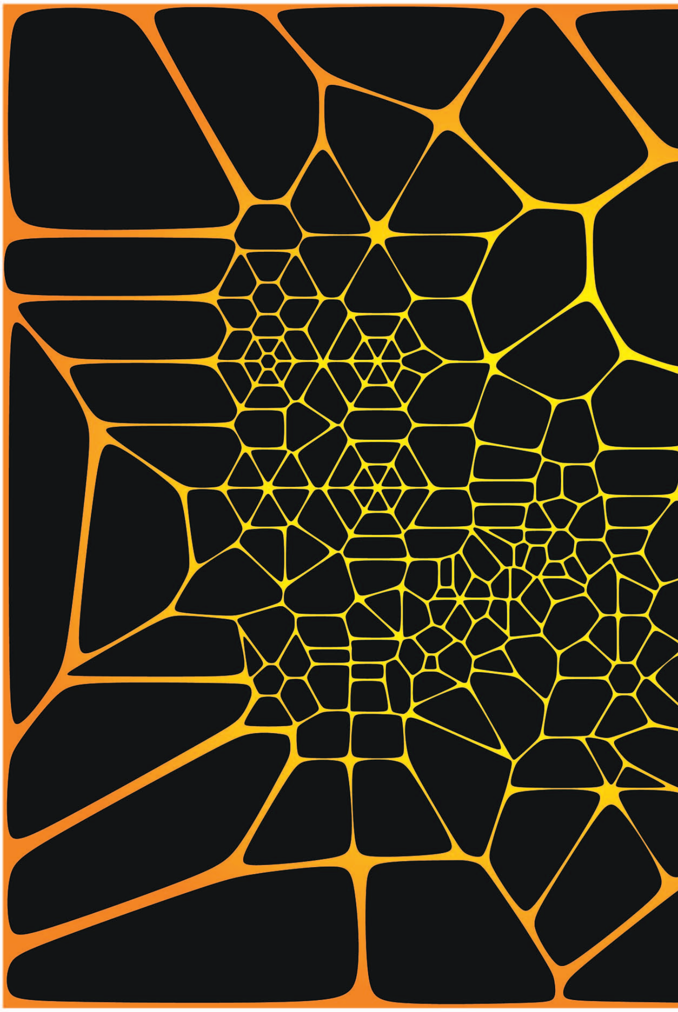
Power in Simplicity with ASM



José E. Rayas-Sánchez

The most widely used space mapping approach to efficient design optimization is the aggressive space mapping (ASM) algorithm. My purpose here is to present both a historical account and a technical reassessment of ASM, starting with the invention of the space mapping concept and continuing with a brief overview of the most fundamental space mapping optimization methods developed until now, within which ASM is framed. The article goes on to review over two decades of ASM evolution, in terms not only of the theoretical contributions directly incorporated into the ASM algorithm but also of the most significant engineering applications documented for ASM to date.

Clearly, ASM is neither the most powerful nor the most advanced space mapping design optimization approach invented up to now. However, the historical evidence proves that it is the most widely adopted space mapping optimization method, both in academia and industry. I believe that two main characteristics account for ASM's popularity: 1) it is simple, and 2) it is very efficient (when it works, it works extremely well). For these reasons, in this article I also revisit the ASM algorithm, emphasizing key steps for its successful implementation, as well as typical scenarios where ASM may fail. Finally, I venture some future directions regarding ASM.

José E. Rayas-Sánchez (erayas@iteso.mx) is with the Department of Electronics, Systems, and Informatics, ITESO—The Jesuit University of Guadalajara, Mexico.

Digital Object Identifier 10.1109/MMM.2015.2514188
Date of publication: 7 March 2016

1527-3342/16©2016IEEE

An Overview of Space Mapping-Based Optimization Methods

Prof. John Bandler invented the space mapping technique in 1994 [1]. A fine description about how it was originated—along with intriguing analogies to human cognition and a qualitative illustration of the multiple faces of space mapping—was presented by its inventor in 2013 [2]. Excellent technical reviews on general space mapping methods for modeling and design optimization are found in [3] and [4], from 2004 and 2008, respectively. The most recent specific review of space mapping-based optimization exploiting artificial neural networks appeared in 2004 [5]. This suggests that an up-to-date review of general space mapping technologies seems particularly pertinent now.

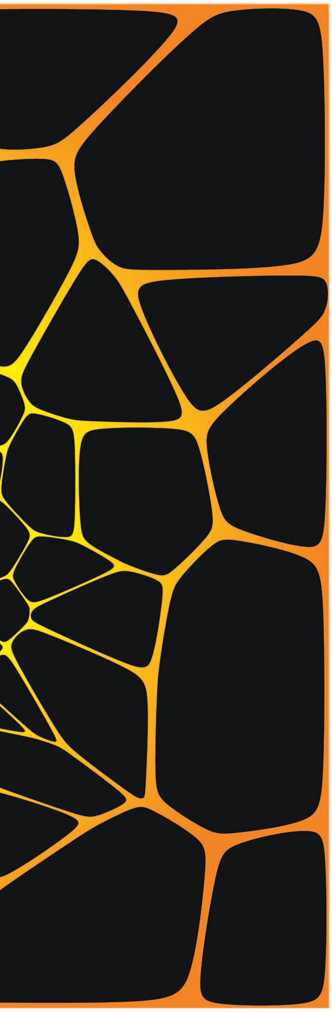


IMAGE LICENSED BY INGRAM PUBLISHING

In spite of that, my aim in this article is not to provide a general review of space mapping but rather to focus on the ASM approach to design optimization. Nevertheless, to place the ASM algorithm into its proper context, Figure 1 briefly illustrates, in a schematic manner, the most fundamental design optimization algorithms to have emerged from the space mapping concept.

Space mapping optimization methods belong to the general class of surrogate-based optimization algorithms [45]. They are specialized for the efficient optimization of computationally expensive objective functions.

ASM emerged in 1995, two decades ago. Since then, many other design optimization algorithms have been proposed, as seen in Figure 1. They aim at making space mapping optimization more general, more robust, and more efficient. Excepting, perhaps, implicit space mapping [16], most of them have a significantly higher complexity than ASM does, making them more difficult to exploit by nonoptimization experts. Any diligent (or even quick) search of IEEE *Xplore* and other recognized digital libraries can confirm that the number of applications using more sophisticated space mapping design optimization methods is significantly smaller than the number using ASM.

The Evolution of ASM Theory and Applications

The most significant theoretical contributions to ASM, as well as its main publicly documented applications in engineering fields, are highlighted in Figures 2 and 3, which also show both the fine and coarse models used for each application case (for a discussion of the fine and coarse models, see "The Beauty of ASM: Simplicity" section).

More detailed descriptions of these advances are summarized in Tables 1 and 2, in which corresponding references are also provided. A number of interesting observations about ASM can be inferred from these two tables.

- *The diversity of engineering disciplines in which ASM is applied.* ASM has been applied not only to electromagnetics-based design optimization of RF and microwave circuits, for which it was originally intended, but also to several other areas, including magnetic circuits, mechanical engineering, materials design, medical instrumentation, environmental sciences, and so forth.
- *The diversity of computer-aided design (CAD) tools employed.* Models of the optimized structures have been implemented using a variety of numerical simulators, including commercially available CAD tools and internal tools. Physical data obtained from direct measurements have also been incorporated as fine models.
- *The diversity of ASM contributors.* A very significant number of theoretical contributions and applications have been made from researchers well beyond the originating group at Ontario's McMaster University—especially for the second decade of ASM evolution.
- *A stable production of applications.* There has been a quite steady generation of engineering applications for ASM, spanning over two decades—with no signs of a proximate end to the development of new applications.

The Beauty of ASM: Simplicity

ASM efficiently finds an approximation for the optimal design of a computationally expensive model (termed a "fine" model) by exploiting a fast but inaccurate surrogate of the original fine model (termed a "coarse" model). It starts with a coarse model optimal design whose coarse-model response satisfies the design specifications and provides a target, or desired, response for the fine model. ASM aims at finding a solution that makes the fine-model

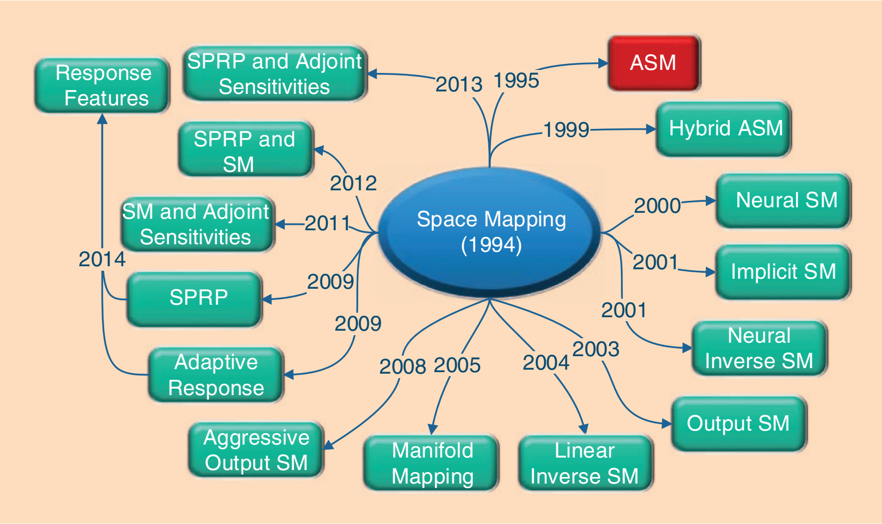


Figure 1. *Fundamental design optimization methods that have emerged from the space mapping (SM) concept: ASM [6], [7]; hybrid ASM [8], [9]; neural SM [10]–[12]; implicit SM [13]–[17]; neural inverse SM [18], [19]; output SM [20], [21]; linear inverse SM [22]–[24]; manifold mapping [25]–[28]; aggressive output SM [29], [30]; adaptive response correction (ARC) [31], [32]; shape-preserving response prediction (SPRP) [33], [34]; SM with adjoint sensitivities [35]–[37]; SPRP exploiting SM [38]; SPRP using adjoint sensitivities [39], [40]; and response features [41]–[44] (which emerged from ARC and SPRP).*

response close enough, from an engineering perspective, to the desired response. ASM is naturally rooted in engineering design practice.

Finding the actual fine-model optimal design $\boldsymbol{x}_f^*$ could be realized by minimizing, with respect to the fine-model design parameters $\boldsymbol{x}_f$, a suitable objective function U that encodes the design specifications in terms of the fine model response $\boldsymbol{R}_f$, by solving

$$\boldsymbol{x}_f^* = \arg\min_{\boldsymbol{x}_f} U(\boldsymbol{R}_f(\boldsymbol{x}_f)).$$

However, the above optimization problem is not feasible in most practical cases given the high computational cost implied by each evaluation of the fine-model response. ASM does not aim at finding $\boldsymbol{x}_f^*$. Instead, ASM aims at finding a solution $\boldsymbol{x}_f^{\mathrm{SM}}$, called the space-mapped solution, that makes the fine-model response close enough to the optimal coarse-model response, $\boldsymbol{R}_f(\boldsymbol{x}_f^{\mathrm{SM}}) \approx \boldsymbol{R}_c^*$.

ASM Flow Diagram

Figure 4 shows a flow diagram for the ASM algorithm. It starts by finding the optimal coarse-model design $\boldsymbol{x}_c^*$ that yields the target response, $\boldsymbol{R}_c(\boldsymbol{x}_c^*) = \boldsymbol{R}_c^*$. This is typically accomplished by directly optimizing the coarse model using classical optimization methods, as in the case of Figure 4. However, analytical procedures can also be applied to find $\boldsymbol{x}_c^*$ using classical engineering design methods on idealized (coarse) models, e.g., classical filter-synthesis procedures.

After obtaining $\boldsymbol{x}_c^*$, the initial guess of the Broyden matrix $\boldsymbol{B}$ is defined, and the fine-model design parameters $\boldsymbol{x}_f$ are initialized. The Broyden matrix $\boldsymbol{B}$ linearly approximates the relationship between both parameters spaces, $\boldsymbol{x}_c$ and $\boldsymbol{x}_f$, as explained in the sections that follow. Using the initial $\boldsymbol{x}_f$, the corresponding fine-model response $\boldsymbol{R}_f$ is calculated at that point. Next, the stopping criteria are tested. If they are fulfilled, the algorithm ends; otherwise, it continues by performing parameter extraction (PE), which consists of finding the coarse-model design that renders the coarse-model response as closely as possible to the current fine-model response.

The difference $\boldsymbol{f}$ between the extracted parameters and the optimal coarse-model design is then calculated. Next, a linear system is solved to calculate the step $\boldsymbol{h}$. The Broyden matrix is updated, and the next iterate is calculated, at which a new fine-model evaluation is realized and the algorithm proceeds.

Initializing the Broyden Matrix and the Fine-Model Design Parameters

When the design parameters in both models, $\boldsymbol{x}_c$ and $\boldsymbol{x}_f$, have the same nature (for instance, both contain the same geometrical dimensions), $\boldsymbol{B}$ should be initialized by the identity matrix, based on the reasonable and implicit assumption that the coarse model does not deviate too much from the fine model. However, if $\boldsymbol{x}_c$ and $\boldsymbol{x}_f$ have different natures (for instance, $\boldsymbol{x}_c$ contains lumped-circuit element values while $\boldsymbol{x}_f$ contains geometrical dimensions), then $\boldsymbol{B}$ can be initialized by estimating the Jacobian of $\boldsymbol{x}_c$ with respect to $\boldsymbol{x}_f$ by finite differences [62], [79].

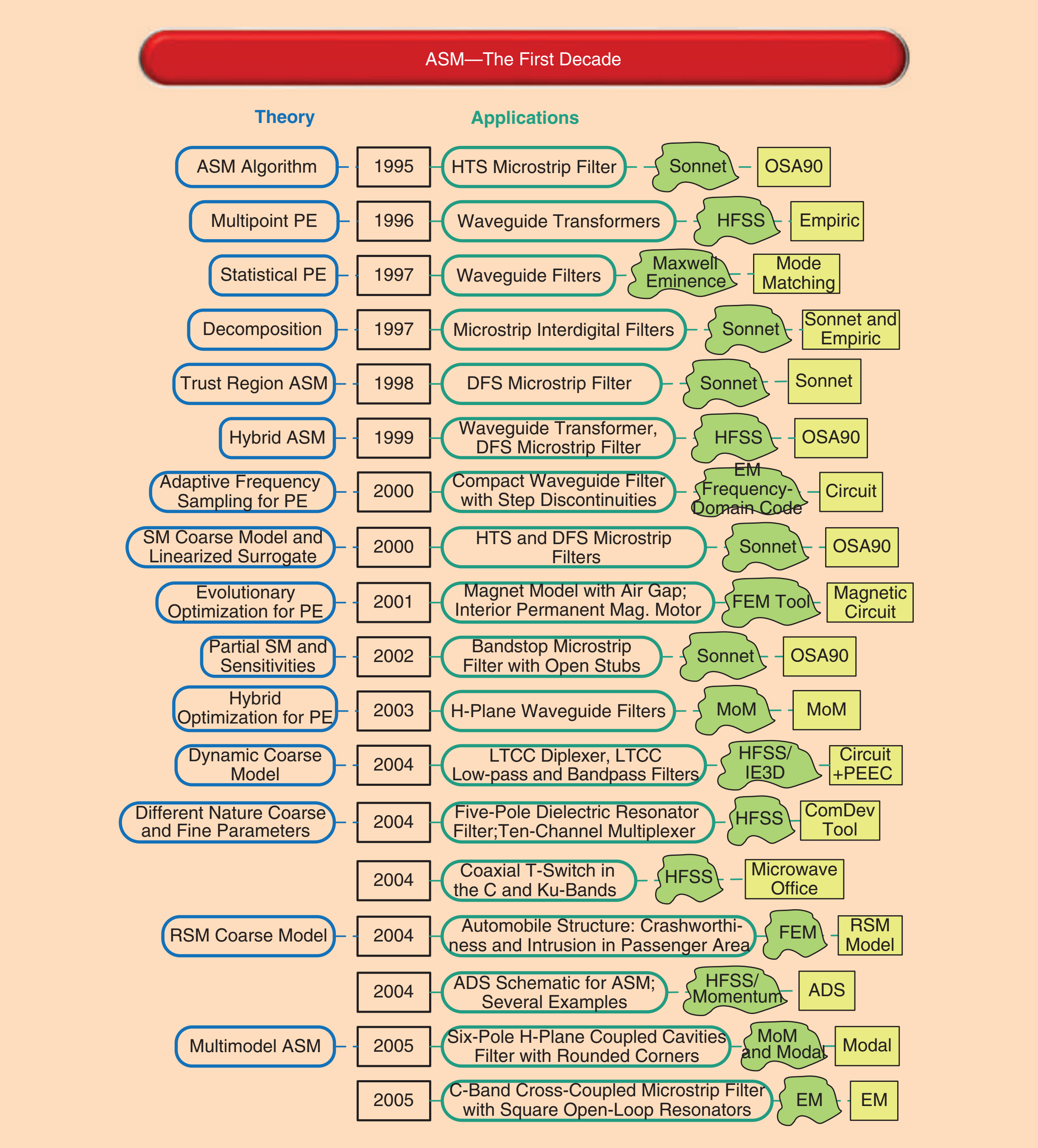
Similarly, if both $\boldsymbol{x}_c$ and $\boldsymbol{x}_f$ have the same nature, the fine-model design parameters are initialized with the optimal coarse-model solution, $\boldsymbol{x}_f = \boldsymbol{x}_c^*$; otherwise, they are initialized as $\boldsymbol{x}_f = \boldsymbol{B}^{-1}\boldsymbol{x}_c^*$.

Parameter Extraction

The PE process is the weakest part of ASM. It is usually formulated as an optimization subproblem that aims at minimizing the differences between coarse- and fine-model responses at the i-th iteration (local alignment) by solving

$$\boldsymbol{x}_c^{(i)} = \arg\min_{\boldsymbol{x}_c} \left\| \boldsymbol{R}_f\left(\boldsymbol{x}_f^{(i)}\right) - \boldsymbol{R}_c(\boldsymbol{x}_c) \right\|.$$

This optimization subproblem may present multiple local minima, some of them yielding a good match



ASM—The First Decade

Theory		Applications		
ASM Algorithm	1995	HTS Microstrip Filter	Sonnet	OSA90
Multipoint PE	1996	Waveguide Transformers	HFSS	Empiric
Statistical PE	1997	Waveguide Filters	Maxwell Eminence	Mode Matching
Decomposition	1997	Microstrip Interdigital Filters	Sonnet	Sonnet and Empiric
Trust Region ASM	1998	DFS Microstrip Filter	Sonnet	Sonnet
Hybrid ASM	1999	Waveguide Transformer, DFS Microstrip Filter	HFSS	OSA90
Adaptive Frequency Sampling for PE	2000	Compact Waveguide Filter with Step Discontinuities	EM Frequency-Domain Code	Circuit
SM Coarse Model and Linearized Surrogate	2000	HTS and DFS Microstrip Filters	Sonnet	OSA90
Evolutionary Optimization for PE	2001	Magnet Model with Air Gap; Interior Permanent Mag. Motor	FEM Tool	Magnetic Circuit
Partial SM and Sensitivities	2002	Bandstop Microstrip Filter with Open Stubs	Sonnet	OSA90
Hybrid Optimization for PE	2003	H-Plane Waveguide Filters	MoM	MoM
Dynamic Coarse Model	2004	LTCC Diplexer, LTCC Low-pass and Bandpass Filters	HFSS/ IE3D	Circuit +PEEC
Different Nature Coarse and Fine Parameters	2004	Five-Pole Dielectric Resonator Filter;Ten-Channel Multiplexer	HFSS	ComDev Tool
	2004	Coaxial T-Switch in the C and Ku-Bands	HFSS	Microwave Office
RSM Coarse Model	2004	Automobile Structure: Crashworthiness and Intrusion in Passenger Area	FEM	RSM Model
	2004	ADS Schematic for ASM; Several Examples	HFSS/ Momentum	ADS
Multimodel ASM	2005	Six-Pole H-Plane Coupled Cavities Filter with Rounded Corners	MoM and Modal	Modal
	2005	C-Band Cross-Coupled Microstrip Filter with Square Open-Loop Resonators	EM	EM

Figure 2. *The first decade of ASM evolution. Shown are the key theoretical elements that have contributed to developing ASM and the main applications for design optimization that use ASM, indicating both the fine and coarse models employed. PE: parameter extraction; HTS: high-temperature superconductor; DFS: double-folded stub; HFSS: high frequency structural simulator; EM: electromagnetic; FEM: finite-element method; MoM: method of moments; PEEC: partial element equivalent circuit; RSM: response-surface model; ADS: Agilent Advanced Design System.*

(several coarse-model designs that are able to approximate, with acceptable accuracy, the current fine-model response). Non-uniqueness of the PE solution may lead to oscillations or even divergence in the ASM algorithm [48], [49]. Several successful strategies have been proposed to overcome this difficulty [3].

When the coarse model consists of an equivalent circuit model, some physics-based analytical approximation, or some metamodel (response-surface model, polynomial model, neural network model, etc.), then solving the PE optimization subproblem is computationally very inexpensive. However, if the coarse model consists of a coarsely discretized full-wave electromagnetic model, its computational cost becomes non-negligible and may exhibit numerical noise and discontinuous behavior [98].

Another interesting approach to performing PE consists of completely avoiding the previously discussed optimization subproblem (and its inherent

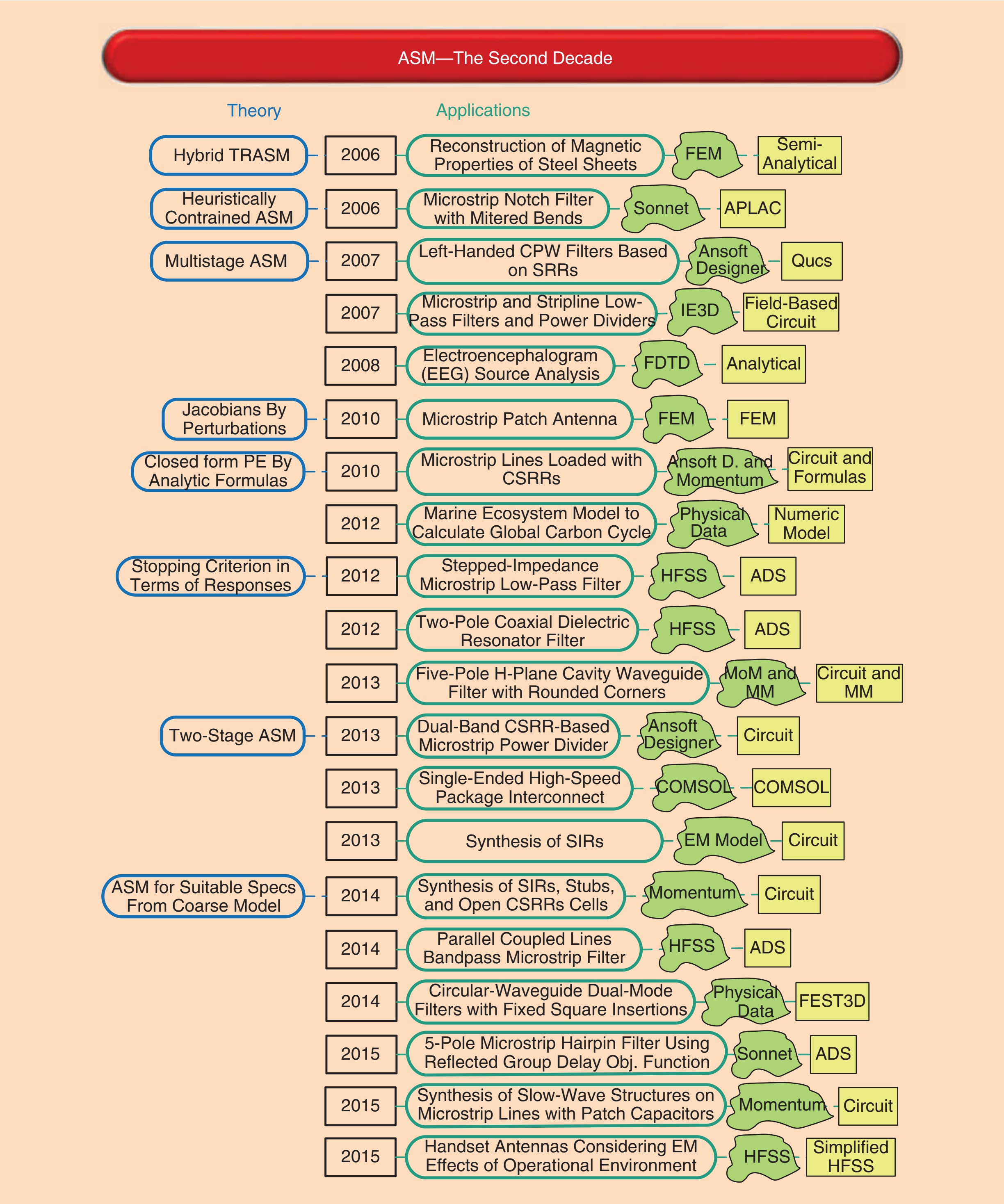


Figure 3. *The second decade of ASM evolution. TRASM: trust-region ASM; SRR: split-ring resonator; CSRR: complementary SRR; SIR; stepped-impedance resonator.*

difficulties) by following a synthesis approach, i.e., by finding in closed form the coarse-model (physics-based) parameter values that synthesize the current fine-model response, as in [79], [90].

The Root of ASM: Finding Roots

The PE problem described in the previous section can be interpreted as a multidimensional vector function $\boldsymbol{P}$ representing the mapping between both design parameter spaces, $\boldsymbol{x}_c^{(i)} = \boldsymbol{P}\left(\boldsymbol{x}_f^{(i)}\right)$. If the current extracted parameters $\boldsymbol{x}_c^{(i)}$ correspond approximately to $\boldsymbol{x}_c^*$, then the current fine-model response approximates the desired response, $\boldsymbol{R}_f\left(\boldsymbol{x}_f^{(i)}\right) \approx \boldsymbol{R}_c^*$. From here, we can see that the ASM algorithm (see Figure 4) iteratively finds a solution to the following system of nonlinear equations,

TABLE 1. A historical account of ASM: The first decade.

Year	Reference	Theoretical Contributions to ASM	ASM Applications	Fine Model	Coarse Model
1995	[6], [7]	Invention of ASM algorithm. Huber norm for parameter extraction (PE). Linear frequency mapping with exact penalty functions for severe model misalignment.	High temperature superconductive (HTS) microstrip filter	Sonnet	Equivalent circuit in OSA90
1996	[46]	Multipoint PE to increase uniqueness of PE solution and to improve ASM convergence.	Waveguide transformers	HFSS	Empirical model
1997	[47], [48]	Statistical approach to PE involving penalty concepts for PE uniqueness and consistency.	H-plane resonator waveguide filters with rounded corners	Maxwell Eminence (Ansoft)	Mode-matching/ equivalent circuit
1997	[49]	Structural decomposition to build accurate coarse model combining EM models with a coarse grid and empirical models for noncritical substructures.	Microstrip interdigital filter	Sonnet	Sonnet with coarse grid/ equivalent circuit
1998	[50], [51]	Trust-region ASM with multipoint PE. Nonconventional quasi-Newton step with an empirical parameter to ensure next candidate is within trust region.	Microstrip double-folded stub filter	Sonnet	Sonnet with coarse grid
1999	[8], [9]	Trust-region ASM combined with direct optimization. Lemma to calculate the fine model response Jacobian as a function of the coarse model response Jacobian and the Broyden matrix.	Waveguide transformer; microstrip double-folded stub filter	High-frequency structural simulator (HFSS)	Equivalent circuit in OSA90
2000	[52], [53]	New surrogate: combination of a mapped coarse model with a linearized fine model. Next iterate accepted if it improves objective function; otherwise, surrogate is enhanced by additional fine data.	HTS filter; two-section impedance transformer; double-folded stub filter	Sonnet	Equivalent circuit in OSA90
2000	[54]	Sampling algorithm to minimize the number and automate the selection of fine model frequency points for ASM.	Low-pass compact rectangular waveguide filter with capacitive step discontinuities	EM frequency-domain code	Equivalent circuit
2001	[55]	Evolutionary optimization method to perform PE (to extract global optimum at each ASM iteration).	Magnet model with air gap; interior permanent magnet motor	Finite-element method (FEM) tool	Magnetic equivalent circuit
2002	[56], [57]	Partial space mapping exploiting fine model exact sensitivities in PE and mapping update.	Bandstop microstrip filter with open stubs; two-section impedance transformer	Sonnet	Equivalent circuit in OSA90
2003	[58], [59]	Geometrical segmentation to decrease design variables. Combination of optimization methods for coarse model optimization and for PE.	Tunable H-plane waveguide filters with tuning posts operating at 11 and 13 GHz.	Method of moments (MoM)	MoM with small number of modes
2004	[60], [61]	Dynamic coarse model: a combination of an evolutionary equivalent-circuit model and quasi-static EM partial-element equivalent circuit (PEEC) model (highly accurate coarse model).	Low temperature co-fired ceramic (LTCC) frequency-selective passive modules: LTCC diplexer; low-pass and bandpass LTCC filters.	HFSS and IE3D	Equivalent-circuit and PEEC model

(continued)

TABLE 1. A historical account of ASM: The first decade. *(continued)*

Year	Reference	Theoretical Contributions to ASM	ASM Applications	Fine Model	Coarse Model
2004	[62]	Coarse and fine design parameters of different nature (coupling coefficients versus geometrical dimensions). Broyden matrix initialized by finite differences.	Double-terminated five-pole dielectric resonator filter; ten-channel manifold-coupled output multiplexer	Ansoft HFSS	ComDev internal circuit analysis tool
2004	[63]	–	Four port electromechanical coaxial T-switch in the C- and Ku-bands	HFSS	Microwave Office
2004	[64]	Response-surface methodologies (RSMs) to develop coarse models for ASM design optimization.	Automobile structure optimized (crashworthiness and intrusion in the passenger compartment)	Large industrial FEM model	RSM model
2004	[21], [65]	–	ADS schematic for ASM: microstrip transformer; H-plane waveguide filter; interdigital microstrip filter	HFSS and Momentum	Equivalent circuit in ADS
2005	[66]	ASM with multiple models of increasing accuracy: coarse model is the fastest, while the finest model is used in the last iteration (gradual mapping to avoid divergence).	Six-pole H-plane coupled cavities filter with rounded corners in the coupling windows due to die casting fabrication	MoM combined with modal techniques	Modal techniques
2005	[67]	–	C-band cross-coupled bandpass microstrip filter using square open-loop resonators.	full-wave EM model	Coarsely discretized EM model

TABLE 2. A historical account of ASM: The second decade.

Year	Reference	Theoretical Contributions to ASM	ASM Applications	Fine Model	Coarse Model
2006	[68]	Trust-region ASM combined with direct optimization; similar to [8], [9].	Reconstruction of magnetic proper-ties of steel sheets by needle probe	FEM model	Semi-analytical model
2006	[69], [70]	Heuristically constrained ASM: if next candidate falls outside predefined limits, step size is decreased in same quasi-Newton direction (using empirical shrinking factor).	Microstrip notch filter with mitered bends.	Sonnet	Equivalent circuit in APLAC
2007	[71]	Multistage ASM to address manufacturing limitations in metamaterial structure. Heuristic constraints used as in [70].	Left-handed coplanar waveguide filters based on split-ring resonators (SRRs)	EM simulator Ansoft Designer	Equivalent circuit in Qucs
2007	[72]–[74]	–	Microstrip and stripline low-pass filters; microstrip and stripline power dividers	IE3D	Field-based equivalent circuit
2008	[75]	–	Localization of electric current sources within the brain from electroencephalograms	Finite-difference time-domain head model	Analytical head model

(continued)

TABLE 2. A historical account of ASM: The second decade. ***(continued)***

Year	Reference	Theoretical Contributions to ASM	ASM Applications	Fine Model	Coarse Model
2010	[76]	Approximation of the Jacobian matrix by perturbations, combined with Broyden update.	Microstrip patch antenna	Fine-mesh FEM model	Coarse-mesh FEM model
2010	[77]–[80]	PE in closed form using analytical design formulas. Coarse and fine design parameters of different nature (lumped elements versus geometrical dimensions).	Resonant-type metamaterial transmission lines: microstrip lines loaded with complementary SSRs (CSRRs)	Ansoft Designer, Agilent Momentum	Lumped circuit and analytical formulas
2012	[81]	—	Marine ecosystem model to calculate global carbon cycle (oceanic CO_2 uptake)	Physical data	Coarse time discretization numerical model
2012	[82]	Additional stopping criterion for ASM: error between target response and fine model response (already used in [12]).	Stepped-impedance microstrip low-pass filter	Ansoft HFSS	Equivalent circuit in ADS
2012	[83]	—	Two-pole coaxial dielectric resonator filter	Ansoft HFSS	Equivalent circuit in ADS
2013	[84]	—	Five-pole H-plane direct-coupled-cavity waveguide bandpass filter with rounded corners for space applications in the C-band	Method of Moments & Mode Matching	Equivalent circuit & Mode Matching
2013	[85], [86]	Two-stage ASM: 1) pre-optimization to determine a convergence region for implementable equivalent circuits; and 2) conventional ASM.	Stopband microstrip filters by cascading CSRR-loaded line unit cells; dual-band CSRR-based power divider	EM simulator Ansoft Designer	Equivalent circuit
2013	[87]	—	Fabricated prototype of a single-ended high-speed package interconnect	COMSOL	Simplified and coarsely meshed COMSOL
2013	[88]	—	Synthesis of stepped impedance resonators (SIRs); fabricated third-order elliptic microstrip low-pass filter	Full-wave EM model	Equivalent circuit
2014	[89]–[91]	Two-stage ASM: 1) pre-optimization to determine suitable design specs from lumped circuit; 2) conventional ASM. PE in closed form using design formulas in terms of characteristic responses.	Synthesis of SIRs, shunt stubs, and open CSRRs in individual cells; wide-band bandpass filters by cascading individual cells (negligible EM interaction)	Agilent Momentum	Equivalent circuit
2014	[92]	—	Parallel coupled lines bandpass microstrip filter	HFSS	Equivalent circuit in ADS
2014	[93]–[94]	—	Circular-waveguide dual-mode filters with fixed square insertions (avoiding tuning screws)	physical data (VNA)	FEST3D
2015	[95]	—	Five-pole microstrip hairpin filter using a reflected group delay objective function	Sonnet	Equivalent circuit in ADS

(continued)

TABLE 2. A historical account of ASM: The second decade. ***(continued)***

Year	Reference	Theoretical Contributions to ASM	ASM Applications	Fine Model	Coarse Model
2015	[96]	—	Synthesis of slow-wave structures based on microstrip lines with patch capacitors	Agilent Momentum	Equivalent circuit
2015	[97]	—	Handset antennas considering EM effects of mobile phone components and human head	HFSS	Simplified HFSS-ignoring environment

$$\boldsymbol{f}(\boldsymbol{x}_f) = \boldsymbol{P}(\boldsymbol{x}_f) - \boldsymbol{x}_c^*,$$

because any root $\boldsymbol{x}_f^{\mathrm{SM}}$ of this system of equations $\boldsymbol{f}(\boldsymbol{x}_f)$ implies that $\boldsymbol{R}_f(\boldsymbol{x}_f^{\mathrm{SM}}) \cong \boldsymbol{R}_c^*$ [either $\boldsymbol{R}_f(\boldsymbol{x}_f^{\mathrm{SM}}) = \boldsymbol{R}_c^*$ or $\boldsymbol{R}_f(\boldsymbol{x}_f^{\mathrm{SM}}) \approx \boldsymbol{R}_c^*$, the latter case resulting from a possible residual in matching the responses during PE].

Therefore, ASM is essentially equivalent to the classical Broyden method for solving systems of nonlinear equations [99], also known as the *method of secants.* ASM makes a linear approximation of $\boldsymbol{f}(\boldsymbol{x}_f)$ at each iteration. It iteratively approximates the Jacobian of the mapping function $\boldsymbol{P}$ by matrix $\boldsymbol{B}$ using the Broyden rank-one updating formula (see Figure 4), where each evaluation of the system $\boldsymbol{f}(\boldsymbol{x}_f)$ implies at least one fine-model simulation and the next iterate is predicted from a quasi-Newton step. It has been shown [100] that ASM is not expected to yield the exact fine-model optimum $\boldsymbol{x}_f^*$, but rather a space-mapped solution $\boldsymbol{x}_f^{\mathrm{SM}}$, the accuracy of which is usually sufficient from a practical engineering perspective.

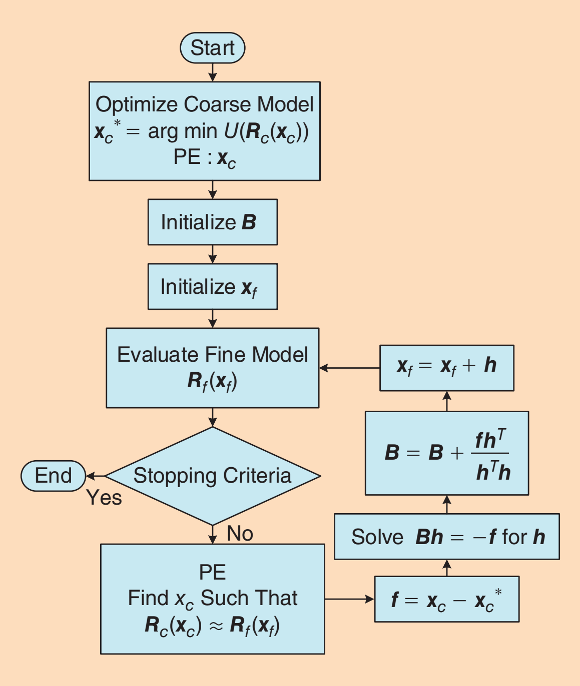


Figure 4. *Flow diagram of the ASM algorithm [6], [7].*

A typical evolution of ASM from the perspective of the system of nonlinear equations associated to the mapping function is illustrated in Figure 5, where the Broyden matrix is initialized with the identity (for purposes of simplicity, a one-dimensional design optimization problem is considered). In this illustration, it is assumed that the initial design is very bad (or a very deviated coarse model), implying a very large value of $\left\| f\left(x_f^{(0)}\right)\right\|$. In spite of that, ASM converges very quickly to a space-mapped solution x_f^{SM}.

The plots in Figure 5 also provide some insight regarding the famous efficiency of ASM, by which many highly complex problems are frequently solved in just a few fine-model evaluations, regardless of the number of optimization variables—even in cases were the initial fine-model response $\boldsymbol{R}_f(\boldsymbol{x}_c^*)$ is very much deviated from the target response $\boldsymbol{R}_c^*$. As can be seen in Figure 5, the efficiency of ASM depends on the degree of nonlinearity of $\boldsymbol{f}(\boldsymbol{x}_f)$, which, in turns, depends on the degree of nonlinearity of the mapping $\boldsymbol{P}$ between both model parameter spaces. If the mapping is relatively linear (perhaps with a large offset), ASM solves the design problem in a few iterations, regardless of the problem's dimensionality and even when the initial fine-model response is significantly deviated from the target, as in [97].

It is clear then, assuming the PE process is correctly implemented, that ASM can face the four scenarios depicted in Figure 6 (again, to simplify matters, a one-dimensional design problem is assumed):

1) *A unique and exact solution exists for the space mapping problem.* ASM finds a fine-model design whose response matches, either exactly or approximately, the desired response. This scenario may occur in practice, usually when the desired response is only approximated by a fine-model response at a unique space-mapped solution.
2) *Several exact solutions exist.* This is a theoretically possible, but infrequent scenario in practice. It implies that several fine-model designs are able to match the optimal coarse-model response. ASM would find only one of those space-mapped solutions, depending on the starting point.
3) *An acceptable solution exists.* This is the most common scenario found in practice for successful ASM design optimizations. Of course, a closely

related scenario can happen when several acceptable solutions exist (though this is less likely to happen).

4) *There is no acceptable space-mapped solution*. In this scenario, ASM fails. It occurs when the coarse model is too inaccurate with respect to the fine model.

At the end of a successful ASM algorithm, we have not only a fine-model design whose corresponding response approximates the desired response, $\boldsymbol{R}_f(\boldsymbol{x}_f^{\mathrm{SM}}) \approx \boldsymbol{R}_c^*$, but also a fast linear input-mapped coarse model that makes a good approximation of the fine model around the space-mapped solution, $\boldsymbol{R}_c(\boldsymbol{P}(\boldsymbol{x}_f)) \approx \boldsymbol{R}_f(\boldsymbol{x}_f)$ for $\boldsymbol{x}_f$ around $\boldsymbol{x}_f^{\mathrm{SM}}$. This final linear mapping is given by $\boldsymbol{P}(\boldsymbol{x}_f) = \boldsymbol{B}\boldsymbol{x}_f + \boldsymbol{c}$, with $\boldsymbol{c} = \boldsymbol{x}_c^* - \boldsymbol{B}\boldsymbol{x}_f^{\mathrm{SM}}$ and $\boldsymbol{B}$ as the final Broyden matrix (see Figure 4).

Stopping Criteria

Because ASM aims at finding the roots of $\boldsymbol{f}(\boldsymbol{x}_f)$, the most natural and widely used stopping criterion is when the maximum absolute error in the solution of the system of nonlinear equations is small enough. However, it has been found in practice that, by incorporating other criteria, ASM performance can be significantly enhanced [12], [82]. Appropriate additional stopping criteria include three other possibilities: when the maximum relative error in the fine-model response with respect to the target response is small enough; when the relative change in the fine-model design parameters is small enough; or when a maximum number of iterations is reached.

There is a trend toward the development of fully automated CAD tools, based on ASM, for efficient and accurate synthesis and design optimization algorithms dedicated to particular structures in specific technologies.

In summary, these four criteria for finalizing ASM at the *i*-th iteration can be implemented as follows:

$$\left\| \boldsymbol{f}\left(\boldsymbol{x}_f^{(i)}\right)\right\|_\infty < \varepsilon_1 \vee \cdots$$
$$\left\| \boldsymbol{R}_f\left(\boldsymbol{x}_f^{(i)}\right) - \boldsymbol{R}_c(\boldsymbol{x}_c^*)\right\|_\infty \le \varepsilon_2\left(\varepsilon_2 + \|\boldsymbol{R}_c(\boldsymbol{x}_c^*)\|_\infty\right) \vee \cdots$$
$$\left\| \boldsymbol{x}_f^{(i+1)} - \boldsymbol{x}_f^{(i)}\right\|_2 \le \varepsilon_3\left(\varepsilon_3 + \left\|\boldsymbol{x}_f^{(i)}\right\|_2\right) \vee \cdots$$
$$i > i_{\max},$$

where ε_1, ε_2, and ε_3 are arbitrary small positive scalars. Because ASM is normally very efficient, a suitable maximum number of iterations, $i_{\max}$, to stop ASM is $3n$ or $4n$, where n is the total number of design variables. Exceeding that amount of iterations is typically a sign of anomalous ASM behavior, most probably caused by an inadequate PE process or by a too coarse model.

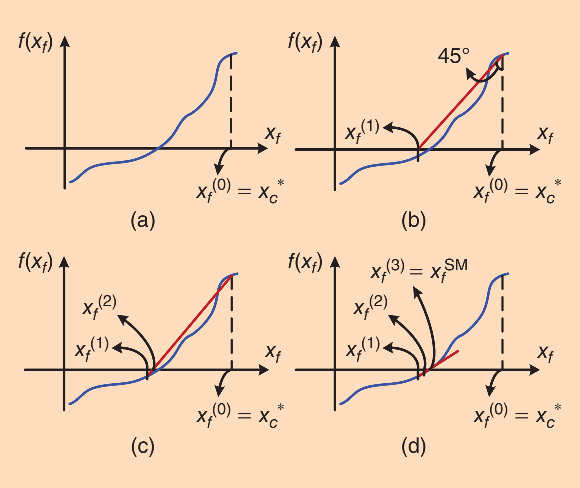


Figure 5. *A typical evolution of ASM, assuming a one-dimensional design optimization problem and a very bad initial design. (a) The initial fine-model response is calculated, with the first extracted parameters being very different from $\boldsymbol{x}_c^*$; (b) a Broyden matrix is initialized with the identity and first iterate predicted; (c) the Broyden matrix is updated with formulae, and the next iterate is calculated; (d) when the Broyden matrix is updated, the next iterate is practically a root (the extracted parameters are practically equal to $\boldsymbol{x}_c^*$).*

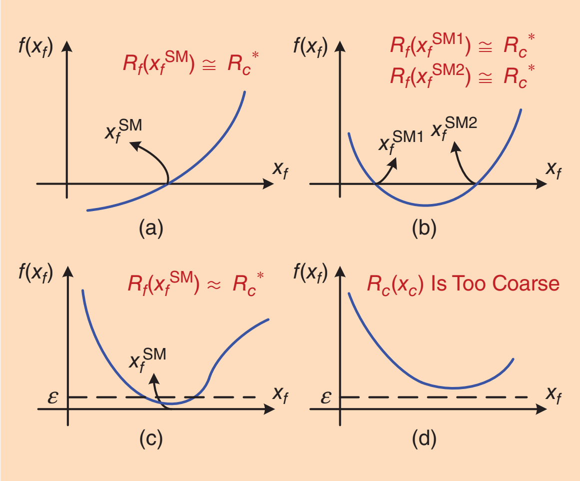


Figure 6. *Four ASM scenarios, all assuming a one-dimensional design optimization problem: (a) a unique and exact space-mapped solution exists; (b) several exact space-mapped solutions exist; (c) an acceptable space-mapped solution exists; and (d) no acceptable space-mapped solution exists due to a coarse model that is too inaccurate. The desired response is denoted as $\boldsymbol{R}_c^*$, which is equal to the coarse-model response at the optimal coarse-model design, $\boldsymbol{R}_c(\boldsymbol{x}_c^*)$. (The symbol "≅" denotes "equal or approximately equal.")*

Final Remarks and Future Directions for ASM

In the context of all the space mapping-based design optimization approaches described in the opening section and considering the essential characteristics of ASM as subsequently described here, a more technical name for this space mapping design technique would be a *Broyden-based input space mapping algorithm*.

Looking in more detail at the most recent applications of ASM listed in Table 2, it seems that there is a trend toward the development of fully automated CAD tools, based on ASM, for efficient and accurate synthesis and design optimization algorithms dedicated to particular structures in specific technologies. This trend might lead to the future incorporation into industrial CAD tools of ASM-based built-in design functions.

As time goes by, perhaps some of the most recent and advanced space mapping-like optimization approaches, indicated in Figure 1, will prove to be as popular as the ASM algorithm has been so far.

References

[1] J. W. Bandler, R. M. Biernacki, S. H. Chen, P. A. Grobelny, and R. H. Hemmers, "Space mapping technique for electromagnetic optimization," *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 12, pp. 2536–2544, Dec. 1994.

[2] J. W. Bandler, "Have you ever wondered about the engineer's mysterious 'feel' for a problem?" *IEEE Canadian Rev.*, no. 70, pp. 50–60, 2013, Summer.

[3] J. W. Bandler, Q. Cheng, S. A. Dakroury, A. S. Mohamed, M. H. Bakr, K. Madsen, and J. Søndergaard, "Space mapping: The state of the art," *IEEE Trans. Microwave Theory Tech.*, vol. 52, no. 1, pp. 337–361, Jan. 2004.

[4] S. Koziel, Q. S. Cheng, and J. W. Bandler, "Space mapping," *IEEE Microwave Mag.*, vol. 9, pp. 105–122, Dec. 2008.

[5] J. E. Rayas-Sánchez, "EM-based optimization of microwave circuits using artificial neural networks: The state of the art," *IEEE Trans. Microwave Theory Tech.*, vol. 52, no. 1, pp. 420–435, Jan. 2004.

[6] J. W. Bandler, R. M. Biernacki, S. H. Chen, R. H. Hemmers, and K. Madsen, "Aggressive space mapping for electromagnetic design," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Orlando, FL, Jun. 1995, pp. 1455–1458.

[7] J. W. Bandler, R. M. Biernacki, S. H. Chen, R. H. Hemmers, and K. Madsen, "Electromagnetic optimization exploiting aggressive space mapping," *IEEE Trans. Microwave Theory Tech.*, vol. 41, no. 12, pp. 2874–2882, Dec. 1995.

[8] M. H. Bakr, J. W. Bandler, N. Georgieva, and K. Madsen, "A hybrid aggressive space mapping algorithm for EM optimization," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Anaheim, CA, Jun. 1999, pp. 265–268.

[9] M. H. Bakr, J. W. Bandler, N. Georgieva, and K. Madsen, "A hybrid aggressive space mapping algorithm for EM optimization," *IEEE Trans. Microwave Theory Tech.*, vol. 47, no. 12, pp. 2440–2449, Dec. 1999.

[10] M. H. Bakr, J. W. Bandler, M. A. Ismail, J. E. Rayas-Sánchez, and Q. J. Zhang, "Neural space mapping optimization for EM-based design," *IEEE Trans. Microwave Theory Tech.*, vol. 48, no. 12, pp. 2307–2315, Dec. 2000.

[11] V. Gutiérrez-Ayala and J. E. Rayas-Sánchez, "High-frequency circuit design using a neural space-mapping algorithm based on a two-layer perceptron with optimized nonlinearity," in *Proc. Int. Conf. Electronic Design*, Veracruz, Mexico, Nov. 2006, pp. 90–95.

[12] V. Gutiérrez-Ayala and J. E. Rayas-Sánchez, "Neural input space mapping optimization based on nonlinear two-layer perceptrons with optimized nonlinearity," *Int. J. RF Microwave CAE*, vol. 20, pp. 512–526, Sept. 2010.

[13] J. W. Bandler, M. A. Ismail, and J. E. Rayas-Sánchez, "Expanded space mapping design framework exploiting preassigned parameters," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Phoenix, AZ, May 2001, pp. 1151–1154.

[14] J. W. Bandler, M. A. Ismail, and J. E. Rayas-Sánchez, "Expanded space mapping EM based design framework exploiting preassigned parameters," *IEEE Trans. Circuits Sys. I*, vol. 49, pp. 1833–1838, Dec. 2002.

[15] J. W. Bandler, Q. S. Cheng, N. Georgieva, and M. A. Ismail, "Implicit space mapping EM-based modeling and design exploiting preassigned parameters," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Seattle, WA, June 2002, pp. 713–716.

[16] J. W. Bandler, Q. S. Cheng, N. Georgieva, and M. A. Ismail, "Implicit space mapping optimization exploiting preassigned parameters," *IEEE Trans. Microwave Theory Tech.*, vol. 52, no. 1, pp. 378–385, Jan. 2004.

[17] S. Koziel, J. W. Bandler, and Q. S. Cheng, "Constrained parameter extraction for microwave design optimisation using implicit space mapping," *IET Microwave Antennas Propagat.*, vol. 5, pp. 1156–1163, July 2011.

[18] J. W. Bandler, M. A. Ismail, J. E. Rayas-Sánchez, and Q. J. Zhang, "Neural inverse space mapping EM-optimization," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Phoenix, AZ, May 2001, pp. 1007–1010.

[19] J. W. Bandler, M. A. Ismail, J. E. Rayas-Sánchez, and Q. J. Zhang, "Neural inverse space mapping (NISM) for EM-based microwave design," *Int. J. RF Microwave CAE*, vol. 13, pp. 136–147, Mar. 2003.

[20] J. W. Bandler, Q. S. Cheng, D. H. Gebre-Mariam, K. Madsen, F. Pedersen, and J. Sondergaard, "EM-based surrogate modeling and design exploiting implicit, frequency and output space mappings," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Philadelphia, PA, June 2003, pp. 1003–1006.

[21] J. W. Bandler, Q. S. Cheng, D. A. Hailu, and N. K. Nikolova, "A space-mapping design framework," *IEEE Trans. Microwave Theory Tech.*, vol. 52, no. 11, pp. 2601–2610, Nov. 2004.

[22] J. E. Rayas-Sánchez, F. Lara-Rojo, and E. Martínez-Guerrero, "A linear inverse space mapping algorithm for microwave design in the frequency and transient domains," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Fort Worth, TX, June 2004, pp. 1847–1850.

[23] J. E. Rayas-Sánchez, F. Lara-Rojo, and E. Martínez-Guerrero, "A linear inverse space mapping (LISM) algorithm to design linear and nonlinear RF and microwave circuits," *IEEE Trans. Microwave Theory Tech.*, vol. 53, pp. 960–968, Mar. 2005.

[24] J. E. Rayas-Sánchez and N. Vargas-Chávez, "A linear regression inverse space mapping algorithm for EM-based design optimization of microwave circuits," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Baltimore, MD, June 2011, pp. 1–4.

[25] D. Echeverria and P. W. Hemker, "Space mapping and defect correction," *Comput. Methods Appl. Math.*, vol. 5, no. 2, pp. 107–136, 2005.

[26] P. W. Hemker and D. Echeverría, "A trust-region strategy for manifold-mapping optimization," *J. Comput. Phys.*, vol. 224, pp. 464–475, Apr. 2007.

[27] D. Echeverría-Ciaurri, "Multi-level optimization: Space mapping and manifold mapping," Ph.D. dissertation, Faculteit der Natuurwetenschappen, Wiskunde en Informatica, Universiteit van Amsterdam, Amsterdam, 2007.

[28] S. Koziel and D. Echeverría-Ciaurri, "Reliable simulation-driven design optimization of microwave structures using manifold mapping," *Progress Electromagn. Res. B*, vol. 26, pp. 361–382, 2010.

[29] L. Encica, J. J. H. Paulides, E. A. Lomonova, and A. J. A. Vandenput, "Aggressive output space-mapping optimization for electromagnetic actuators," *IEEE Trans. Magn.*, vol. 44, no. 6, pp. 1106–1109, June 2008.

[30] L. Encica, J. J. H. Paulides, E. A. Lomonova, and A. J. A. Vandenput, "Electromagnetic and thermal design of a linear actuator using output polynomial space mapping," *IEEE Trans. Ind. Appl.*, vol. 44, no. 2, pp. 534–542, Mar.–Apr. 2008.

[31] S. Koziel, J. W. Bandler, and K. Madsen, "Space mapping with adaptive response correction for microwave design optimization," *IEEE Trans. Microwave Theory Tech.*, vol. 57, no. 2, pp. 478–486, 2009.

[32] S. Koziel and S. Ogurtsov, "Design optimisation of antennas using electromagnetic simulations and adaptive response correction technique," *IET Microwave Antennas Propagat.*, vol. 8, no. 3, pp. 180–185, Feb. 2014.

[33] S. Koziel, "Efficient optimization of microwave circuits using shape-preserving response prediction," in *IEEE MTT-S Int. Microw. Symp. Dig.*, Boston, MA, June 2009, pp. 1569–1572.

[34] S. Koziel, "Shape-preserving response prediction for microwave design optimization," *IEEE Trans. Microwave Theory Tech.*, vol. 58, no. 11, pp. 2829–2837, Nov. 2010.

[35] A. Khalatpour, R. K. Amineh, Q. S. Cheng, M. H. Bakr, N. K. Nikolova, and J. W. Bandler, "Accelerating space mapping optimization with adjoint sensitivities," *IEEE Microwave Wireless Comp. Lett.*, vol. 21, no. 6, pp. 280–282, June 2011.

[36] Q. S. Cheng, J. W. Bandler, N. K. Nikolova, and S. Koziel, "Fast space mapping modeling with adjoint sensitivity," in *IEEE MTT-S Int. Microw. Symp.*, Baltimore, MD, June 2011, pp. 1–4.

[37] S. Koziel, S. Ogurtsov, J. W. Bandler, and Q. S. Cheng, "Reliable space-mapping optimization integrated with em-based adjoint sensitivities," *IEEE Trans. Microw. Theory Tech.*, vol. 61, no. 10, pp. 3493–3502, Oct. 2013.

[38] S. Koziel, "Derivative-free microwave design optimisation using shape-preserving response prediction and space mapping," *IET Sci., Meas. Technol.*, vol. 6, no. 1, pp. 13–20, Jan. 2012.

[39] S. Koziel, J. W. Bandler, and Q. S. Cheng, "Shape-preserving response prediction with adjoint sensitivities for microwave design optimization," in *IEEE MTT-S Int. Microw. Symp. Dig.*, Seattle, WA, June 2013, pp. 1–3.

[40] S. Koziel, S. Ogurtsov, Q. S. Cheng, and J. W. Bandler, "Rapid electromagnetic-based microwave design optimisation exploiting shape-preserving response prediction and adjoint sensitivities," *IET Microwaves Antennas Propagat.*, vol. 8, no. 10, pp. 775–781, July 2014.

[41] S. Koziel, "Expedite design optimization of narrow-band antennas using response features," in *IEEE Int. Symp. Antennas Propagat. (APSURSI)*, Memphis, TN, July 2014, pp. 1958–1959.

[42] S. Koziel and J. W. Bandler, "Rapid yield estimation and optimization of microwave structures exploiting feature-based statistical analysis," *IEEE Trans. Microwave Theory Tech.*, vol. 63, no. 1, pp. 107–114, Jan. 2015.

[43] S. Koziel and J. W. Bandler, "Fast EM-driven design optimization of microwave filters using adjoint sensitivity and response features," in *IEEE MTT-S Int. Microw. Symp.*, Phoenix, AZ, May 2015, pp. 1–3.

[44] C. Zhang, F. Feng, V.-M.-R. Gongal-Reddy, Q. J. Zhang, and J. W. Bandler, "Cognition-driven formulation of space mapping for equal-ripple optimization of microwave filters," *IEEE Trans. Microwave Theory Tech.*, vol. 63, no. 7, pp. 2154–2165, July 2015.

[45] A. J. Booker, J. E. Dennis Jr., P. D. Frank, D. B. Serafini, V. Torczon, and M. W. Trosset, "A rigorous framework for optimization of expensive functions by surrogates," *Struct. Optim.*, vol. 17, pp. 1–13, Feb. 1999.

[46] J. W. Bandler, R. M. Biernacki, and S. H. Chen, "Fully automated space mapping optimization of 3D structures," in *IEEE MTT-S Int. Microwave Symp. Dig.*, San Francisco, CA, June 1996, pp. 753–756.

[47] J. W. Bandler, R. M. Biernacki, S. H. Chen, and D. Omeragic, "Space mapping optimization of waveguide filters using finite element and mode-matching electromagnetic simulators," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Denver, CO, June 1997, pp. 635–638.

[48] J. W. Bandler, R. M. Biernacki, S. H. Chen, and D. Omeragic, "Space mapping optimization of waveguide filters using finite element and mode-matching electromagnetic simulators," *Int. J. RF Microw. Computer-Aided Eng.*, vol. 9, no. 1, pp. 54–70, Jan. 1999.

[49] J. W. Bandler, R. M. Biernacki, S. H. Chen, and Y. F. Huang, "Design optimization of interdigital filters using aggressive space mapping and decomposition," *IEEE Trans. Microwave Theory Tech.*, vol. 45, no. 5, pp. 761–769, May 1997.

[50] M. H. Bakr, J. W. Bandler, R. M. Biernacki, S. H. Chen, and K. Madsen, "A trust region aggressive space mapping algorithm for EM optimization," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Baltimore, MD, June 1998, pp. 1759–1762.

[51] M. H. Bakr, J. W. Bandler, R. M. Biernacki, S. H. Chen, and K. Madsen, "A trust region aggressive space mapping algorithm for EM optimization," *IEEE Trans. Microwave Theory Tech.*, vol. 46, no. 12, pp. 2412–2425, Dec. 1998.

[52] M. H. Bakr, J. W. Bandler, K. Madsen, J. E. Rayas-Sánchez, and J. Søndergaard, "Space mapping optimization of microwave circuits exploiting surrogate models," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Boston, MA, June 2000, pp. 1785–1788.

[53] M. H. Bakr, J. W. Bandler, K. Madsen, J. E. Rayas-Sánchez, and J. Søndergaard, "Space mapping optimization of microwave circuits exploiting surrogate models," *IEEE Trans. Microwave Theory Tech.*, vol. 48, no. 12, pp. 2297–2306, Dec. 2000.

[54] R. Lehmensiek and P. Meyer, "An efficient adaptive frequency sampling algorithm for model-based parameter estimation as applied to aggressive space mapping," *Microw. Opt. Technol. Lett.*, vol. 24, no. 1, pp. 71–78, Jan. 2000.

[55] H.-S. Choi, D.-H. Kim, I.-H. Park, and S.-Y. Hahn, "A new design technique of magnetic systems using space mapping algorithm," *IEEE Trans. Magn.*, vol. 37, no. 5, pp. 3627–3630, Sept. 2001.

[56] J. W. Bandler, A. S. Mohamed, M. H. Bakr, K. Madsen, and J. Søndergaard, "Space mapping optimization of microwave circuits exploiting surrogate models," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Seattle, WA, June 2002, pp. 2101–2104.

[57] J. W. Bandler, A. S. Mohamed, M. H. Bakr, K. Madsen, and J. Søndergaard, "Space mapping optimization of microwave circuits exploiting surrogate models," *IEEE Trans. Microwave Theory Tech.*, vol. 50, no. 12, pp. 2741–2750, Dec. 2002.

[58] J. V. Morro, H. Esteban, P. Soto, V. E. Boria, C. Bachiller, S. Cogollos, and B. Gimeno, "Automated design of waveguide filters using aggressive space mapping with a segmentation strategy and hybrid optimization techniques," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Philadelphia, PA, June 2005, pp. 1215–1218.

[59] J. V. Morro-Ros, P. Soto-Pacheco, H. Esteban-González, V. E. Boria-Esbert, C. Bachiller-Martín, M. Taroncher-Calduch, S. Cogollos-Borrás, and B. Gimeno-Martínez, "Fast automated design of waveguide filters using aggressive space mapping with a new segmentation strategy and a hybrid optimization algorithm," *IEEE Trans. Microwave Theory Tech.*, vol. 53, no. 4, pp. 1130–1142, Apr. 2005.

[60] K. L. Wu, Y. J. Zhao, J. Wang, and M. K. K. Cheng, "An effective dynamic coarse model for optimization design of LTCC RF circuits with aggressive space mapping," *IEEE Trans. Microwave Theory Tech.*, vol. 52, no. 1, pp. 393–402, Jan. 2004.

[61] Y. J. Zhao, Y. Z. Yin, K. L. Wu, and J. Wang "An efficient design method of LTCC filters with aggressive space mapping technique," *Microw. Optical Technol. Lett.*, vol. 41, pp. 24–28, Apr. 2004.

[62] M. A. Ismail, D. Smith, A. Panariello, Y. Wang, and M. Yu, "EM-based design of large-scale dielectric-resonator filters and multiplexers by space mapping," *IEEE Trans. Microw. Theory Tech.*, vol. 52, no. 1, pp. 386–392, Jan. 2004.

[63] M. A. Ismail, K. G. Engel, and Y. Ming, "Multiple space mapping for RF T-switch design," in *IEEE MTT-S Int. Microw. Symp. Dig.*, Fort Worth, TX, June 2004, pp. 1569–1572.

[64] M. Redhe and L. Nilsson, "Optimization of the new Saab 9-3 exposed to impact load using a space mapping technique," *Struct. Multidiscip. Optim.*, vol. 27, no. 5, pp. 411–420, July 2004.

[65] Q. S. Cheng, J. W. Bandler, and S. Koziel, "A simple ADS schematic for space mapping," in *IEEE MTT-S Int. Microw. Workshop Series Signal Integrity High-Speed Interconnects*, Guadalajara, Mexico, Feb. 2009, pp. 35–38.

[66] J. V. Morro, H. Esteban, V. E. Boria, C. Bachiller, and A. Coves, "Automated design of waveguide filters using aggressive space mapping with a segmentation strategy and hybrid optimization techniques," in *IEEE MTT-S Int. Microwave Symp. Dig.*, Long Beach, CA, June 2005, pp. 1–4.

[67] H. Wang, A.-J. Zhang, and D.-G. Fang, "The design of cross-coupled microstrip filter exploiting aggressive space mapping technique," in *Proc. Asia-Pacific Microwave Conf.*, Suzhou, China, Dec. 2005, vol. 5, pp. 1–3.

[68] G. Crevecoeur, L. Dupré, L. Vandenbossche, and R. Van de Walleb, “Reconstruction of local magnetic properties of steel sheets by needle probe methods using space mapping techniques,” *J. Appl. Phys.*, vol. 99, pp. 08H905-08H905-3, Apr. 2006.

[69] J. E. Rayas-Sánchez and V. Gutiérrez-Ayala, “EM-based statistical analysis and yield estimation using linear-input and neural-output space mapping,” in *IEEE MTT-S Int. Microwave Symp. Dig.*, San Francisco, CA, June 2006, pp. 1597–1600.

[70] J. E. Rayas-Sánchez and V. Gutiérrez-Ayala, “EM-based Monte Carlo analysis and yield prediction of microwave circuits using linear-input neural-output space mapping,” *IEEE Trans. Microwave Theory Tech.*, vol. 54, no. 12, pp. 4528–4537, Dec. 2006.

[71] L. J. Roglá, J. E. Rayas-Sánchez, V. E. Boria, and J. Carbonell, “EM-based space mapping optimization of left-handed coplanar waveguide filters with split ring resonators,” in *IEEE MTT-S Int. Microwave Symp. Dig.*, Honolulu, HI, June 2007, pp. 111–114.

[72] W. Tang, P. Zhou, X. Chen, and Y. L. Chow, “Design and optimization of microstrip passive circuits by field-based equivalent circuit model with aggressive space mapping,” in Proc. *Int. Symp. Microwave Antenna, Propagation EMC Technologies Wireless Communication*, Hangzhou, China, Aug. 2007, pp. 909–912.

[73] W. Tang, P. Zhou, X. Chen, and Y. L. Chow, “Design and optimization of stripline passive components with trust-region aggressive space mapping,” in *Proc. Asia-Pacific Microwave Conf.*, Bangkok, Thailand, Dec. 2007, pp. 1–4.

[74] W. Tang, P. Zhou, X. Chen, and Y. L. Chow, “Design and optimization of microstrip passive circuits by field-based equivalent circuit model with aggressive space mapping,” *Microw. Opt. Technol. Lett.*, vol. 49, no. 12, pp. 2889–2893, Dec. 2007.

[75] G. Crevecoeur, H. Hallez, P. Van Hese, Y. D’Asseler, L. Dupré, and R. Van de Walleb, “EEG source analysis using space mapping techniques,” *J. Comput. Appl. Math.*, vol. 215, no. 2, pp. 339–347, June 2008.

[76] F. Chen, Z. Shi, X. Deng, J. Song, and Y. Liu, “The application of aggressive space mapping algorithm in microstrip patch antennna design,” in *Int. Conf. Microw. Millimeter Wave Technol.*, Chengdu, China, May 2010, pp. 835–838.

[77] A. Rodriguez, J. Selga, M. Gil, J. Carbonell, V. E. Boria, and F. Martin, “Automated synthesis of resonant-type metamaterial transmission lines using aggressive space mapping,” in *IEEE MTT-S Int. Microwave Symp. Dig.*, Anaheim, CA, May 2010, pp. 209–212.

[78] J. Selga, A. Rodríguez, M. Gil, J. Carbonell, V. E. Boria, and F. Martín, “Towards the automatic layout synthesis in resonant-type metamaterial transmission lines,” *IET Microw. Antennas Propagat.*, vol. 4, no. 8, pp. 1007–1015, Aug. 2010.

[79] J. Selga, A. Rodríguez, M. Gil, J. Carbonell, V. E. Boria, and F. Martín, “Synthesis of planar microwave circuits through aggressive space mapping using commercially available software packages,” *Int. J. RF Microw. Computer-Aided Eng.*, vol. 20, no. 5, pp. 527–534, Sept. 2010.

[80] J. Selga, A. Rodriguez, V. E. Boria, and F. Martin, “Application of aggressive space mapping to the synthesis of composite right/left handed (CRLH) transmission lines based on complementary split ring resonators (CSRRs),” in *Proc. 41st European Microwave Conf.*, Manchester, U.K., Oct. 2011, pp. 968–971.

[81] M. Prieß and T. Slawig, “Aggressive space mapping for optimisation of a marine ecosystem model,” *Int. J. Math. Model. Numer. Optim.*, vol. 3, no. 1–2, pp. 98–116, 2012.

[82] J. Wu, Y. Dong, J. He, and J. Deng, “An ameliorated aggressive space mapping based on responses error,” in *Proc. Int. Workshop Microwave Millimeter Wave Circuits System Technology*, Chengdu, China, Apr. 2012, pp. 1–4.

[83] J. Xu, X. Luo, L. Z. Cao, and R. S. Chen, “Optimization of coaxial dielectric resonator filter with aggressive space mapping,” in *Proc. Asia-Pacific Microwave Conf.*, Kaohsiung, Taiwan, Dec. 2012, pp. 229–231.

[84] E. Diaz-Caballero, J. Morro, A. Belenguer, H. Esteban, and V. Boria, “CAD technique for designing H-plane waveguide filters considering rounded corners,” in *IEEE MTT-S Int. Microwave Symp. Dig.*, Seattle, WA, June 2013, pp. 1–3.

[85] J. Selga, A. Rodriguez, V. E. Boria, and F. Martin, “Synthesis of split-rings-based artificial transmission lines through a new two-step, fast converging, and robust aggressive space mapping (ASM) algorithm,” *IEEE Trans. Microwave Theory Tech.*, vol. 61, no. 6, pp. 2295–2308, June 2013.

[86] F. Martín, “Appendix H: Synthesis of resonant-type metamaterial transmission lines by means of aggressive space mapping,” in Artificial Transmission Lines for RF and Microwave Applications. Hoboken, NJ: Wiley, 2015.

[87] J. C. Cervantes-González, C. A. López, J. E. Rayas-Sánchez, Z. Brito-Brito, and G. Hernández-Sosa, “Return-loss minimization of package interconnects through input space mapping using FEM-based models,” in *Proc. SBMO/IEEE MTT-S Int. Microwave Optoelectronics Conf.*, Rio de Janeiro, Brazil, Aug. 2013, pp. 1–4.

[88] J. Selga, L. Naqui, M. Duran-Sindreu, F. Martin, A. Rodriguez, and V. E. Boria, “Application of aggressive space mapping (ASM) to the efficient synthesis of stepped impedance resonators (SIRs),” in *Proc. European Microwave Conf.*, Nuremberg, Germany, Oct. 2013, pp. 636–639.

[89] J. Selga, M. Sans, J. Bonache, A. Rodriguez, V. E. Boria, and F. Martin, “Automated synthesis of planar wideband bandpass filters based on stepped impedance resonators (SIRs) and shunt stubs through aggressive space mapping (ASM),” in *IEEE MTT-S Int. Microwave Symp. Dig.*, Tampa, FL, June 2014, pp. 1–4.

[90] M. Sans, J. Selga, A. Rodriguez, J. Bonache, V. E. Boria, and F. Martin, “Design of planar wideband bandpass filters from specifications using a two-step aggressive space mapping (ASM) optimization algorithm,” *IEEE Trans. Microwave Theory Tech.*, vol. 62, no. 12, pp. 3341–3350, Dec. 2014.

[91] A. Rodriguez, V. E. Boria, J. Selga, M. Sans, and F. Martin, “Synthesis of open complementary split ring resonators (OCSRRs) through aggressive space mapping (ASM) and application to bandpass filters,” in *Proc. 44th European Microwave Conf.*, Rome, Italy, Oct. 2014, pp. 323–326.

[92] Q. Deng, T. Fan, and S. Lu, “The design of Ka band filter with active space mapping algorithm,” in *Proc. Asia-Pacific Conf. Antennas Propagation*, Harbin, China, July 2014, pp. 1140–1143.

[93] M. Brumos, V. E. Boria, M. Guglielmi, and S. Cogollos, “Correction of manufacturing deviations in circular-waveguide dual-mode filters using aggressive space mapping,” in *Proc. 44th European Microwave Conf.*, Rome, Italy, Oct. 2014, pp. 624–627.

[94] S. Cogollos, C. Carceller, M. Taroncher, V. E. Boria, M. Gugleilmi, C. Vicente, and M. Brumos, “Correction of manufacturing deviations in waveguide filters and manifold multiplexers using metal insertions,” *Int. J. Microw. Wireless Technol.*, vol. 7 no. 3–4, pp. 219–227, June 2015.

[95] X. Fan and P. D. Laforge, “Design of a chebyshev microstrip filter using the reflected group delay method and the aggressive space mapping technique,” in *Proc. IEEE MTT-S Int. Microwave Symp.*, Phoenix, AZ, May 17–22, 2015, pp. 1–4.

[96] M. Orellana, J. Selga, M. Sans, A. Rodríguez, V. Boria, and F. Martín, “Synthesis of slow-wave structures based on capacitive-loaded lines through aggressive space mapping (ASM),” *Int. J. RF Microw. Computer-Aided Eng.*, vol. 25, pp. 629–638, Sept. 2015.

[97] J. C. Cervantes-González, J. E. Rayas-Sánchez, C. A. López, J. R. Camacho-Pérez, Z. Brito-Brito, and J. L. Chavez-Hurtado, “Space mapping optimization of handset antennas considering EM effects of mobile phone components and human body,” *Int. J. RF Microw. CAE*, Oct. 21, 2015, {online} http://onlinelibrary.wiley.com/doi/10.1002/mmce.20945/abstract

[98] S. Koziel and L. Leifsson, Ed., *Surrogate-Based Modeling and Optimization: Applications in Engineering*, Berlin, Germany: Springer-Verlag, 2013.

[99] C. G. Broyden, “A class of methods for solving non-linear simultaneous equations,” *Math. Comput.*, vol. 19, no. 92, pp. 577–593, Oct. 1965.

[100] J. Søndergaard, “Optimization using surrogate models by the Space mapping technique,” Ph.D. dissertation, Dept. of Informatics and Mathematical Modeling, Technical Univ. of Denmark, Lyngby, Denmark, 2003.