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## Optimizing a Simple Microstrip Line: A Solution

Consider a conventional microstrip line whose physical structure is shown below. The width of the microstrip line is $W$, the length is $L$ and its metal thickness is $t$. The microstrip line is on a dielectric substrate with relative dielectric constant $\varepsilon_{r}$ and loss tangent $\tan (\delta)$. The substrate height is $H$. Bellow the substrate there is a metallic ground plane whose thickness is also $t$. Metal has a conductivity $\sigma$.


This microstrip line uses the following parameters:
Substrate parameters: $\mathcal{E}_{\mathrm{r}}=3.6, \tan (\delta)=0.01, H=16 \mathrm{mil}$.
Metals: $t=0.65 \mathrm{mil}$ (half-once copper), $\sigma=5.8 \times 10^{7} \mathrm{~S} / \mathrm{m}$.
Trace: $L=800$ mil, $W=45 \mathrm{mil}$.

## 1. Optimization Variables and Starting Point

The optimization variable is $\boldsymbol{x}=[W(\mathrm{mil})]$, with a starting point $\boldsymbol{x}^{(0)}=[45]$.
Circuit responses at $\boldsymbol{x}^{(0)}$


## 2. Design Specifications

Design specs are:

$$
\left|S_{11}\right| \rightarrow 0 \text { in the complete simulated frequency band }
$$

## 3. Formulation of the Optimization Problem

$$
\boldsymbol{x}^{*}=\arg \min _{\boldsymbol{x}} \max \left\{\ldots e_{k}(\boldsymbol{x}) \ldots\right\}
$$

where the $k$-th error function is given by

$$
e_{k}(\boldsymbol{x})=\frac{\left|S_{11}\right|(\boldsymbol{x})}{\left|S_{11}\right|_{\max }}-1 \text { for } f_{\mathrm{L}}<f_{k}<f_{\mathrm{H}}
$$

where $f_{\mathrm{L}}$ and $f_{\mathrm{H}}$ are the lowest and highest simulated frequencies, respectively, $f_{k}$ is the $k$-th simulated frequency point, and $\left|S_{11}\right|_{\max }$ is a maximum acceptable level of reflection. For instance, $\left|S_{11}\right|_{\max }=0.1$.

## 4. Objective Function Implementation

```
% ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
% Objective Function for a Microstrip Line Simulated in APLAC
function MaxError = OF_mcsLine_APLAC(X)
mS11max = 0.1;
% Pre-assigned Parameters, Xp = [H L epsr losstan thck rho]
H = 16; % Substrate height (units).
L = 800; % Length of the microstrip line (units).
epsr = 3.6; % Substrate relative dielectric constant.
losstan = 0.01; % Substrate loss tangent.
thck = 0.65; % Metal thickness (units).
rho = 0.7066; % Metal resistivity normalized to that of gold.
Xp = [H L epsr losstan thck rho];
% Parameters of the Simulator, Ps
units = 1; % Units for APLAC lengths: mil(1), mm(2), um(3).
IF = 2e6; % Initial frequency (Hz).
FF = 20e9; % Final frequency (Hz).
FP = 301; % Number of frequencies per sweep.
Ps = [units IF FF FP];
% Calculate Circuit Response
[f,S] = mcsLine_APLAC(X,Xp,Ps);
mS11 = S(:,1);
% Calculate Error Functions
e = mS11/mS11max - 1; % Upper bound for mS11.
% Calculating Objective Function Value
MaxError = max(e);
```


## 5. Optimization Results

$\boldsymbol{x}^{(0)}=[45]$.

## A) Using the Nelder-Mead Method

options = optimset('MaxFunEvals',1000,' ${ }^{(M a x I t e r ', 1000, ' T o l X ', 1 e-2) ; ~}$
[Xopt,FunVal,EF,output] = fminsearch('OF_MFBP2',Xo,options);

| FP | $\boldsymbol{x}^{* T}$ | $u\left(\boldsymbol{x}^{*}\right)$ | iter | OFE |
| :---: | :---: | :---: | :---: | :---: |
| 151 | 34.9233 | -0.94703 | 14 | 28 |
| 301 | 34.9211 | -0.94702 | 14 | 28 |
| 1000 | 34.9211 | -0.94702 | 14 | 28 |
| 50 | 34.9233 | -0.94703 | 13 | 26 |

iter: number of iterations.
OFE: number of objective function evaluations $=$ WinSpice simulations.
It is seen that the objective function formulation is quite insensitive to FP , as expected.
Using 'TolX' $=1 \mathrm{e}-5$ and $\mathrm{FP}=301$, Nelder-Mead requires much more iterations and function evaluations: $x^{*}=[34.9222], u\left(x^{*}\right)=-0.94715$, iter $=22, \mathrm{OFE}=47$.
B) Using Gradient-Based Methods (gradients calculated by finite central differences, $h=1 e-4$ ) MaxIter = 1000; epsg = 1e-5; epsx = 1e-5;
$\mathrm{FP}=301$

| Method | $\boldsymbol{x}^{* T}$ | $u\left(\boldsymbol{x}^{*}\right)$ | iter | OFE |
| :---: | :---: | :---: | :---: | :---: |
| Steepest Descent | 34.9222 | -0.94715 | 2 | 53 |
| Conjugate Gradient | 34.9222 | -0.94715 | 2 | 53 |
| Quasi-Newton (BFGS) | 34.9222 | -0.94715 | 2 | 53 |



