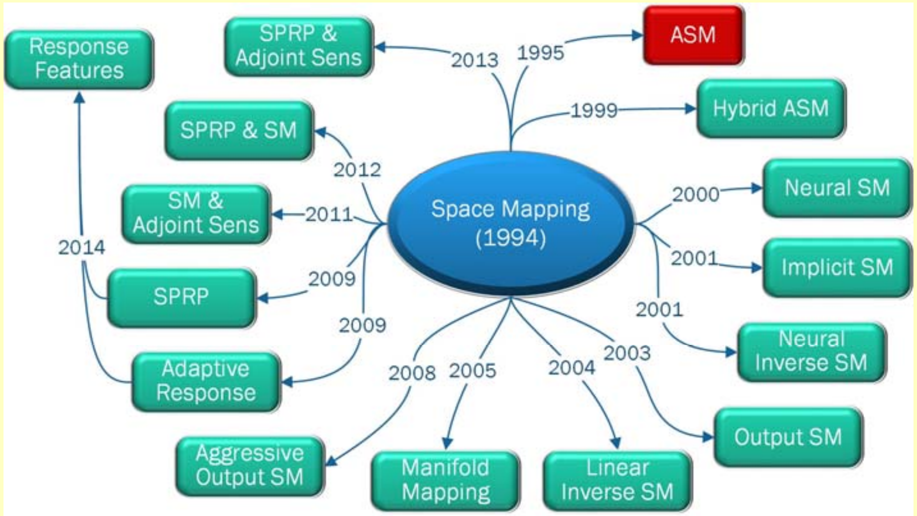
Space Mapping Origin (cont.)

- SM belongs to a class of surrogate-based optimization
- ASM, or Broyden-based input SM, emerged in 1995



J. W. Bandler, “Have you ever wondered about the engineer's mysterious ‘feel’ for a problem?,” *IEEE Canadian Rev.*, no. 70, pp. 50-60, 2013, Summer.

Space Mapping Evolution



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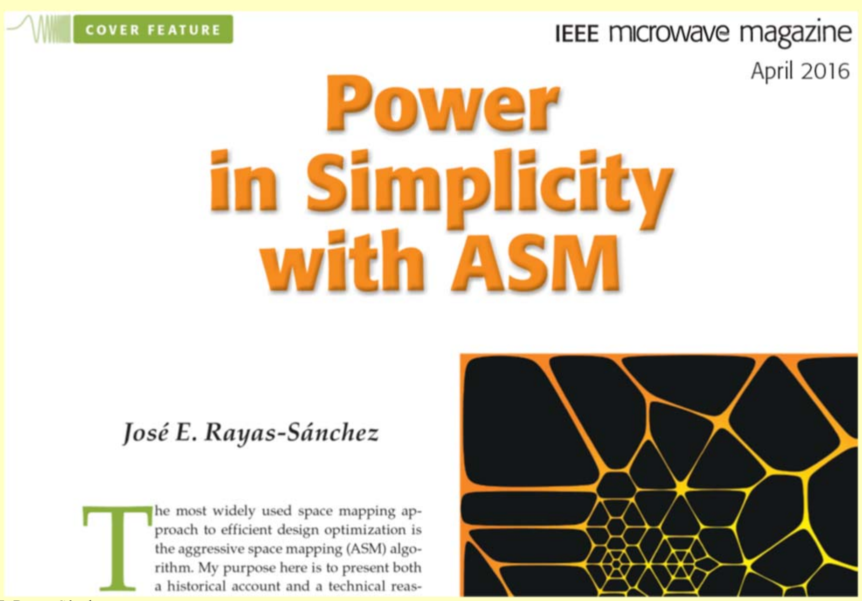
(Rayas-Sánchez, 2016) 5

ASM Review


COVER FEATURE

IEEE microwave magazine

April 2016

Power in Simplicity with ASM

José E. Rayas-Sánchez

The most widely used space mapping approach to efficient design optimization is the aggressive space mapping (ASM) algorithm. My purpose here is to present both a historical account and a technical reas-

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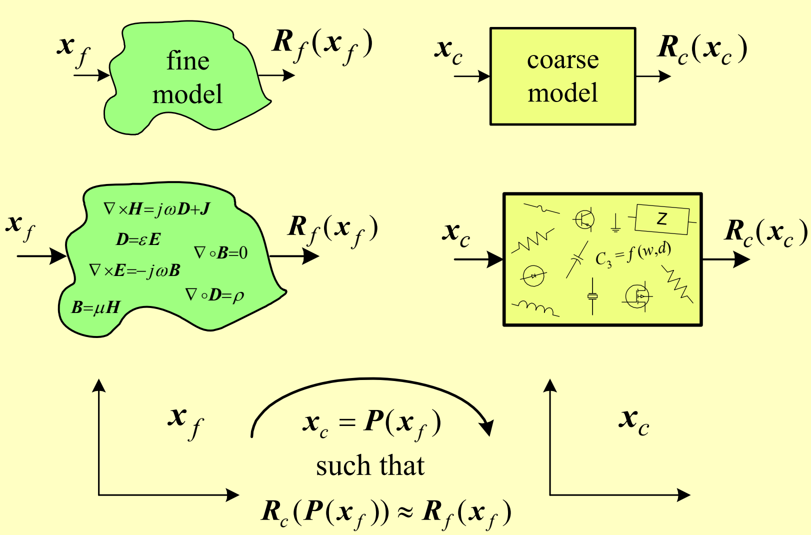
Space Mapping Terminology

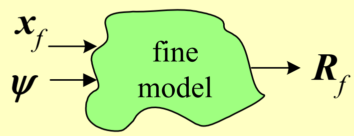
- SM requires at least 2 models for the device to be designed: a fine model and a coarse model
- $\boldsymbol{x}_f, \boldsymbol{x}_c \in \Re^n$: fine and coarse model design variables
- $\boldsymbol{R}_f, \boldsymbol{R}_c \in \Re^r$: fine and coarse model optimizable responses
- $\boldsymbol{\psi} \in \Re^s$: independent variable samples
- $\boldsymbol{R}_f(\boldsymbol{x}_f, \boldsymbol{\psi})$ is very accurate but computationally very expensive
- $\boldsymbol{R}_c(\boldsymbol{x}_c, \boldsymbol{\psi})$ is not accurate but computationally very efficient

The Aim of Input Space Mapping



(Bandler et al., 1999)

Designing with a Fine Model



- $\boldsymbol{x}_f \in \Re^n$: design parameters
- $\boldsymbol{\psi} \in \Re^s$: independent variable samples
- $\boldsymbol{R}_f \in \Re^r$: optimizable responses
- U: $\Re^n \to \Re$ is the objective function (usually minimax) expressed in terms of the design specifications

$$\boldsymbol{x}_f^* = \arg\min_{\boldsymbol{x}_f} U(\boldsymbol{R}_f(\boldsymbol{x}_f, \boldsymbol{\psi}))$$

solving the above problem is usually prohibitive

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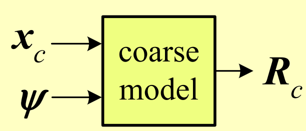
Designing with a Coarse Model



- $\boldsymbol{x}_c \in \Re^n$: design parameters
- $\boldsymbol{\psi} \in \Re^s$: independent variable samples
- $\boldsymbol{R}_c \in \Re^r$: optimizable responses
- U: $\Re^n \to \Re$ is the same objective function

$$\boldsymbol{x}_c^* = \arg\min_{\boldsymbol{x}_c} U(\boldsymbol{R}_c(\boldsymbol{x}_c, \boldsymbol{\psi}))$$

the above problem is usually solved by classical optimization methods

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Typical Design Scenario

- ☺ Designing with a coarse model (finding $\boldsymbol{x}_c^*$) is fast and relatively easy
- ☺ The optimal coarse model solution provides a target response that satisfies all our design specs

$$\boldsymbol{R}_c(\boldsymbol{x}_c^*, \psi) = \boldsymbol{R}_c^* \;\rightarrow \text{target}$$

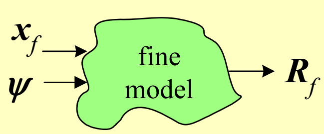
- ☹ However, the fine model evaluated at $\boldsymbol{x}_c^*$ typically deviates from the target response and violates the specs

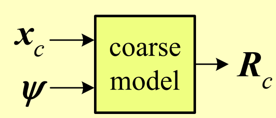
$$\boldsymbol{R}_f(\boldsymbol{x}_c^*, \psi) \neq \boldsymbol{R}_c^*$$

- We use our engineering knowledge and experience to adjust the solution

$$\boldsymbol{R}_f(\boldsymbol{x}_c^* + \Delta\boldsymbol{x}, \psi) \approx \boldsymbol{R}_c^*$$

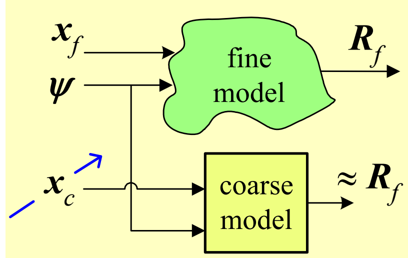
Input Space Mapping Approach to Design





$$\boldsymbol{x}_c^* = \arg\min_{\boldsymbol{x}_c} U(\boldsymbol{R}_c(\boldsymbol{x}_c, \boldsymbol{\psi}))$$

$$\boldsymbol{f}(\boldsymbol{x}_f) = \boldsymbol{P}(\boldsymbol{x}_f) - \boldsymbol{x}_c^*$$



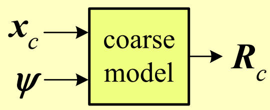
$$\boldsymbol{P}(\boldsymbol{x}_f) = \arg\min_{\boldsymbol{x}_c} \left\| \boldsymbol{R}_f(\boldsymbol{x}_f, \boldsymbol{\psi}) - \boldsymbol{R}_c(\boldsymbol{x}_c, \boldsymbol{\psi}) \right\|$$

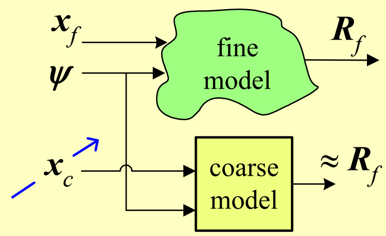
A SM solution $\boldsymbol{x}_f^{SM}$ is found when

$$\boldsymbol{f}(\boldsymbol{x}_f^{SM}) \approx \boldsymbol{0}$$

$$\rightarrow \boldsymbol{R}_f(\boldsymbol{x}_f^{SM}) \approx \boldsymbol{R}_c(\boldsymbol{x}_c^*)$$

Input Space Mapping Approach to Design





$$x_c^* = \arg\min_{x_c} U(R_c(x_c,\psi))$$

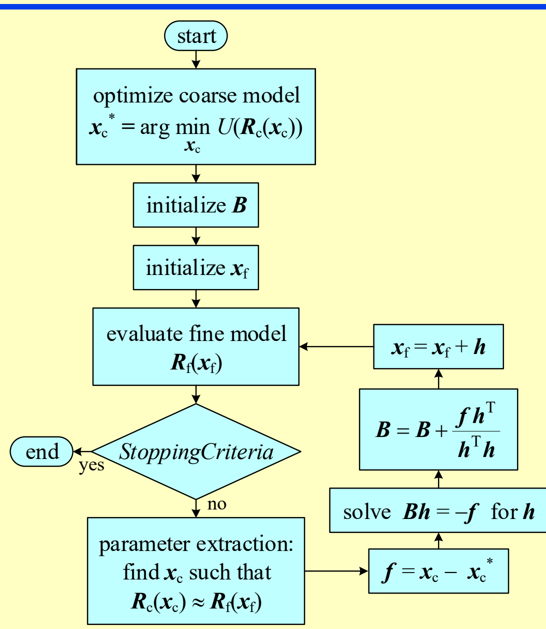
$$P(x_f) = \arg\min_{x_c} \left\| R_f(x_f,\psi) - R_c(x_c,\psi) \right\|$$

$$f(x_f) = P(x_f) - x_c^*$$

A SM solution x_f^{SM} is found when

$$f(x_f^{SM}) \approx 0 \quad \rightarrow \quad R_f(x_f^{SM}) \approx R_c(x_c^*)$$

Broyden-based Input Space Mapping Algorithm



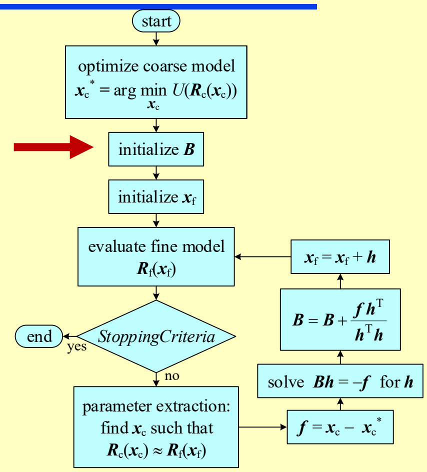
Initializing Matrix $\boldsymbol{B}$

- If $\boldsymbol{x}_\mathrm{f}$ and $\boldsymbol{x}_\mathrm{c}$ have the same nature and dimension

$$\boldsymbol{B} = \boldsymbol{I}$$

- If $\boldsymbol{x}_\mathrm{f}$ and $\boldsymbol{x}_\mathrm{c}$ have different nature or dimension:

$$\boldsymbol{B} \approx \boldsymbol{J}(\boldsymbol{x}_\mathrm{c}(\boldsymbol{x}_\mathrm{f})) = \frac{\partial \boldsymbol{x}_\mathrm{c}}{\partial \boldsymbol{x}_\mathrm{f}}$$



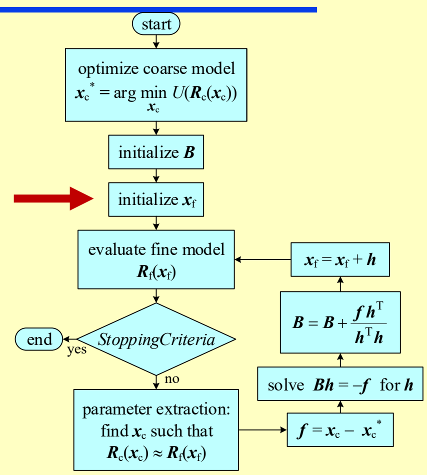
Initializing Vector $\boldsymbol{x}_\mathrm{f}$

- If $\boldsymbol{x}_\mathrm{f}$ and $\boldsymbol{x}_\mathrm{c}$ have the same nature and dimension

$$\boldsymbol{x}_\mathrm{f} = \boldsymbol{x}_\mathrm{c}^*$$

- If $\boldsymbol{x}_\mathrm{f}$ and $\boldsymbol{x}_\mathrm{c}$ have different nature or dimension:

$$\text{solve} \quad \boldsymbol{B}\boldsymbol{x}_\mathrm{f} = \boldsymbol{x}_\mathrm{c}^* \quad \text{for} \quad \boldsymbol{x}_\mathrm{f}$$

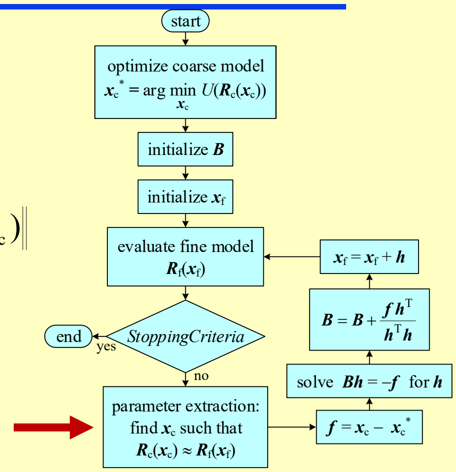


Parameter Extraction

- Weakest part of ASM
- Local alignment at the i-th iteration:

$$\boldsymbol{x}_{\mathrm{c}}^{(i)} = \arg\min_{\boldsymbol{x}_{\mathrm{c}}} \left\| \boldsymbol{R}_{\mathrm{f}}(\boldsymbol{x}_{\mathrm{f}}^{(i)}) - \boldsymbol{R}_{\mathrm{c}}(\boldsymbol{x}_{\mathrm{c}}) \right\|$$

- Multiple local minima may lead ASM to oscillations or divergence



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Stopping Criteria

$$\left\| \boldsymbol{f}(\boldsymbol{x}_{\mathrm{f}}^{(i)}) \right\|_\infty < \varepsilon_1 \quad \vee \ldots$$

$$\left\| \boldsymbol{R}_{\mathrm{f}}(\boldsymbol{x}_{\mathrm{f}}^{(i)}) - \boldsymbol{R}_{\mathrm{c}}(\boldsymbol{x}_{\mathrm{c}}^{*}) \right\|_\infty \le \varepsilon_2(\varepsilon_2 + \left\| \boldsymbol{R}_{\mathrm{c}}(\boldsymbol{x}_{\mathrm{c}}^{*}) \right\|_\infty) \quad \vee \ldots$$

$$\left\| \boldsymbol{x}_{\mathrm{f}}^{(i+1)} - \boldsymbol{x}_{\mathrm{f}}^{(i)} \right\|_2 \le \varepsilon_3(\varepsilon_3 + \left\| \boldsymbol{x}_{\mathrm{f}}^{(i)} \right\|_2) \quad \vee \ldots$$

$$i > i_{\max}$$

ε_1, ε_2, and ε_3 are arbitrary small positive scalars

$i_{\max}$ is typically $3n$ or $4n$ $\quad (\boldsymbol{x}_{\mathrm{f}} \in \Re^n)$

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The Root of ASM: Finding Roots

$$\boldsymbol{x}_{\mathrm{c}}^{(i)} = \arg\min_{\boldsymbol{x}_{\mathrm{c}}} \left\| \boldsymbol{R}_{\mathrm{f}}(\boldsymbol{x}_{\mathrm{f}}^{(i)}) - \boldsymbol{R}_{\mathrm{c}}(\boldsymbol{x}_{\mathrm{c}}) \right\| \quad \rightarrow \quad \boldsymbol{x}_{\mathrm{c}}^{(i)} = \boldsymbol{P}(\boldsymbol{x}_{\mathrm{f}}^{(i)})$$

ASM iteratively solves

$$\boldsymbol{f}(\boldsymbol{x}_{\mathrm{f}}) = \boldsymbol{P}(\boldsymbol{x}_{\mathrm{f}}) - \boldsymbol{x}_{\mathrm{c}}^{*} = \boldsymbol{0}$$

Any root $\boldsymbol{x}_{\mathrm{f}}^{\mathrm{SM}}$ of $\boldsymbol{f}(\boldsymbol{x}_{\mathrm{f}})$ implies

$$\boldsymbol{R}_{\mathrm{f}}(\boldsymbol{x}_{\mathrm{f}}^{\mathrm{SM}}) \approx \boldsymbol{R}_{\mathrm{c}}^{*}$$

ASM ≡ Broyden method for solving SNLEq

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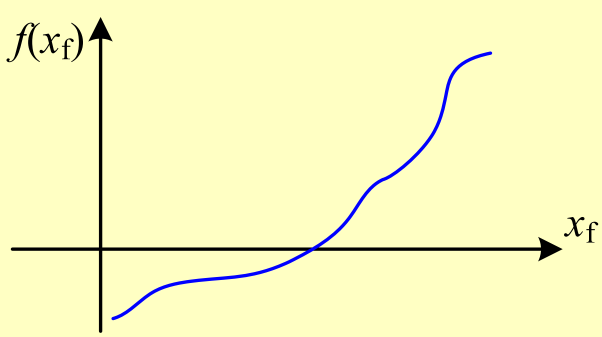
Typical Evolution of ASM



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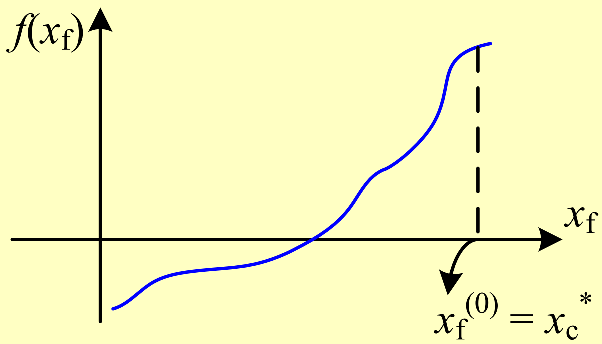
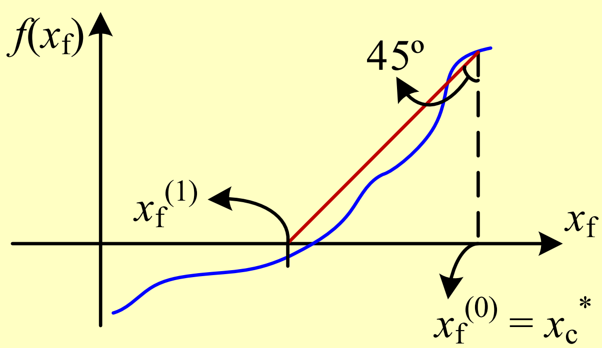
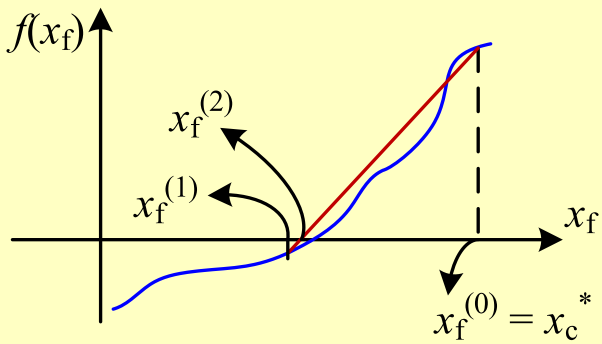
Typical Evolution of ASM (cont.)



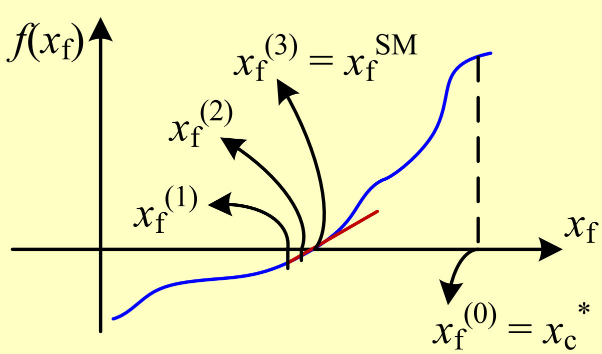
Typical Evolution of ASM (cont.)



Typical Evolution of ASM (cont.)



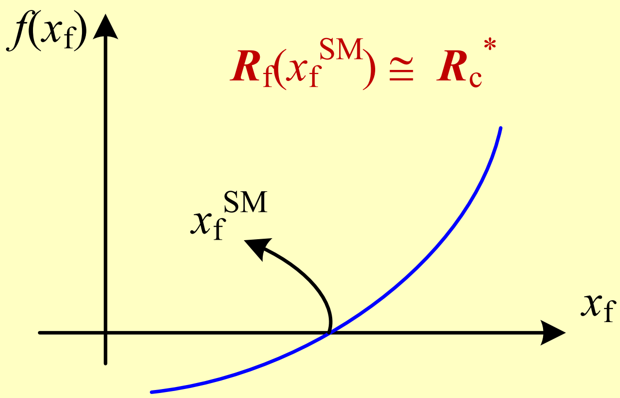
Typical Evolution of ASM (cont.)



ASM efficiency depends on the nonlinearity of ***P***

ASM Scenarios

- A unique and exact solution exists



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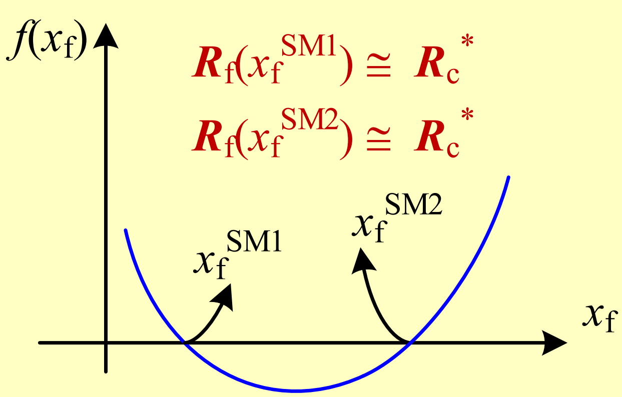
ASM Scenarios (cont.)

- Several exact solutions exists



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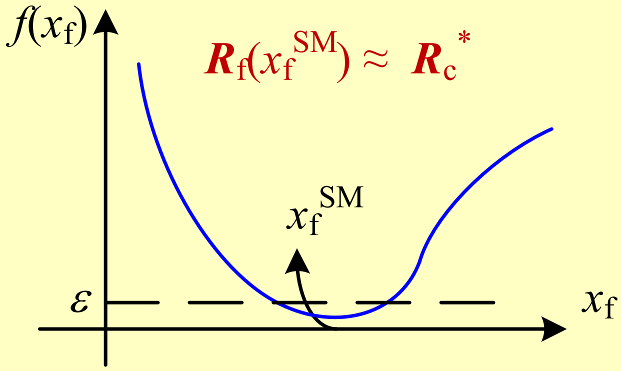
ASM Scenarios (cont.)

- An acceptable solution exists



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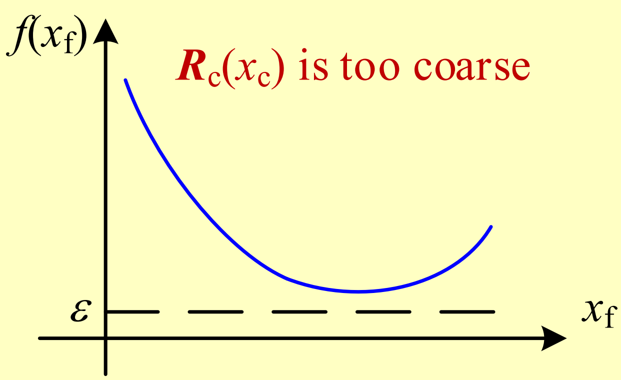
ASM Scenarios

- There is no acceptable solution



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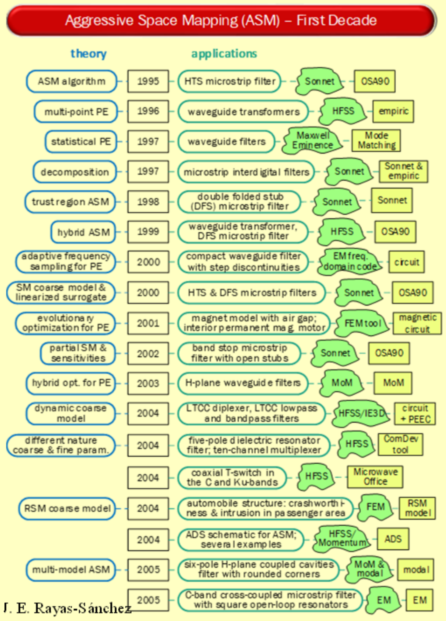
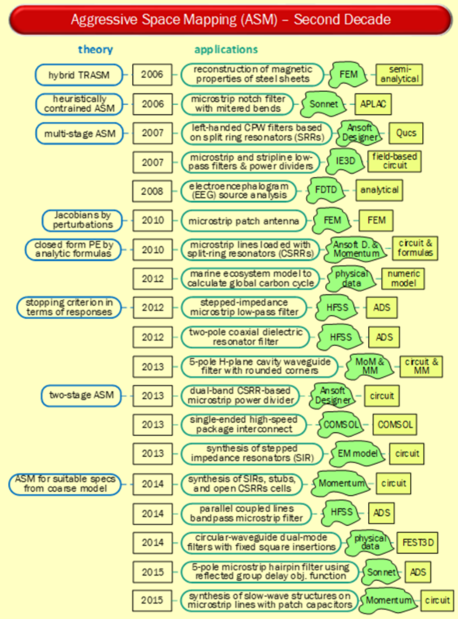
2.3 Decades of ASM





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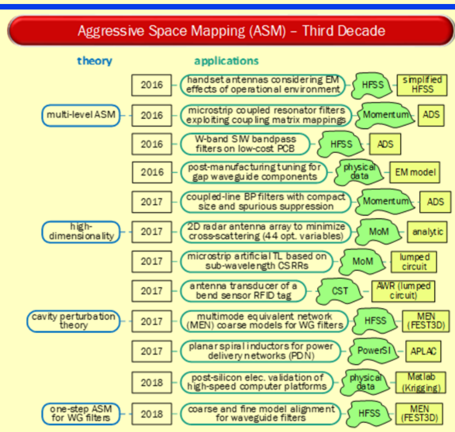
2.3 Decades of ASM (cont.)



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First Decade of ASM

theory		applications		
ASM algorithm	1995	HTS microstrip filter	Sonnet	OSA90
multi-point PE	1996	waveguide transformers	HFSS	empiric
statistical PE	1997	waveguide filters	Maxwell Eminence	Mode Matching
decomposition	1997	microstrip interdigital filters	Sonnet	Sonnet & empiric
trust region ASM	1998	double folded stub (DFS) microstrip filter	Sonnet	Sonnet
hybrid ASM	1999	waveguide transformer, DFS microstrip filter	HFSS	OSA90

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First Decade of ASM (cont.)

theory		applications		
adaptive frequency sampling for PE	2000	compact waveguide filter with step discontinuities	EM freq. domain code	circuit
SM coarse model & linearized surrogate	2000	HTS & DFS microstrip filters	Sonnet	OSA90
evolutionary optimization for PE	2001	magnet model with air gap; interior permanent mag. motor	FEM tool	magnetic circuit
partial SM & sensitivities	2002	band stop microstrip filter with open stubs	Sonnet	OSA90
hybrid opt. for PE	2003	H-plane waveguide filters	MoM	MoM
dynamic coarse model	2004	LTCC diplexer, LTCC lowpass and bandpass filters	HFSS/IE3D	circuit + PEEC

magnetic circuits

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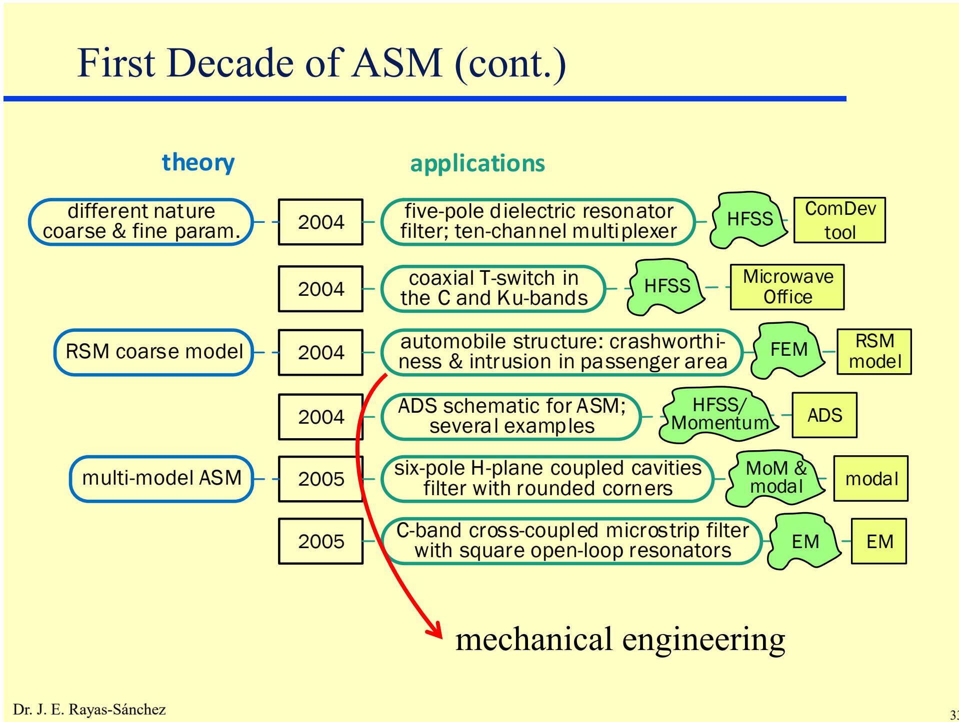
First Decade of ASM (cont.)
theory
applications
different nature coarse & fine param.
2004
five-pole dielectric resonator filter; ten-channel multiplexer
HFSS
ComDev tool
2004
coaxial T-switch in the C and Ku-bands
HFSS
Microwave Office
RSM coarse model
2004
automobile structure: crashworthiness & intrusion in passenger area
FEM
RSM model
2004
ADS schematic for ASM; several examples
HFSS/ Momentum
ADS
multi-model ASM
2005
six-pole H-plane coupled cavities filter with rounded corners
MoM & modal
modal
2005
C-band cross-coupled microstrip filter with square open-loop resonators
EM
EM
mechanical engineering
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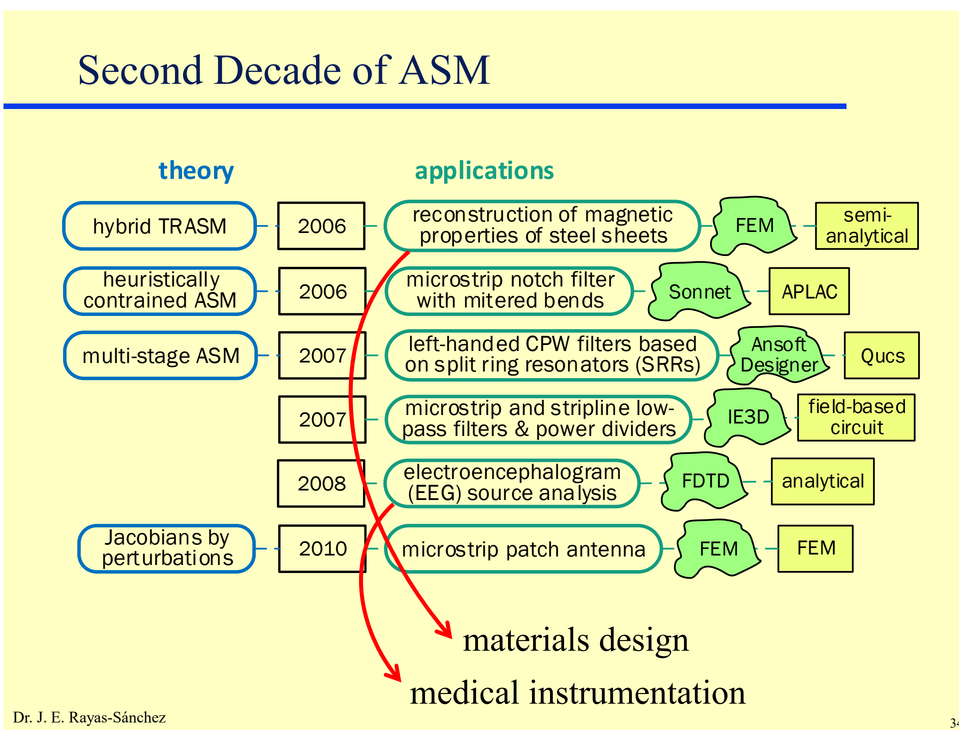
Second Decade of ASM
theory
applications
hybrid TRASM
2006
reconstruction of magnetic properties of steel sheets
FEM
semi-analytical
heuristically contrained ASM
2006
microstrip notch filter with mitered bends
Sonnet
APLAC
multi-stage ASM
2007
left-handed CPW filters based on split ring resonators (SRRs)
Ansoft Designer
Qucs
2007
microstrip and stripline low-pass filters & power dividers
IE3D
field-based circuit
2008
electroencephalogram (EEG) source analysis
FDTD
analytical
Jacobians by perturbations
2010
microstrip patch antenna
FEM
FEM
materials design
medical instrumentation
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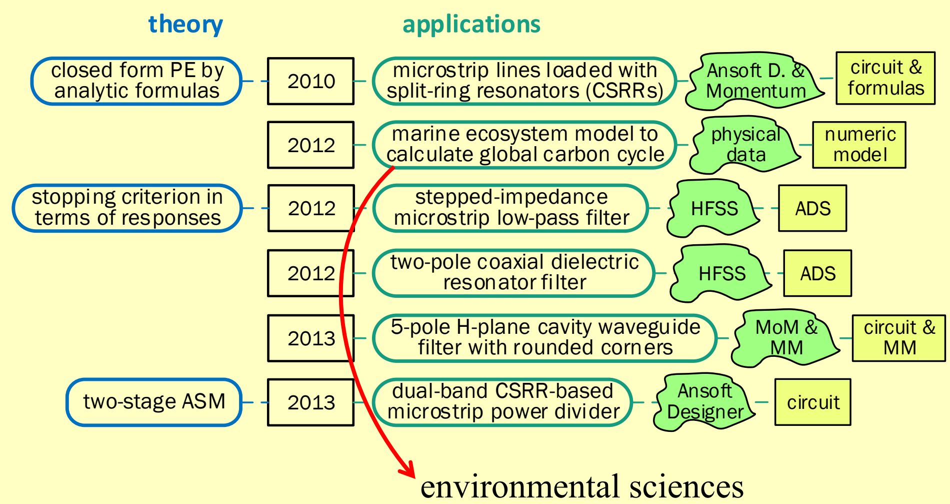
Second Decade of ASM (cont.)



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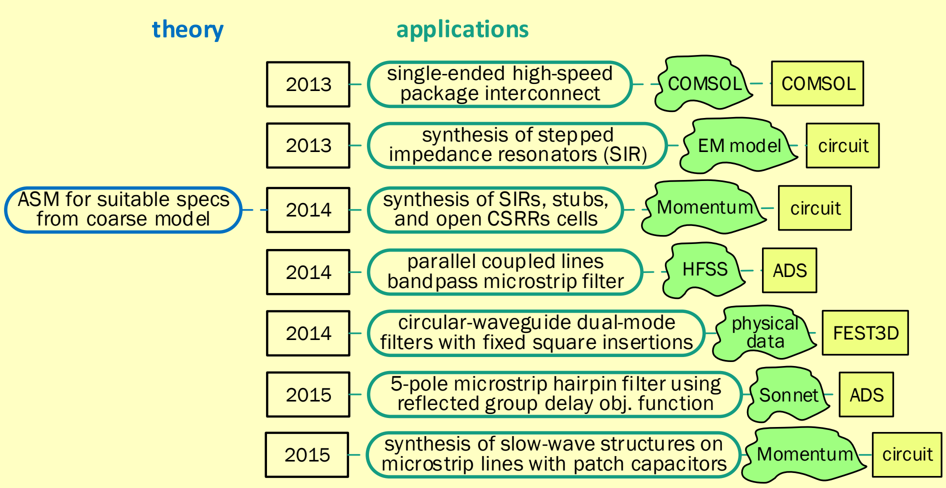
Second Decade of ASM (cont.)



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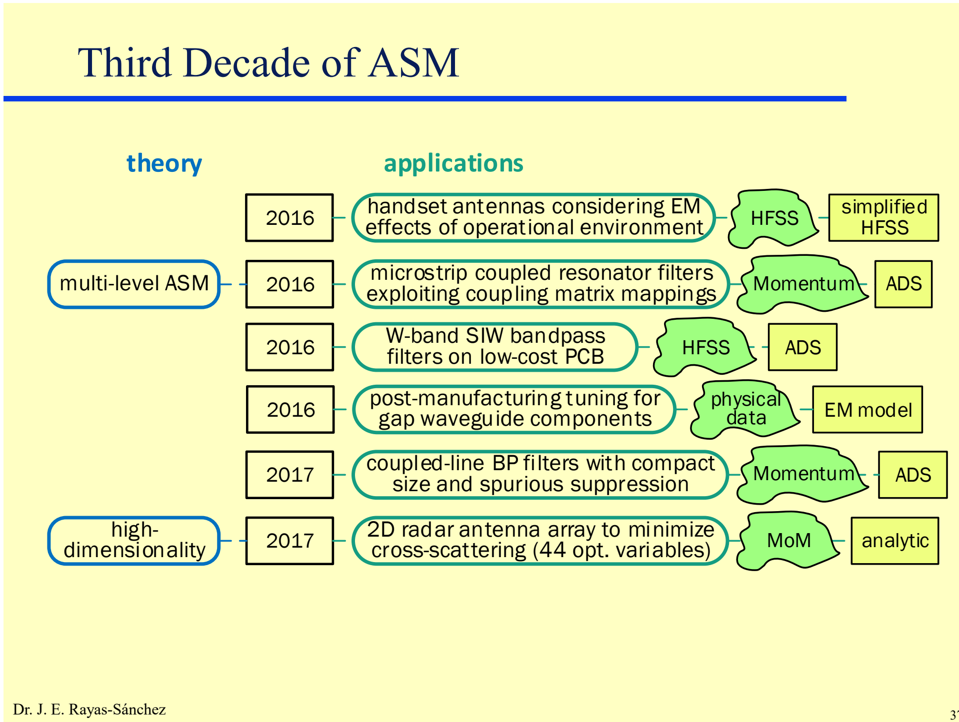
Third Decade of ASM
theory
applications
2016
handset antennas considering EM effects of operational environment
HFSS
simplified HFSS
multi-level ASM
2016
microstrip coupled resonator filters exploiting coupling matrix mappings
Momentum
ADS
2016
W-band SIW bandpass filters on low-cost PCB
HFSS
ADS
2016
post-manufacturing tuning for gap waveguide components
physical data
EM model
2017
coupled-line BP filters with compact size and spurious suppression
Momentum
ADS
high-dimensionality
2017
2D radar antenna array to minimize cross-scattering (44 opt. variables)
MoM
analytic
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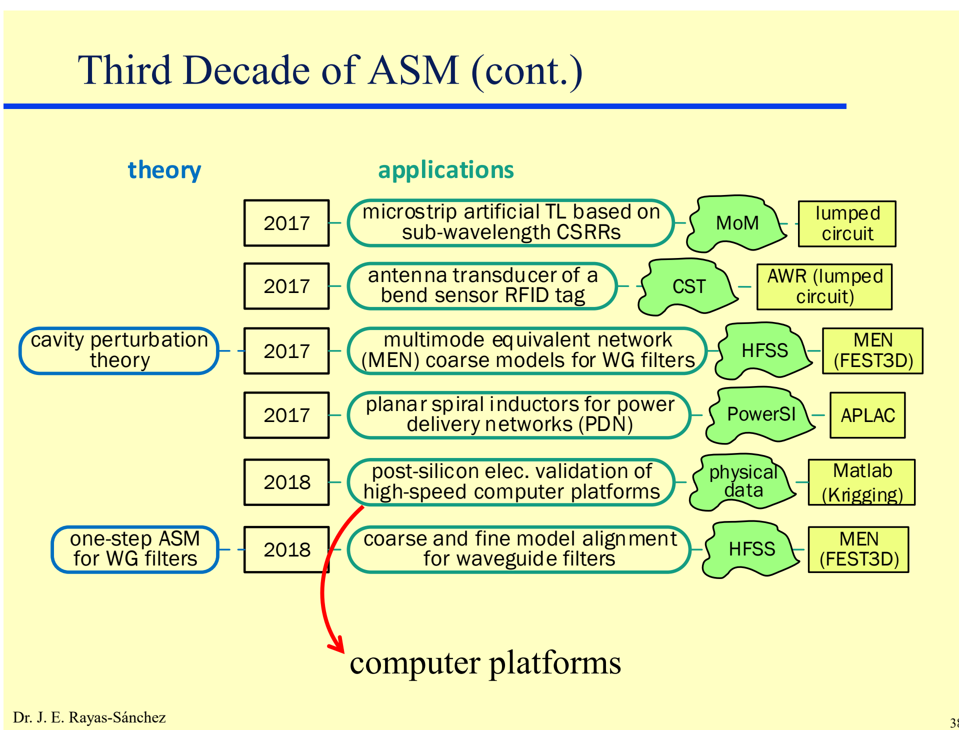
Third Decade of ASM (cont.)
theory
applications
2017
microstrip artificial TL based on sub-wavelength CSRRs
MoM
lumped circuit
2017
antenna transducer of a bend sensor RFID tag
CST
AWR (lumped circuit)
cavity perturbation theory
2017
multimode equivalent network (MEN) coarse models for WG filters
HFSS
MEN (FEST3D)
2017
planar spiral inductors for power delivery networks (PDN)
PowerSI
APLAC
2018
post-silicon elec. validation of high-speed computer platforms
physical data
Matlab (Krigging)
one-step ASM for WG filters
2018
coarse and fine model alignment for waveguide filters
HFSS
MEN (FEST3D)
computer platforms
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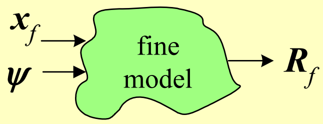
The Fine Model Solution



$$\boldsymbol{x}_f^* = \arg\min_{\boldsymbol{x}_f} U(\boldsymbol{R}_f(\boldsymbol{x}_f, \boldsymbol{\psi}))$$

When solving

$$\boldsymbol{f}(\boldsymbol{x}_f) = \boldsymbol{P}(\boldsymbol{x}_f) - \boldsymbol{x}_c^* = \boldsymbol{0}$$

we do not attempt to find $\boldsymbol{x}_f^*$

$$\boldsymbol{x}_f^{SM} \neq \boldsymbol{x}_f^*$$

Obtaining $\boldsymbol{P}$ and $\boldsymbol{x}_f^{SM}$

We apply a constrained Broyden-based algorithm to solve the following system of nonlinear equations

$$\boldsymbol{f}(\boldsymbol{x}_f) = \boldsymbol{P}(\boldsymbol{x}_f) - \boldsymbol{x}_c^* = \boldsymbol{0}$$

where $\boldsymbol{x}_c = \boldsymbol{P}(\boldsymbol{x}_f)$ is evaluated through

$$\boldsymbol{P}(\boldsymbol{x}_f) = \arg\min_{\boldsymbol{x}_c} \left\| \boldsymbol{e}_1^T \quad \ldots \quad \boldsymbol{e}_p^T \right\|_2^2$$

p is the number of points of the independent variable and the j-th parameter extraction error vector is given by

$$\boldsymbol{e}_j(\boldsymbol{x}_f) = \boldsymbol{R}_{fs}(\boldsymbol{x}_f, \psi_j) - \boldsymbol{R}_{cs}(\boldsymbol{x}_c, \psi_j)$$

Algorithm for Constrained Broyden-Based i-SM

Begin
find $\boldsymbol{x}_c^*$ solving (1)
$i = 0$, $\boldsymbol{x}_f^{(i)} = \boldsymbol{x}_c^*$, $\boldsymbol{B}^{(i)} = \boldsymbol{I}$, $\delta = 0.3$
$\boldsymbol{f}^{(i)} = \boldsymbol{P}(\boldsymbol{x}_f^{(i)}) - \boldsymbol{x}_c^*$ using (2)
repeat until *StoppingCriteria*
solve $\boldsymbol{B}^{(i)}\boldsymbol{h}^{(i)} = -\boldsymbol{f}^{(i)}$ for $\boldsymbol{h}^{(i)}$
$\boldsymbol{x}_f^{(test)} = \boldsymbol{x}_f^{(i)} + \boldsymbol{h}^{(i)}$
while $\boldsymbol{x}_f^{(test)} \prec \boldsymbol{x}_f^{\min} \vee \boldsymbol{x}_f^{(test)} \succ \boldsymbol{x}_f^{\max}$
$\boldsymbol{h}^{(i)} = \delta\, \boldsymbol{h}^{(i)}$
$\boldsymbol{x}_f^{(test)} = \boldsymbol{x}_f^{(i)} + \boldsymbol{h}^{(i)}$
end
$\boldsymbol{x}_f^{(i+1)} = \boldsymbol{x}_f^{(test)}$
$\boldsymbol{f}^{(i+1)} = \boldsymbol{P}(\boldsymbol{x}_f^{(i+1)}) - \boldsymbol{x}_c^*$ using (2)
$$\boldsymbol{B}^{(i+1)} = \boldsymbol{B}^{(i)} + \frac{\boldsymbol{f}^{(i+1)}\boldsymbol{h}^{(i)T}}{\boldsymbol{h}^{(i)T}\boldsymbol{h}^{(i)}}, \quad i = i+1$$
end

(1)
$$\boldsymbol{x}_c^* = \arg\min_{\boldsymbol{x}_c} U(\boldsymbol{R}_c(\boldsymbol{x}_c, \psi))$$

(2)
$$\boldsymbol{P}(\boldsymbol{x}_f) = \arg\min_{\boldsymbol{x}_c} \left\| \boldsymbol{e}_1^T \quad \dots \quad \boldsymbol{e}_p^T \right\|_2^2$$
$$\boldsymbol{e}_j(\boldsymbol{x}_f) = \boldsymbol{R}_{fs}(\boldsymbol{x}_f, \psi_j) - \boldsymbol{R}_{cs}(\boldsymbol{x}_c, \psi_j)$$

Algorithm for Constrained Broyden-Based i-SM

Begin
find $\boldsymbol{x}_c^*$ solving (1)
$i = 0$, $\boldsymbol{x}_f^{(i)} = \boldsymbol{x}_c^*$, $\boldsymbol{B}^{(i)} = \boldsymbol{I}$, $\delta = 0.3$
$\boldsymbol{f}^{(i)} = \boldsymbol{P}(\boldsymbol{x}_f^{(i)}) - \boldsymbol{x}_c^*$ using (2)
repeat until *StoppingCriteria*
solve $\boldsymbol{B}^{(i)}\boldsymbol{h}^{(i)} = -\boldsymbol{f}^{(i)}$ for $\boldsymbol{h}^{(i)}$
$\boldsymbol{x}_f^{(test)} = \boldsymbol{x}_f^{(i)} + \boldsymbol{h}^{(i)}$
while $\boldsymbol{x}_f^{(test)} \prec \boldsymbol{x}_f^{\min} \vee \boldsymbol{x}_f^{(test)} \succ \boldsymbol{x}_f^{\max}$
$\boldsymbol{h}^{(i)} = \delta\, \boldsymbol{h}^{(i)}$
$\boldsymbol{x}_f^{(test)} = \boldsymbol{x}_f^{(i)} + \boldsymbol{h}^{(i)}$
end
$\boldsymbol{x}_f^{(i+1)} = \boldsymbol{x}_f^{(test)}$
$\boldsymbol{f}^{(i+1)} = \boldsymbol{P}(\boldsymbol{x}_f^{(i+1)}) - \boldsymbol{x}_c^*$ using (2)
$$\boldsymbol{B}^{(i+1)} = \boldsymbol{B}^{(i)} + \frac{\boldsymbol{f}^{(i+1)}\boldsymbol{h}^{(i)T}}{\boldsymbol{h}^{(i)T}\boldsymbol{h}^{(i)}}, \quad i = i+1$$
end

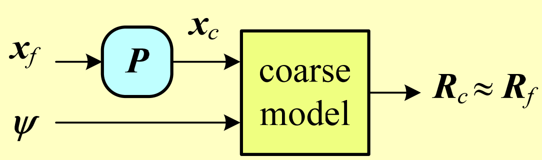
(1)
$$\boldsymbol{x}_c^* = \arg\min_{\boldsymbol{x}_c} U(\boldsymbol{R}_c(\boldsymbol{x}_c, \psi))$$

(2)
$$\boldsymbol{P}(\boldsymbol{x}_f) = \arg\min_{\boldsymbol{x}_c} \left\| \boldsymbol{e}_1^T \quad \dots \quad \boldsymbol{e}_p^T \right\|_2^2$$
$$\boldsymbol{e}_j(\boldsymbol{x}_f) = \boldsymbol{R}_{fs}(\boldsymbol{x}_f, \psi_j) - \boldsymbol{R}_{cs}(\boldsymbol{x}_c, \psi_j)$$

$$\boldsymbol{x}_f^{SM} = \boldsymbol{x}_f^{(i)}$$
$$\boldsymbol{P}(\boldsymbol{x}_f) = \boldsymbol{B}\boldsymbol{x}_f + \boldsymbol{c}$$
where $\boldsymbol{B} = \boldsymbol{B}^{(i)}$ and $\boldsymbol{c} = \boldsymbol{x}_c^* - \boldsymbol{B}\boldsymbol{x}_f^{SM}$

After Broyden-Based Input Space Mapping

☺ $\boldsymbol{R}_f(\boldsymbol{x}_f^{SM}, \psi) \approx \boldsymbol{R}_c(\boldsymbol{x}_c^*, \psi)$



$$\boldsymbol{R}_c(\boldsymbol{P}(\boldsymbol{x}_f), \psi) \approx \boldsymbol{R}_f(\boldsymbol{x}_f, \psi)$$

for all $\boldsymbol{x}_f$ around $\boldsymbol{x}_f^{SM}$

$\boldsymbol{P}(\boldsymbol{x}_f)$ is built iteratively as we approach $\boldsymbol{x}_f^{SM}$

Stopping Criteria for Broyden-Based SM

- A maximum number of iterations (fine model evaluations) has been reached

$$i > i_{\max}$$

- The extracted coarse model parameters are practically the same as the optimal coarse model solution

$$\| \boldsymbol{f}^{(i)} \|_\infty < \varepsilon_1$$

- The relative change in the fine model variables is small enough

$$\| \boldsymbol{x}_f^{(i+1)} - \boldsymbol{x}_f^{(i)} \|_2 < \varepsilon_2 (\| \boldsymbol{x}_f^{(i)} \|_2 + \varepsilon_2)$$

Stopping Criteria for Broyden-Based SM (cont)

- The fine model response is close enough to the optimal coarse model response (target)

$$\| \boldsymbol{R}_f(\boldsymbol{x}_f^{(i)}) - \boldsymbol{R}_c(\boldsymbol{x}_c^*) \|_\infty < \varepsilon_3 (\| \boldsymbol{R}_c(\boldsymbol{x}_c^*) \|_\infty + \varepsilon_3)$$

- To facilitate the notation,

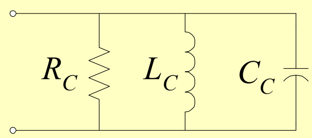
$$\varepsilon_1 \coloneqq \varepsilon_f$$
$$\varepsilon_2 \coloneqq \varepsilon_x$$
$$\varepsilon_3 \coloneqq \varepsilon_R$$

Examples

- Parallel resonant lumped circuit
- 10:1 Two-Section impedance transformer
- Microstrip Notch filter with mitered bends

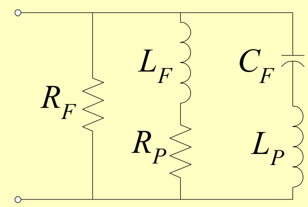
Parallel Resonant Lumped Circuit

"Coarse" model



$\boldsymbol{x}_c = [R_C\ (\Omega)\ \ L_C\ (\text{nH})\ \ C_C\ (\text{pF})]^T$

"Fine" model



$\boldsymbol{x}_f = [R_F\ (\Omega)\ \ L_F\ (\text{nH})\ \ C_F\ (\text{pF})]^T$

$R_P = 0.5\ \Omega$

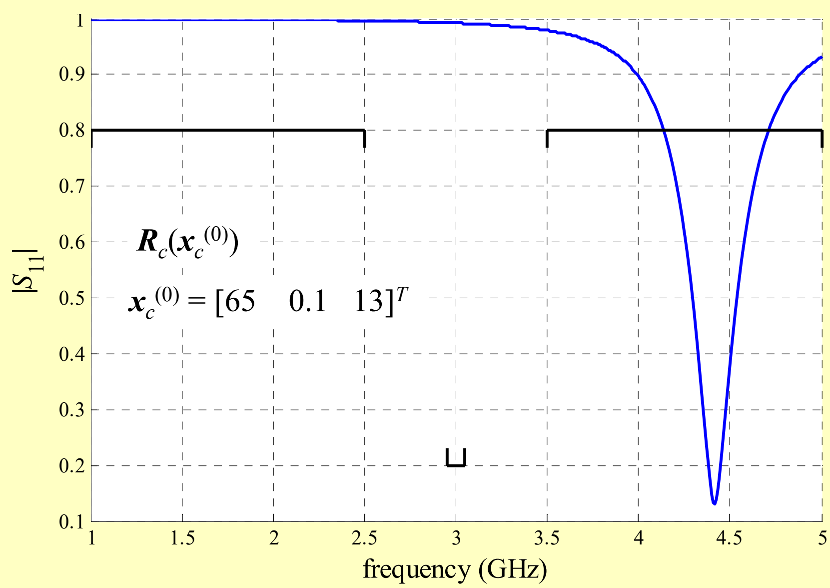
$L_P = 0.13$ nH

Specifications ($Z_o = 50\ \Omega$)

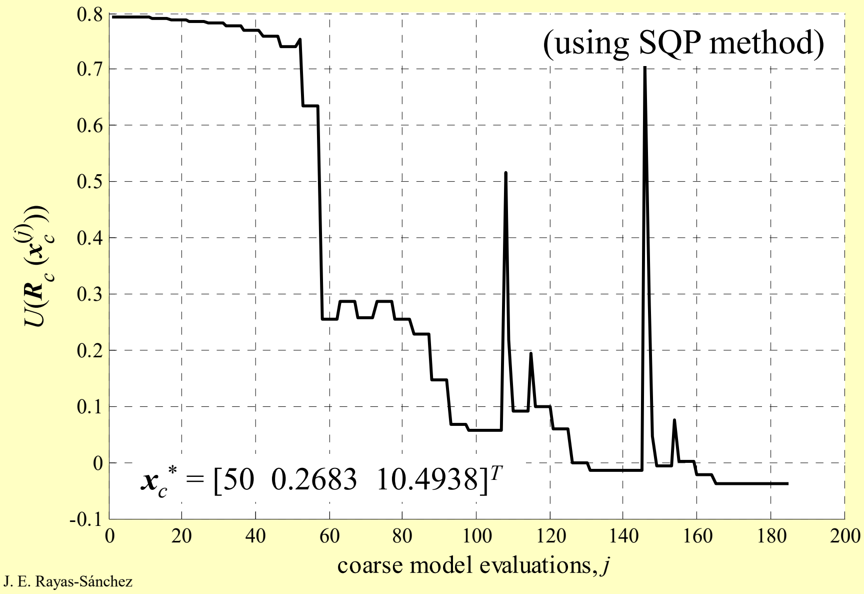
$|S_{11}| > 0.8$ from 1 GHz to 2.5 GHz and from 3.5 GHz to 5 GHz

$|S_{11}| < 0.2$ from 2.95 GHz to 3.05 GHz

Resonant Lumped Circuit – Coarse Opt.



Resonant Lumped Circuit – Coarse Opt. (cont)



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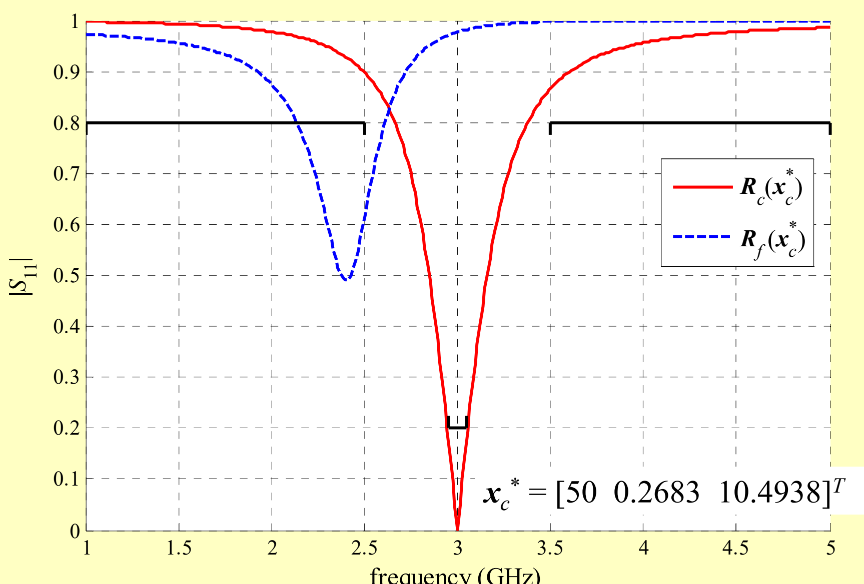
Resonant Lumped Circuit – SM Starting Point



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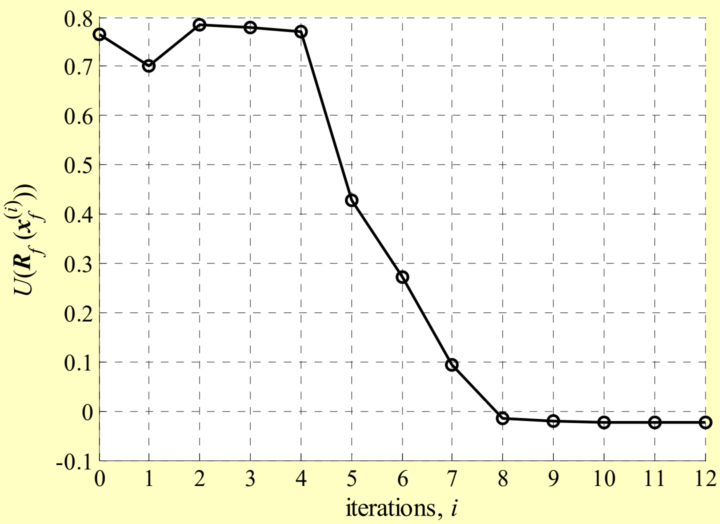
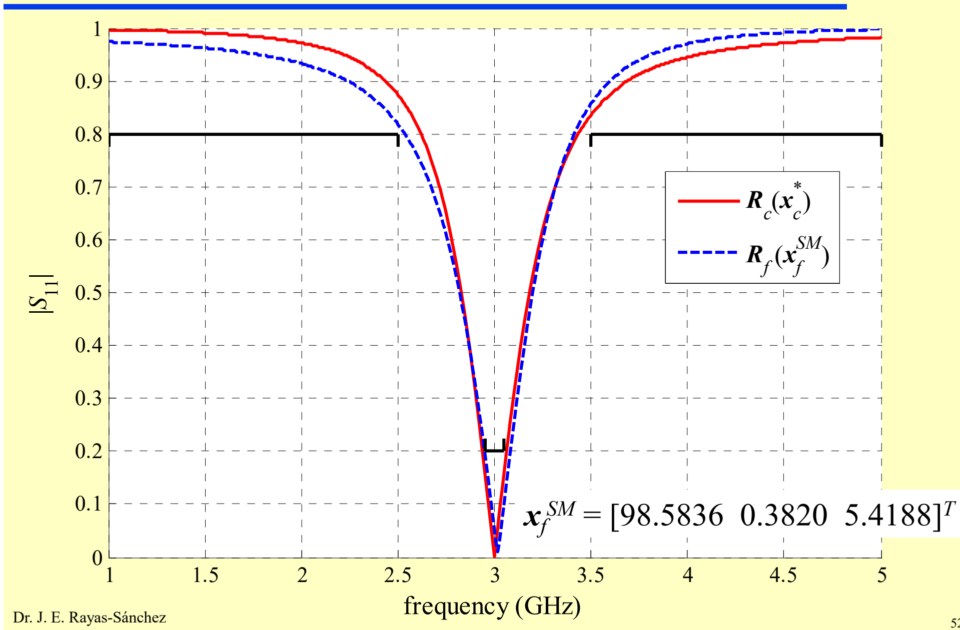
Resonant Lumped Circuit – SM Iterations



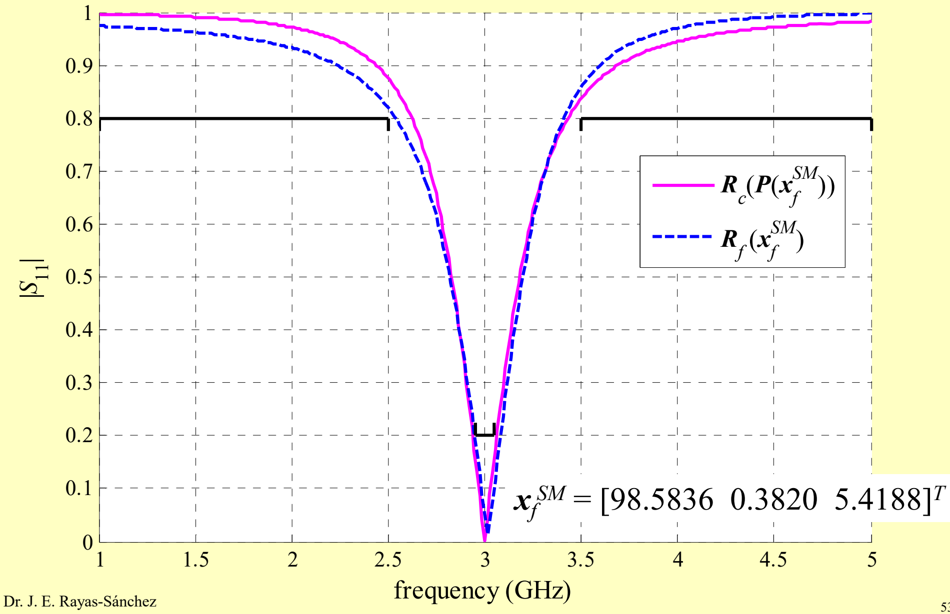
$\varepsilon_x = 1\times10^{-5}$
$\varepsilon_f = 1\times10^{-4}$
$\varepsilon_R = 1\times10^{-2}$

Active termination criterion: $\boldsymbol{f}(\boldsymbol{x}_f^{SM}) \approx \boldsymbol{0}$

Resonant Lumped Circuit – SM Solution



Resonant Lumped Circuit – SM Solution



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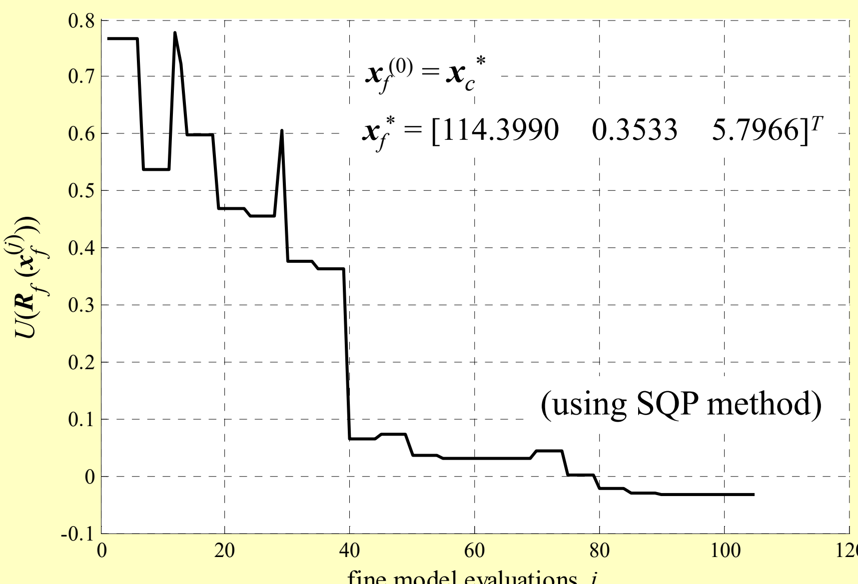
Resonant Lumped Circuit – Direct Optimization



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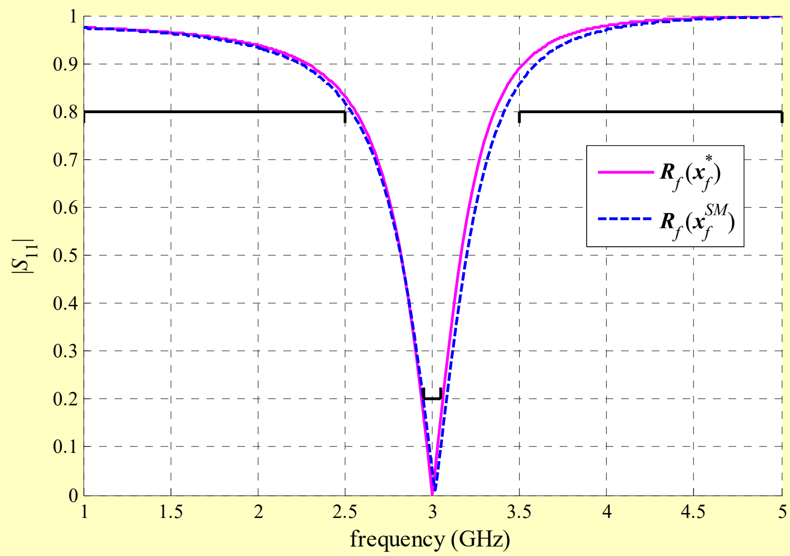
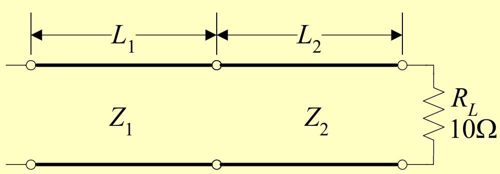
Resonant Lumped Circuit – $\boldsymbol{x}_f^*$ vs $\boldsymbol{x}_f^{SM}$



$\boldsymbol{x}_f^* = [114.3990 \;\; 0.3533 \;\; 5.7966]^T$ $\quad \boldsymbol{x}_f^{SM} = [98.5836 \;\; 0.3820 \;\; 5.4188]^T$

10:1 Two-Section Impedance Transformer

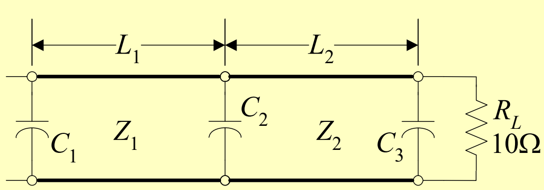
Coarse Model



$C_1 = C_2 = C_3 = 10\text{pF}$
$Z_1 = 2.23615\Omega$, $Z_2 = 4.47230\Omega$

Specs
$|S_{11}| \leq 0.5$ for
$0.5 \text{ GHz} \leq f \leq 1.5 \text{ GHz}$

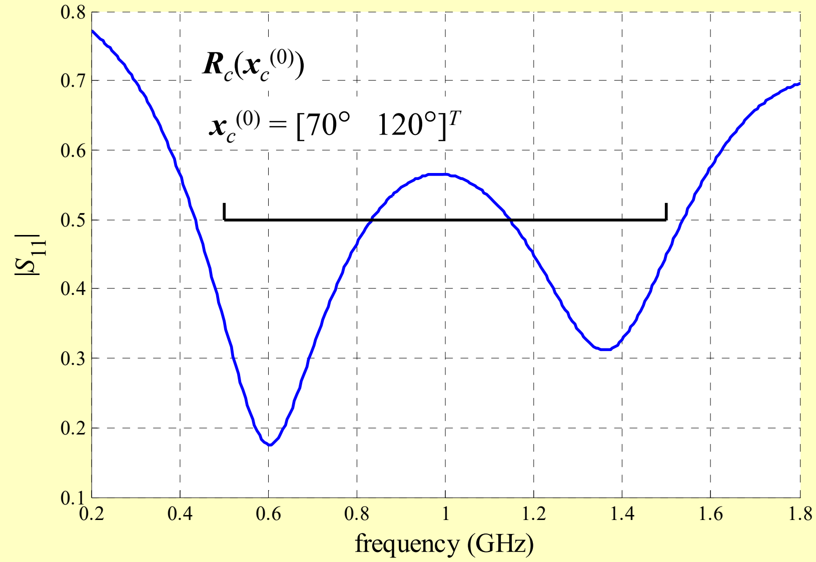
"Fine" Model



$\boldsymbol{x}_f = [L_1 \;\; L_2]^T$

(electrical lengths of the transmission lines at 1GHz, in degrees)

Impedance Transformer – Coarse Opt.



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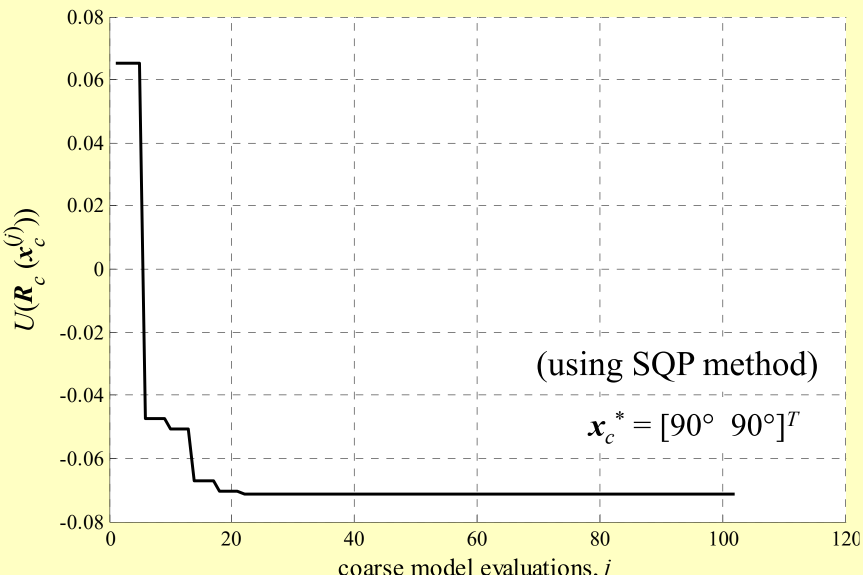
Impedance Transformer – Coarse Opt. (cont)



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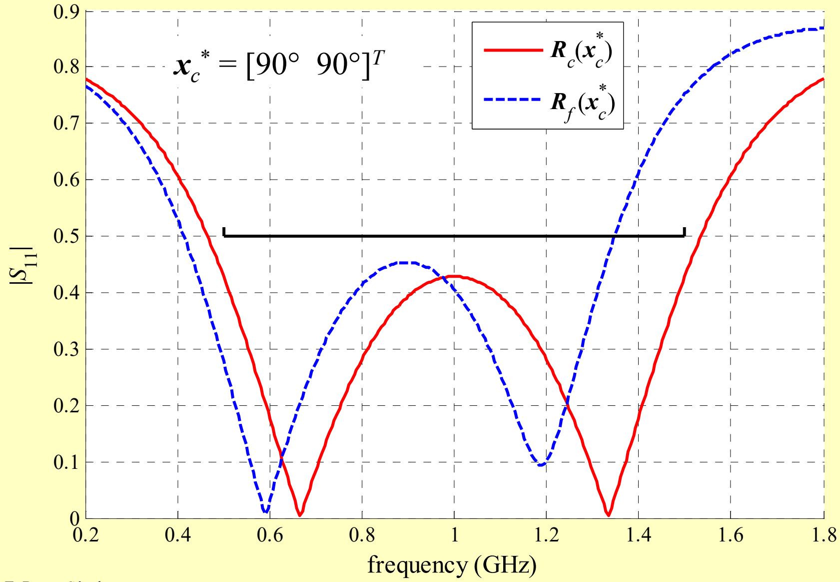
Impedance Transformer – Starting Point



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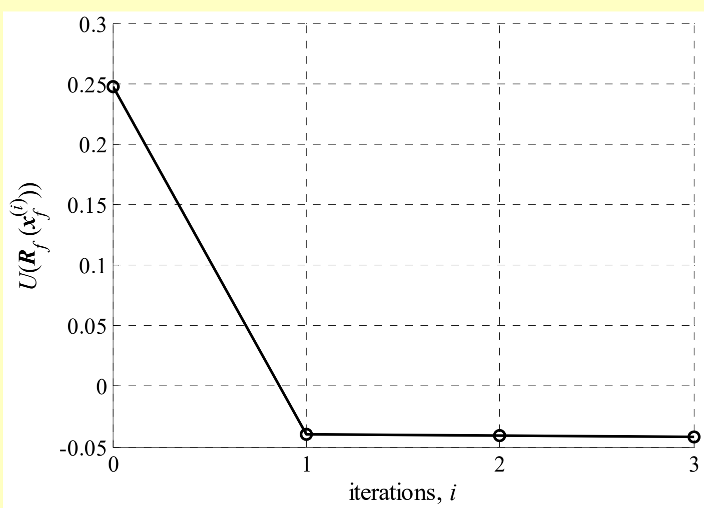
Impedance Transformer – SM Iterations



$\varepsilon_x = 1\times10^{-4}$
$\varepsilon_f = 1\times10^{-4}$
$\varepsilon_R = 1\times10^{-2}$

Active termination criterion: $x_f^{(i+1)} \approx x_f^{(i)}$

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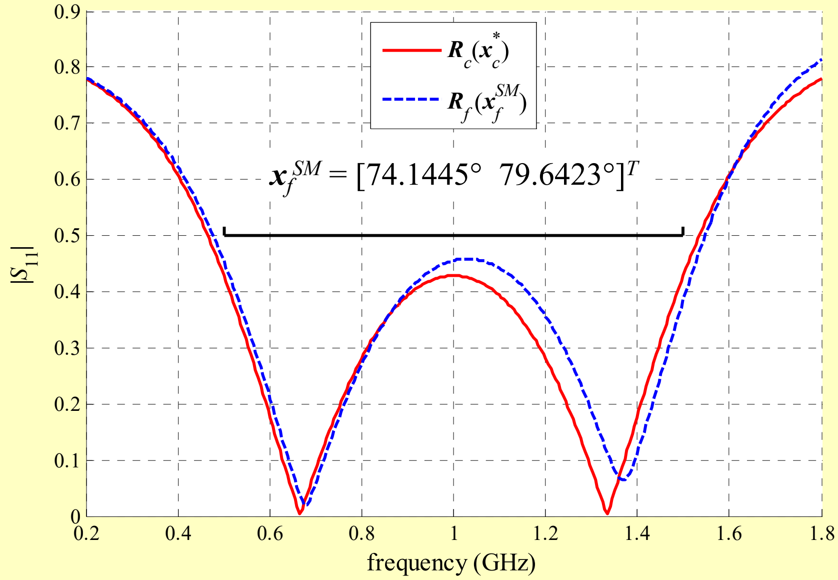
Impedance Transformer – SM Solution



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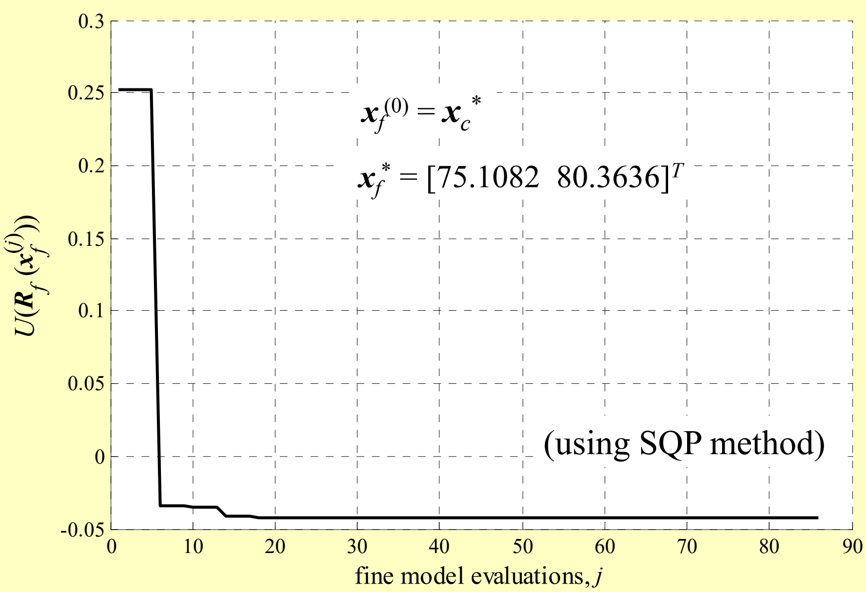
Impedance Transformer – Direct Optimization



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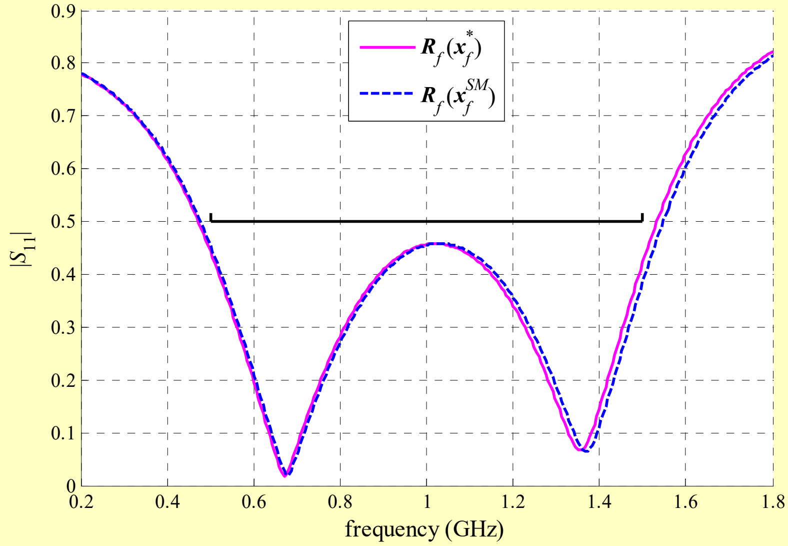
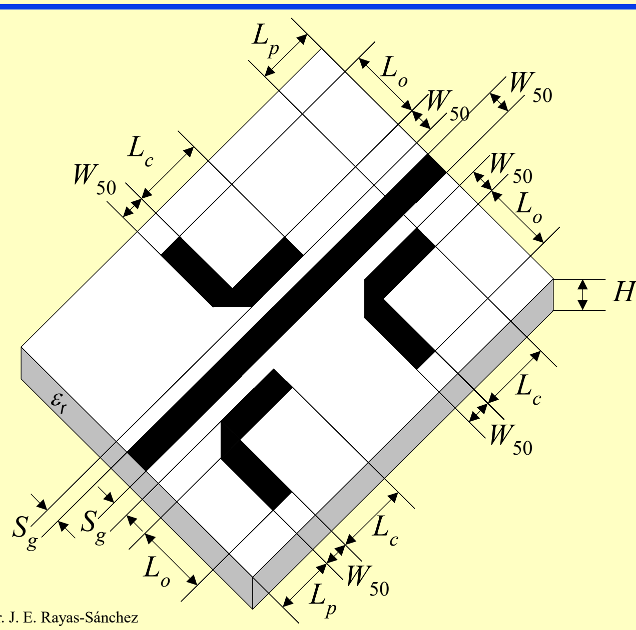
Impedance Transformer – $\boldsymbol{x}_f^*$ vs $\boldsymbol{x}_f^{SM}$



$\boldsymbol{x}_f^* = [75.1082° \;\; 80.3636°]^T$ $\quad$ $\boldsymbol{x}_f^{SM} = [74.1445° \;\; 79.6423°]^T$

Microstrip Notch Filter



$H = 10$mil
$W_{50} = 31$mil
$\varepsilon_r = 2.2$
loss tan = 0.0009
(RT Duroid 5880)

$\boldsymbol{x}_f = [L_c \;\; L_o \;\; S_g]^T$

Microstrip Notch Filter (cont)

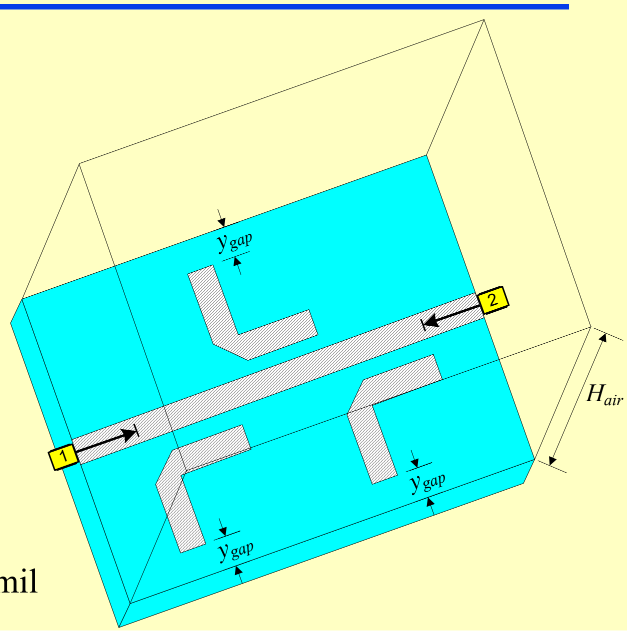
Specifications

$|S_{21}| \leq 0.05$ for 13.19GHz $\leq f \leq$ 13.21GHz

$|S_{21}| \geq 0.95$ for $f \leq$ 13GHz and $f \geq$ 13.4GHz

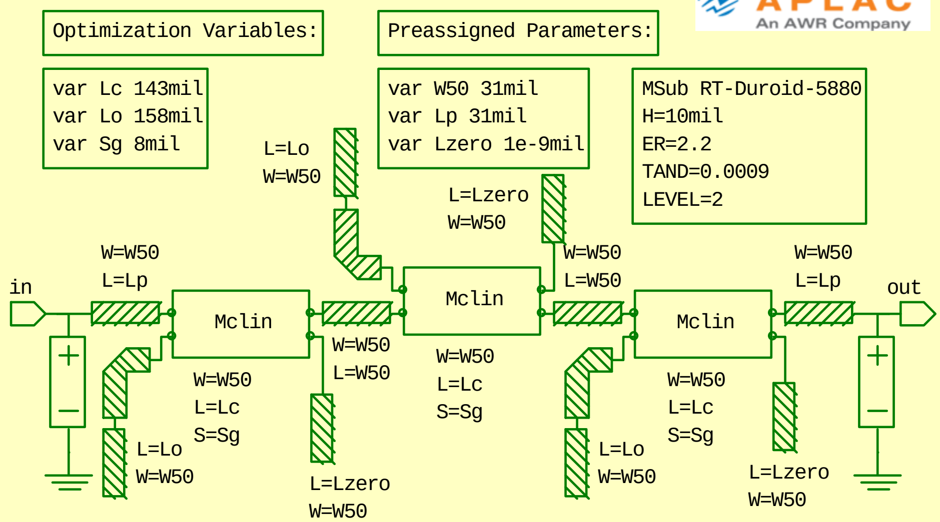
Microstrip Notch Filter – Fine Model

SONNET

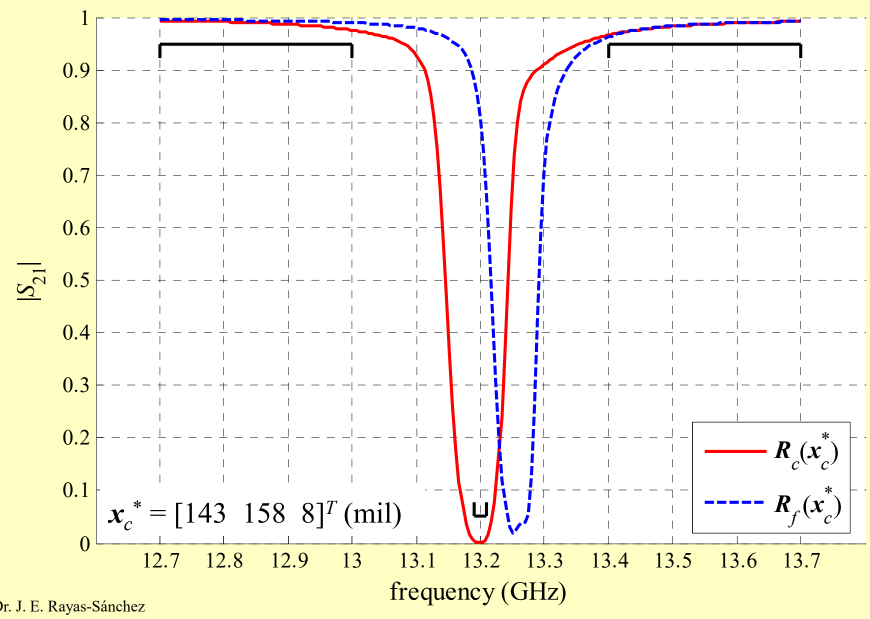


H_{air} = 60 mil
$L_p = ½(L_o+L_c)$
$Y_{gap} = L_o$
grid = 0.5mil×0.5mil

Microstrip Notch Filter – Coarse Model



Microstrip Notch Filter – Starting Point



Microstrip Notch Filter – SM Iterations

$\varepsilon_x = 1\times10^{-5}$
$\varepsilon_f = 1\times10^{-4}$
$\varepsilon_R = 1\times10^{-2}$

Active termination criterion: $\boldsymbol{f}(\boldsymbol{x}_f^{SM}) \approx \mathbf{0}$

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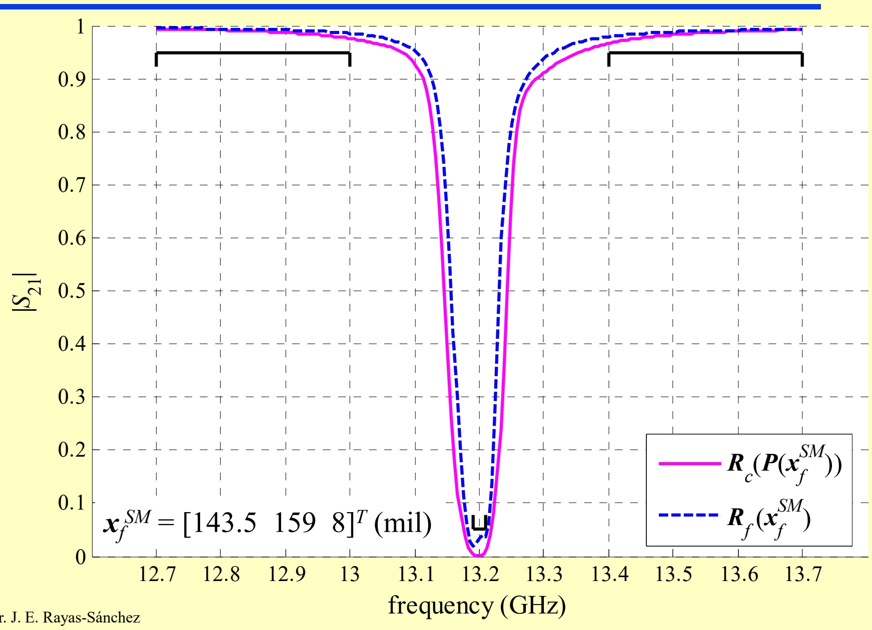
Microstrip Notch Filter – SM Solution



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