CIRCUIT DESIGN BY OPTIMIZATION

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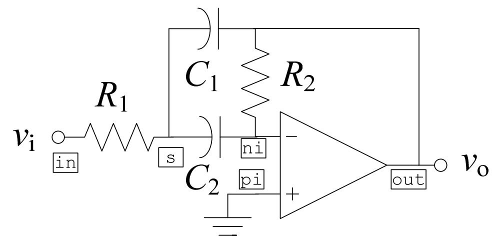
OPTIMIZING A MULTIPLE-FEEDBACK 2ND ORDER BAND-PASS FILTER: A SOLUTION

Consider the following multiple-feedback 2nd order band-pass active filter. At a resonant frequency $f_0 = 15$ KHz, the magnitude of the voltage gain should be $|A_v| = 800$, with a bandwidth $BW = 750$ Hz.



Assuming an ideal Op-Amp, an exact analytical solution can be found following [1]. If $C_1 = C_2 = 1$nF, and $f_0 = 15$ KHz with $BW = 750$ Hz, then $Q = 20$, $|A_v| = 800$ at f_0, yielding $R_1 = 265.26\ \Omega$ and $R_2 = 424.41$ KΩ.

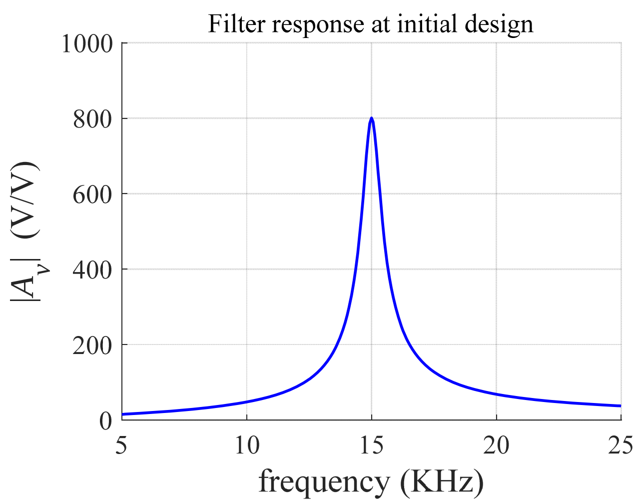
1. Optimization Variables and Starting Point

The selected vector of optimization variables is $\boldsymbol{x} = [R_1(\Omega) \quad R_2(\text{K}\Omega) \quad C(\text{nF})]^T$, with $C = C_1 = C_2$.

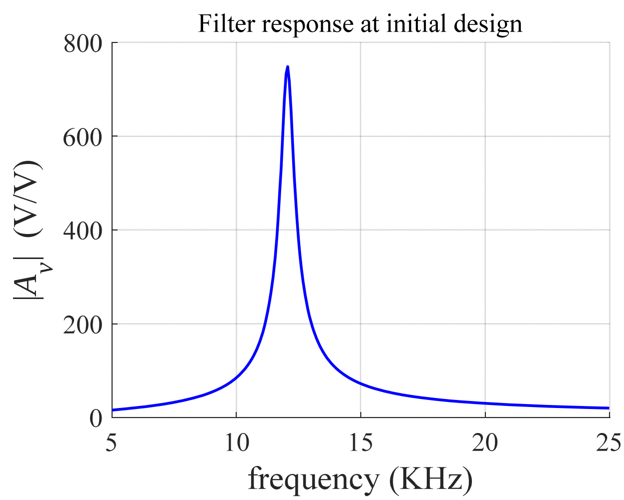
The starting point for optimization is $\boldsymbol{x}^{(0)} = [265.26 \quad 424.41 \quad 1]^T$.

Filter response at $\boldsymbol{x}^{(0)}$

Using ideal Op-Amp



Using Op-Amp uA741



2. Design Specifications

Let $|A_v|_{3dB} = 800/\sqrt{2}$, $f_L = f_0 - \dfrac{BW}{2}$, and $f_H = f_0 + \dfrac{BW}{2}$.

Design specs are:

$$|A_v| = 800 \text{ at } f = f_0$$
$$|A_v| \le |A_v|_{3dB} \quad \text{for } f < f_L$$
$$|A_v| \le |A_v|_{3dB} \quad \text{for } f > f_H$$

3. Formulation of the Optimization Problem

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} \max\{\ldots e_k(\boldsymbol{x})\ldots\}$$

where the *k*-th error function is given by

$$e_k(\boldsymbol{x}) = \begin{cases} \dfrac{|A_v|(\boldsymbol{x})}{|A_v|_{3\text{dB}}} - 1 & \text{for } f_k < f_{\text{L}} \\ \dfrac{|(|A_v|(\boldsymbol{x}) - 800)|}{\varepsilon} - 1 & \text{for } f_0 - \Delta f < f_k < f_0 + \Delta f \\ \dfrac{|A_v|(\boldsymbol{x})}{|A_v|_{3\text{dB}}} - 1 & \text{for } f_k > f_{\text{H}} \end{cases}$$

where f_k is the *k*-th simulated frequency point, $\varepsilon = 0.01(800)$ and $\Delta f = 10$ Hz.

4. Objective Function Implementation

```
% ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
%    Objective Function for a Multiple-Feedback 2nd Order Band-Pass Filter

function MaxError = OF_MFBP2(x)

Avr = 800;
Avr_3dB = Avr/sqrt(2);
fo = 15e3;
BW = 750;
fL = fo - BW/2;
fH = fo + BW/2;

% Calculate Filter Response in Frequency Band of Interest
IF = 12.5e3;
FF = 17.5e3;
FP = 501;
[f,mAv,pAv] = MFBP2_SPICE(x,IF,FF,FP);

% Calculate Error Functions
DeltaAv = Avr/100;
e1 = []; % Initialize error e1.
e2 = []; % Initialize error e2.
e3 = []; % Initialize error e3.
for k = 1:FP
    if f(k) < fL
        e1n = mAv(k)/Avr_3dB - 1; % Upper bound for low frequencies.
        e1 = [e1 e1n];
    elseif f(k) > fH
        e3n = mAv(k)/Avr_3dB - 1; % Upper bound for high frequencies.
        e3 = [e3 e3n];
    else
        if f(k)>fo-10 && f(k)<fo+10
            e2n = abs(mAv(k)-Avr)/DeltaAv - 1; % Equality constraint
            e2 = [e2 e2n];                     % for resonant frequency.
        end
    end
end

% Calculating Objective Function Value
e = [e1 e2 e3];
MaxError = max(e);
```

5. Optimization Results

$\boldsymbol{x}^{(0)} = [265.26 \quad 424.41 \quad 1]^T$.

A) Using the Nelder-Mead Method

```
options = optimset('MaxFunEvals',1000,'MaxIter',1000,'TolX',1e-2);
[Xopt,FunVal,EF,output] = fminsearch('OF_MFBP2',Xo,options);
```

FP	$\boldsymbol{x}^{*T}$	$u(\boldsymbol{x}^*)$	iter	OFE
151	[273.373 507.51 0.673232]	-0.15099	91	163
501	[273.291 507.078 0.673642]	-0.12368	94	175
1001	[272.493 505.297 0.675872]	-0.12365	83	153
50	[5.0e+12 3.7e+12 3.4e+10]	-1	120	377
51	[273.373 507.51 0.673232]	-0.15099	91	163

iter: number of iterations.
OFE: number of objective function evaluations = WinSpice simulations.

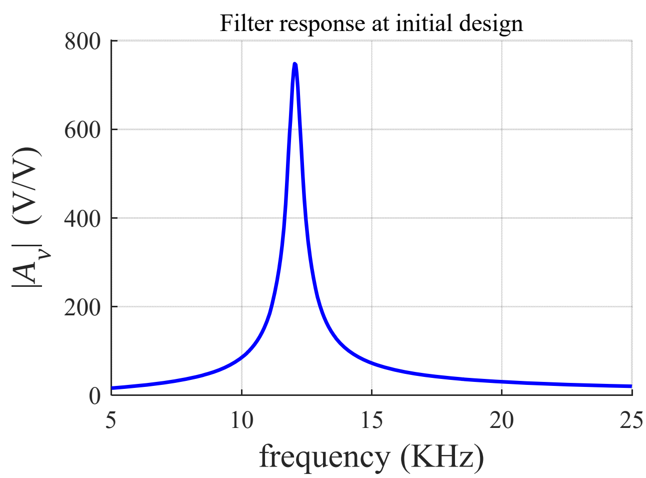
It is seen that the case with 50 frequency points fails because the error function at the resonant frequency is not computed in the objective function.
Using `'TolX'= 1e-8` and FP = 501, Nelder-Mead requires much more iterations and function evaluations: $\boldsymbol{x}^* = [269.718 \quad 497.799 \quad 0.684895]^T$, $u(\boldsymbol{x}^*) = -0.1247$, iter = 496, OFE = 891.

B) Using Gradient-Based Methods (gradients calculated by finite central differences, `h = 1e-5`)

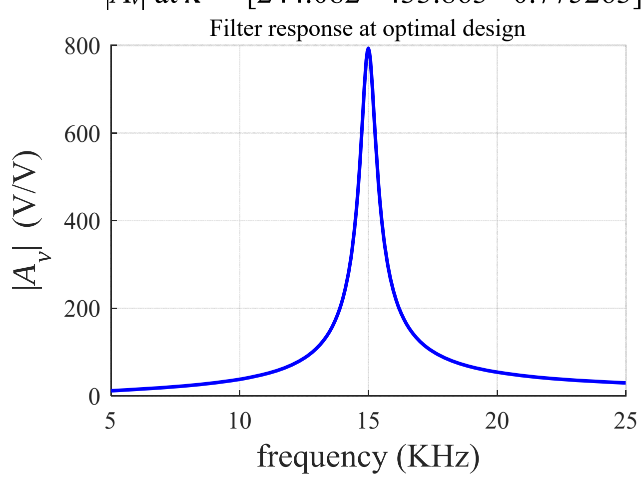
```
MaxIter = 1000; epsg = 1e-8; epsx = 1e-8;
```

FP = 501

Method	$\boldsymbol{x}^{*T}$	$u(\boldsymbol{x}^*)$	iter	OFE
Steepest Descent	[244.082 433.863 0.775263]	-0.13169	12	387
Conjugate Gradient	[243.651 434.58 0.77494]	-0.13005	5	195
Quasi-Newton (BFGS)	[244.152 434.33 0.773367]	-0.10364	4	155



$|A_v|$ at $\boldsymbol{x}^* = [244.082 \quad 433.863 \quad 0.775263]^T$



[1] S. Franco, *Design with Operational Amplifiers and Analog Integrated Circuits*. New York, NY: McGraw-Hill, 1988, pp. 130-131.