Circuit Parameter Extraction using Classical Optimization Methods

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Outline

- Nominal parameter extraction
- Basic definitions
- Non-linear least squares formulation to parameter extraction (PE)
- General *p*-th norm fomulations to PE
- Example
- Local minima in parameter extraction (PE)
- Statistical parameter extraction

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Nominal Parameter Extraction (PE)

It is assumed that

- The topology of the circuit and the component types are already selected by the designer and are fixed
- There is already available a reasonable starting point for PE

Additionally, for nominal PE it is assumed that

- The parameters to be extracted are not subject to statistical fluctuations, i.e., manufacturing tolerances are neglected

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Basic Definitions

- The parameters to be extracted are restricted to a region X of valid parameters
- $\boldsymbol{x} \in X \subseteq \Re^n$ represent the n parameters to be extracted (optimization variables) of the electronic circuit
- $\boldsymbol{z} \in \Re^m$ represent the m pre-assigned parameters of the electronic circuit (usually fixed)
- Vector $\boldsymbol{\psi}$ contains all the independent variables
- The electronic circuit responses are denoted by $\boldsymbol{R} \in \Re^r$ where r is the number of responses of interest
- $\boldsymbol{R}^{\mathrm{t}} \in \Re^r$ is the vector of target responses

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LS Formulation to Parameter Extraction

- The problem of parameter extraction can be formulated as a nonlinear least-squares problem

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} \left\| \boldsymbol{R}(\boldsymbol{x}) - \boldsymbol{R}^{\mathrm{t}} \right\|_2^2$$

where $\boldsymbol{x}^*$ contains the extracted parameters

- $\boldsymbol{x}^*$ is the solution that makes $\boldsymbol{R}(\boldsymbol{x}^*)$ as close as possible to $\boldsymbol{R}^{\mathrm{t}}$
- In a PE problem, we usually want to make zero the error between the circuit response and the target,

$$\boldsymbol{e}(\boldsymbol{x}) = \boldsymbol{R}(\boldsymbol{x}) - \boldsymbol{R}^{\mathrm{t}}$$

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General *p*-th Norm Formulations to PE

- The problem of circuit design can be expressed as a *p*-th norm minimization problem

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} \left\| \boldsymbol{e}(\boldsymbol{x}) \right\|_p$$

- l_1 formulation (Manhattan)

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} \sum_{i=1}^{k} \left| e_i(\boldsymbol{x}) \right|$$

- l_2 formulation (Least Squares or Euclidean)

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} \left\| \boldsymbol{e}(\boldsymbol{x}) \right\|_2^2 = \arg\min_{\boldsymbol{x}} \boldsymbol{e}(\boldsymbol{x})^T \boldsymbol{e}(\boldsymbol{x}) = \arg\min_{\boldsymbol{x}} \sum_{i=1}^{k} e_i^2(\boldsymbol{x})$$

- Chebyshev formulation

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} \max_{i} \{\cdots \quad |e_i(\boldsymbol{x})| \quad \cdots\}$$

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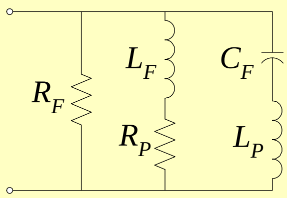
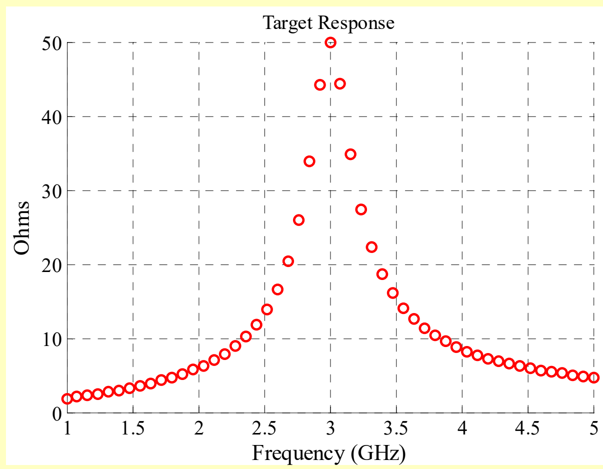
PE Example – Problem Description

Find R_F, L_F and C_F such that $|Z_{in}|$ is as close as possible to the target impedance magnitude shown below



$R_P = 0.5\Omega$ and
$L_P = 0.1\text{nH}$
(they are fixed)



PE Example – Problem Formulation

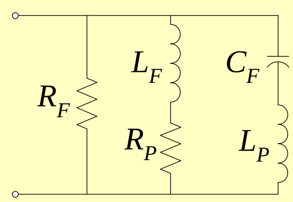
$\boldsymbol{x} = [R_F(\Omega) \quad L_F(\text{nH}) \quad C_F(\text{pF})]^{\text{T}}$

$\boldsymbol{z} = [R_P(\Omega) \quad L_P(\text{nH})]^T = [0.5 \quad 0.1]^{\text{T}}$

$\boldsymbol{R}^{\text{t}}$: target response

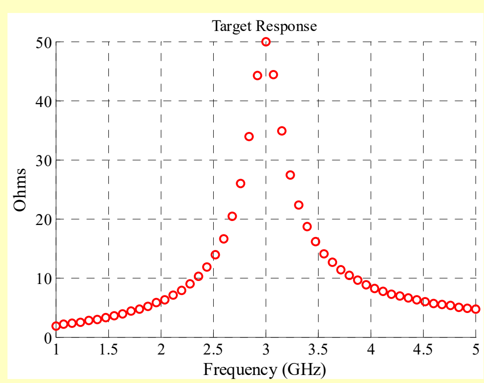
$\boldsymbol{\psi} = [1\text{GHz} \quad \ldots \quad 5\text{GHz}]$

$\boldsymbol{\psi} \in \Re^{51}$



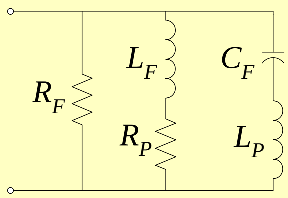
$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} \left\| \boldsymbol{R}(\boldsymbol{x}) - \boldsymbol{R}^{\text{t}} \right\|$$

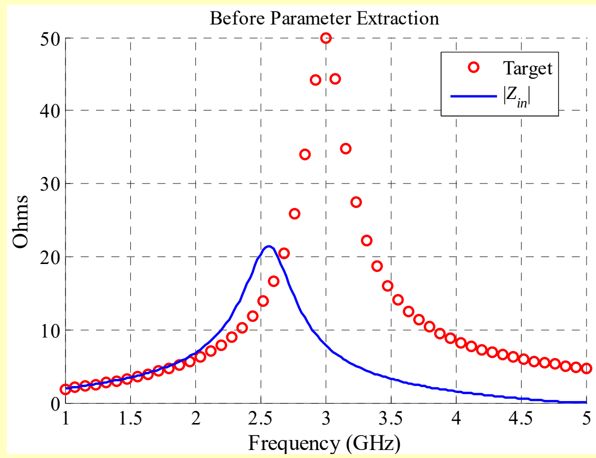
$$\boldsymbol{R}(\boldsymbol{x}) = |\boldsymbol{Z}_{in}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\psi})|$$



PE Example – Starting Point

$\boldsymbol{x}_0 = [50 \quad 0.2683 \quad 10.4938]^T$





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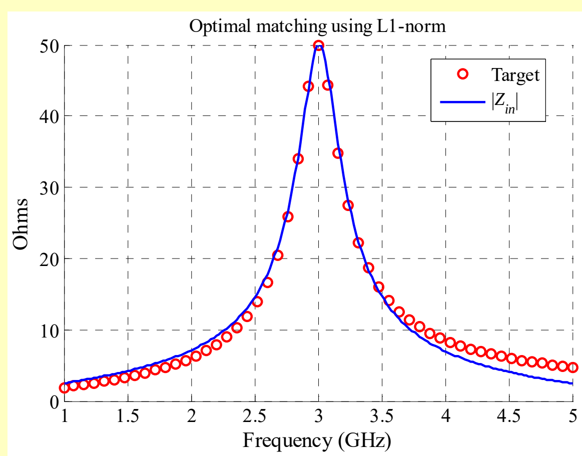
PE Example – Solution A: Manhattan Norm

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} \left\| \boldsymbol{R}(\boldsymbol{x}) - \boldsymbol{R}^t \right\|_1$$

$$\boldsymbol{x}^* = [113.5570 \quad 0.3536 \quad 6.1751]^T$$

- Using Nelder-Mead method:
- Iterations: 189
- Circuit evaluations needed: 335



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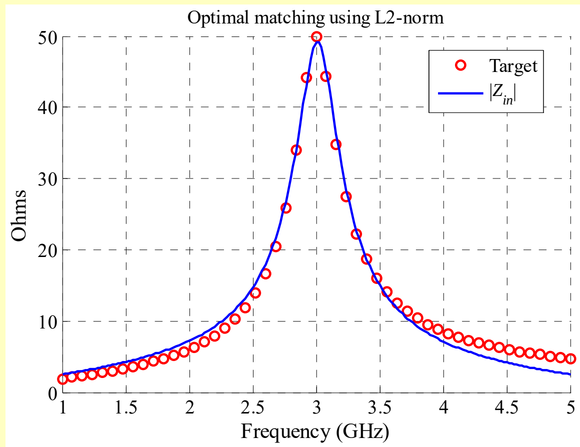
PE Example – Solution B: Euclidean Norm

$$x^* = \arg\min_{x} \left\| R(x) - R^{t} \right\|_2$$

$$x^* = [105.6749 \quad 0.3582 \quad 6.1074]^T$$

- Using Nelder-Mead method
- Iterations: 112
- Circuit evaluations needed: 200



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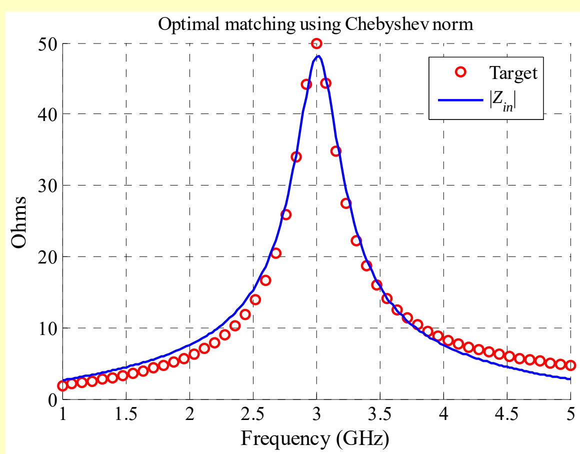
PE Example – Solution C: Chebyshev Norm

$$x^* = \arg\min_{x} \left\| R(x) - R^{t} \right\|_\infty$$

$$x^* = [92.8499 \quad 0.3735 \quad 5.9067]^T$$

- Using Nelder-Mead method
- Iterations: 190
- Circuit evaluations needed: 338



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Local Minima in PE

- There are usually many local minima in the PE problem
- Some of the local minima can be good solutions
- A "poor" or "bad" local minimum is a solution such that

$$\left\| \boldsymbol{R}(\boldsymbol{x}^{*}) - \boldsymbol{R}^{\mathrm{t}} \right\|_{\infty} > \varepsilon_{\mathrm{PE}}$$

  where ε_{PE} is the maximum acceptable error in matching the responses
- There are some advanced strategies to escape from poor local minima

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Statistical PE

- It consists of randomly selecting a different starting point when a poor local minimum is found
- The new starting point is calculated by perturbing the original starting point by some amount
- Several strategies can be followed to calculate this perturbation

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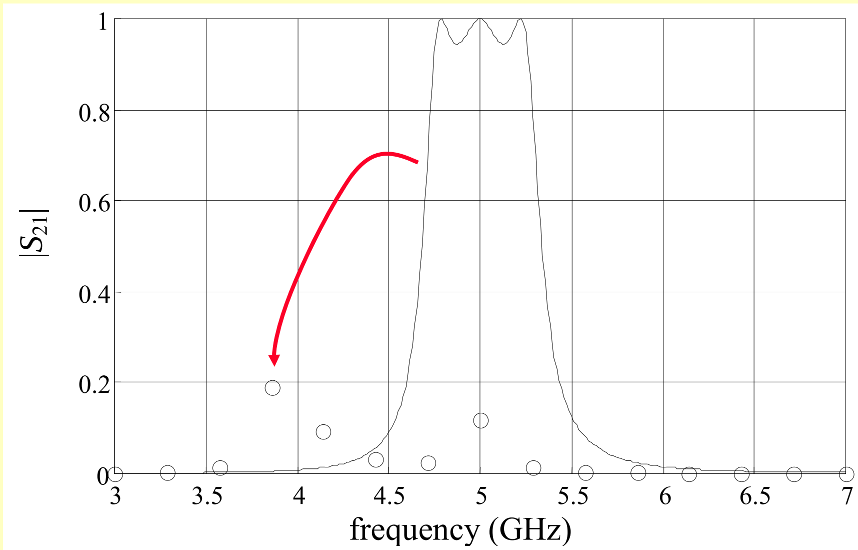
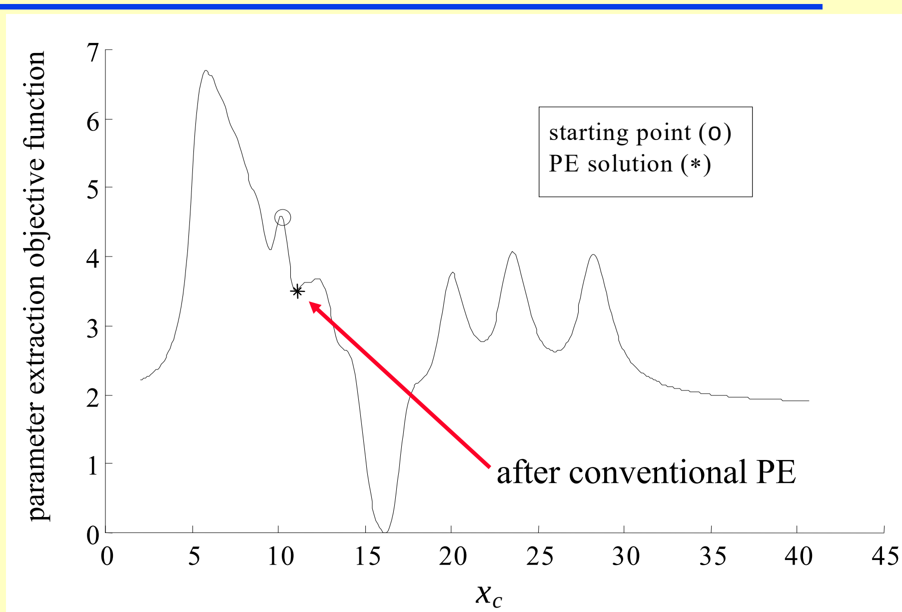
Illustrating the Need of Statistical PE

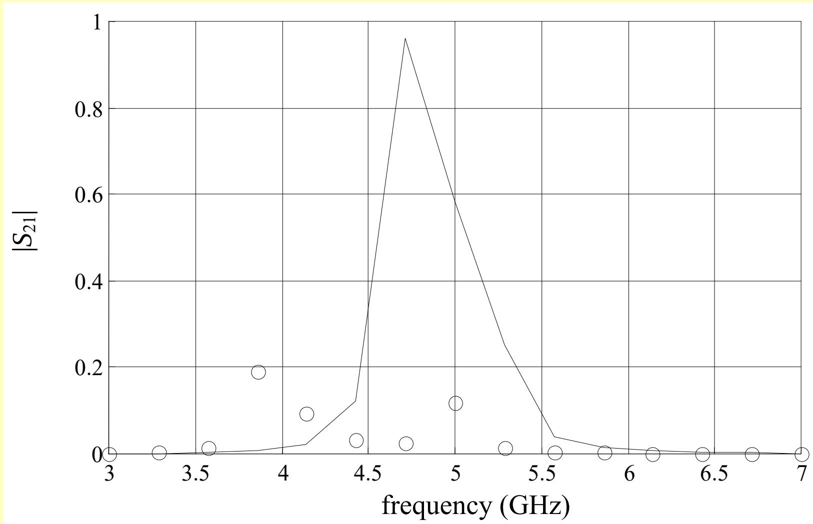


Illustrating the Need of Statistical PE (cont)



Illustrating the Need of Statistical PE (cont)

after conventional PE



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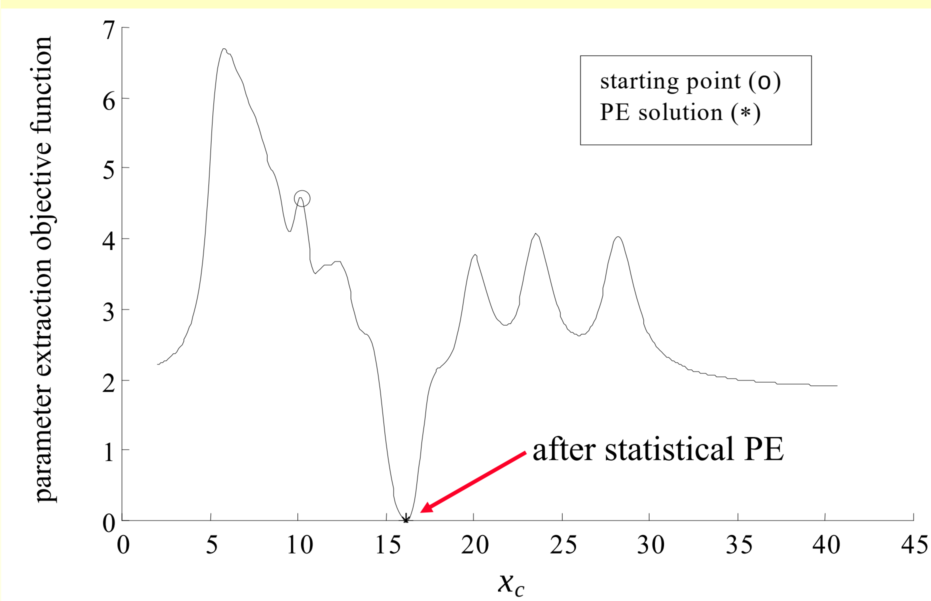
Illustrating the Need of Statistical PE (cont)



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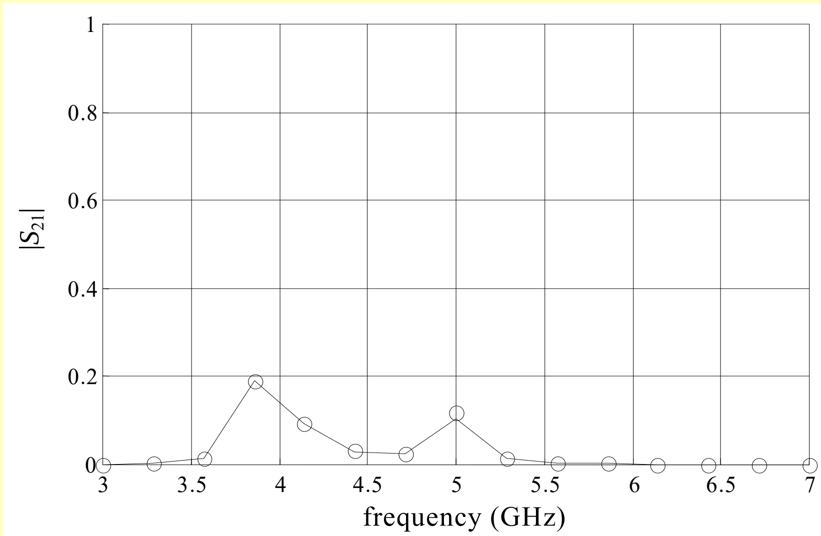
Illustrating the Need of Statistical PE (cont)

after statistical PE



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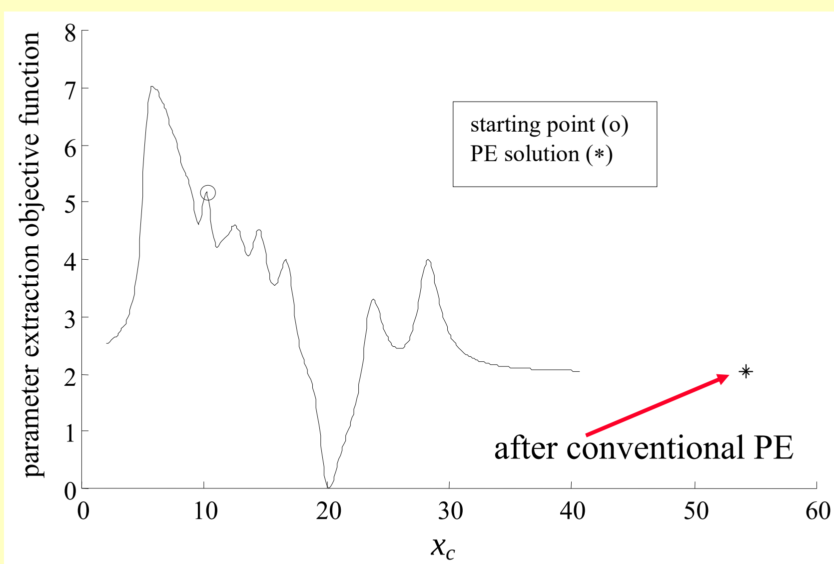
Illustrating the Need of Statistical PE (cont)



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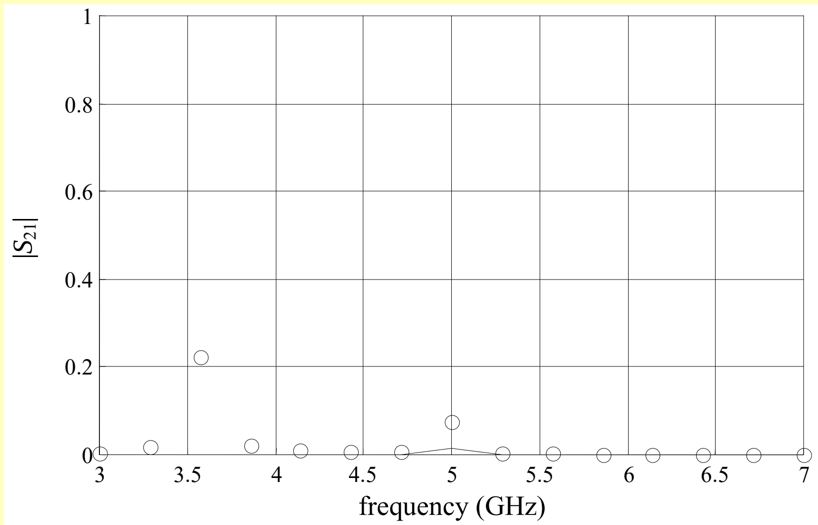
Illustrating the Need of Statistical PE (cont)

after conventional PE



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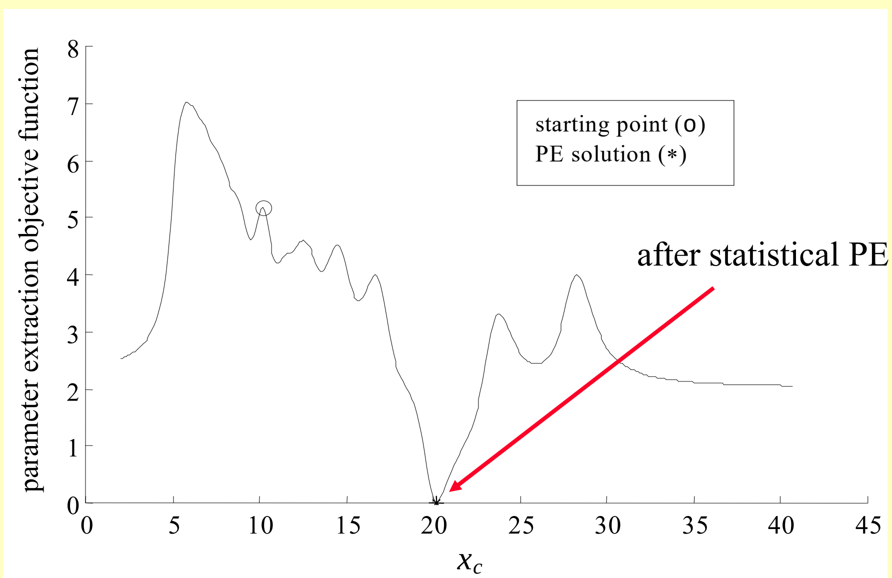
Illustrating the Need of Statistical PE (cont)



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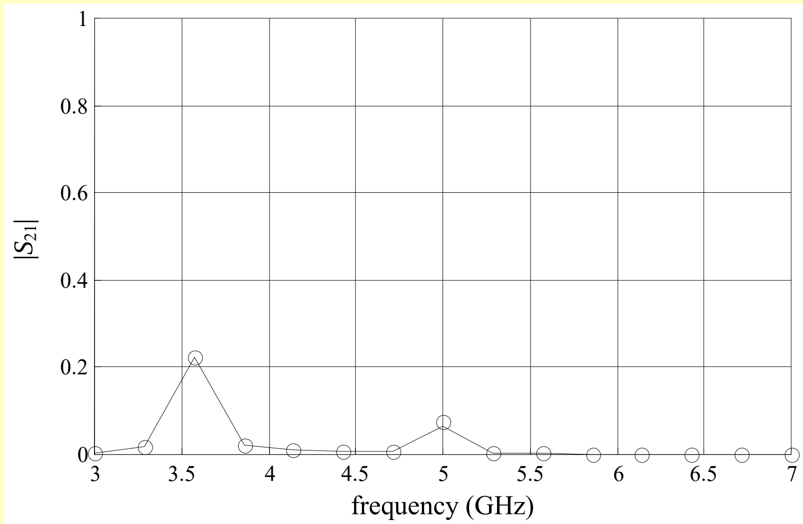
Illustrating the Need of Statistical PE (cont)

after statistical PE



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A Basic Statistical PE Algorithm

$\boldsymbol{x}^* = \text{SPE}(\boldsymbol{R}, \boldsymbol{R}^{\text{t}}, \boldsymbol{x}_0)$
$\boldsymbol{R}: \Re^n \rightarrow \Re^r$; $\boldsymbol{x}_0, \boldsymbol{x}^* \in \Re^n$; $\boldsymbol{R}^{\text{t}} \in \Re^r$

begin

$k = 0$, **set** $\varepsilon_{\text{PE}}, N_{\text{a}}, \alpha$

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} \left\| \boldsymbol{R}(\boldsymbol{x}) - \boldsymbol{R}^{\text{t}} \right\|$$

while $\dfrac{\left\| \boldsymbol{R}(\boldsymbol{x}^*) - \boldsymbol{R}^{\text{t}} \right\|_\infty}{\left\| \boldsymbol{R}^{\text{t}} \right\|_\infty} > \varepsilon_{\text{PE}} \quad \wedge \quad k < N_{\text{a}}$

$$\boldsymbol{x}_0 = \boldsymbol{x}_0 .(\boldsymbol{1} + \alpha \boldsymbol{r})$$

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} \left\| \boldsymbol{R}(\boldsymbol{x}) - \boldsymbol{R}^{\text{t}} \right\|$$

$$k = k + 1$$

end

end

N_{a}: Maximum number of attempts to escape from a poor local minimum

$\boldsymbol{r} \in \Re^n$: Random vector with element values between −1 and +1

$0 < \alpha < 1$: Maximum relative perturbation

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