A REVIEW ON MATRIX COMPUTATIONS

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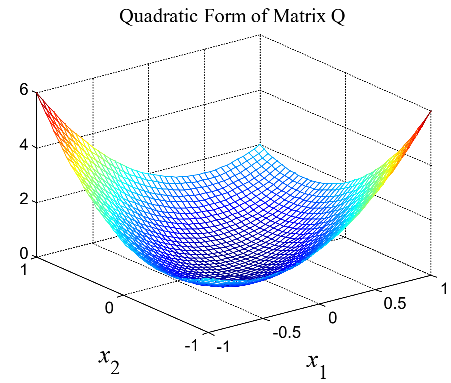
EXAMPLES OF POSITIVE DEFINITE MATRICES AND QUADRATIC FORMS

Example 1:

$$\boldsymbol{Q} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$q(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x} = 2x_1^2 - 2x_1x_2 + 2x_2^2$$

Matrix $\boldsymbol{Q}$ is positive definite. Function $q(\boldsymbol{x})$ corresponds to a convex function.

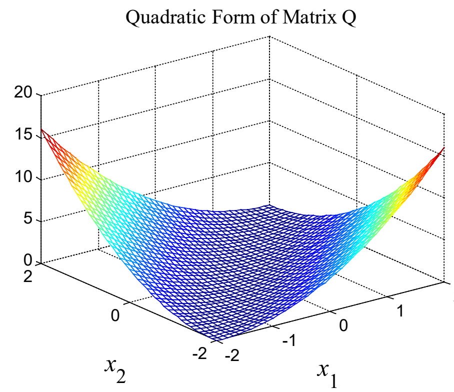


Example 2:

$$\boldsymbol{Q} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$q(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x} = x_1^2 - 2x_1x_2 + x_2^2$$

Matrix $\boldsymbol{Q}$ is positive semi-definite. Function $q(\boldsymbol{x})$ corresponds to a convex function.

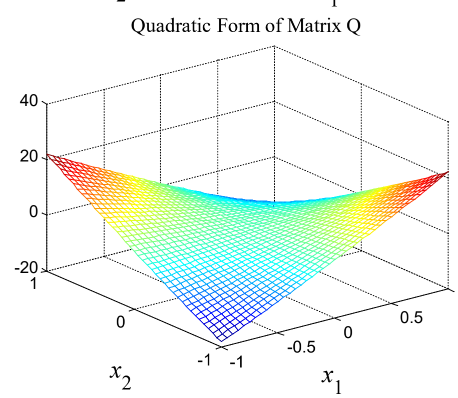


Example 3:

$$\boldsymbol{Q} = \begin{bmatrix} 1 & -10 \\ -10 & 1 \end{bmatrix}$$

$$q(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x} = x_1^2 - 20x_1x_2 + x_2^2$$

Matrix $\boldsymbol{Q}$ is not positive definite nor positive semi-definite. Function $q(\boldsymbol{x})$ corresponds to a non-convex function.

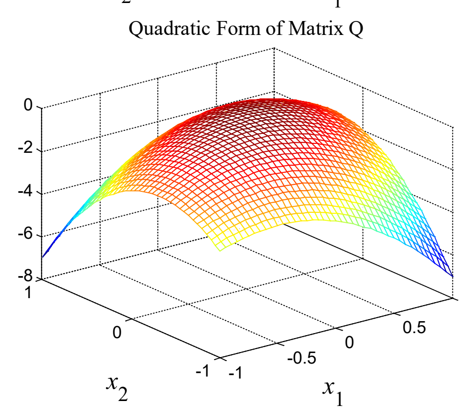


Example 4:

$$\boldsymbol{Q} = \begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix}$$

$$q(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{x} = -2x_1^2 + 2x_1x_2 - 3x_2^2$$

Matrix $\boldsymbol{Q}$ is not positive definite nor positive semi-definite. Function $q(\boldsymbol{x})$ corresponds to a non-convex function.



Examples of Positive Definite Matrix and Quadratic Forms

1) $\underset{\sim}{Q} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$; $\quad \underset{\sim}{Q}\,\underline{x} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_1 - x_2 \\ -x_1 + 2x_2 \end{bmatrix}$

$$\underline{x}^T \underset{\sim}{Q}\,\underline{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} 2x_1 - x_2 \\ -x_1 + 2x_2 \end{bmatrix} = 2x_1^2 - x_1x_2 - x_1x_2 + 2x_2^2$$
$$= \boxed{2x_1^2 - 2x_1x_2 + 2x_2^2}$$

2) $\underset{\sim}{Q} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$; $\quad \underset{\sim}{Q}\,\underline{x} = \begin{bmatrix} x_1 - x_2 \\ -x_1 + x_2 \end{bmatrix}$

$$\underline{x}^T \underset{\sim}{Q}\,\underline{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} x_1 - x_2 \\ -x_1 + x_2 \end{bmatrix} = x_1^2 - x_1x_2 - x_1x_2 + x_2^2$$
$$= \boxed{x_1^2 - 2x_1x_2 + x_2^2}$$

3) $\underset{\sim}{Q} = \begin{bmatrix} 1 & -10 \\ -10 & 1 \end{bmatrix}$; $\quad \underline{x}^T \underset{\sim}{Q}\,\underline{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} x_1 - 10x_2 \\ -10x_1 + x_2 \end{bmatrix}$

$$\underline{x}^T \underset{\sim}{Q}\,\underline{x} = x_1^2 - 10x_1x_2 - 10x_1x_2 + x_2^2$$
$$= \boxed{x_1^2 - 20x_1x_2 + x_2^2}$$

4) $\underset{\sim}{Q} = \begin{bmatrix} -2 & 1 \\ 1 & -3 \end{bmatrix}$; $\quad \underline{x}^T \underset{\sim}{Q}\,\underline{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} -2x_1 + x_2 \\ x_1 - 3x_2 \end{bmatrix}$

$$\underline{x}^T \underset{\sim}{Q}\,\underline{x} = -2x_1^2 + x_1x_2 + x_1x_2 - 3x_2^2$$
$$\underline{x}^T \underset{\sim}{Q}\,\underline{x} = \boxed{-2x_1^2 + 2x_1x_2 - 3x_2^2}$$

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