Circuit Parameter Extraction using Classical Optimization Methods
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March 27, 2019

Outline

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- Basic definitions
- Non-linear least squares formulation to parameter extraction (PE)
- General $p$-th norm formulations to PE
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- Statistical parameter extraction
Nominal Parameter Extraction (PE)

It is assumed that

- The topology of the circuit and the component types are already selected by the designer and are fixed
- There is already available a reasonable starting point for PE

Additionally, for nominal PE it is assumed that

- The parameters to be extracted are not subject to statistical fluctuations, i.e., manufacturing tolerances are neglected

Basic Definitions

- The parameters to be extracted are restricted to a region $X$ of valid parameters
- $\mathbf{x} \in X \subseteq \mathbb{R}^n$ represent the $n$ parameters to be extracted (optimization variables) of the electronic circuit
- $\mathbf{z} \in \mathbb{R}^m$ represent the $m$ pre-assigned parameters of the electronic circuit (usually fixed)
- Vector $\mathbf{\psi}$ contains all the independent variables
- The electronic circuit responses are denoted by $\mathbf{R} \in \mathbb{R}^r$ where $r$ is the number of responses of interest
- $\mathbf{R}^t \in \mathbb{R}^r$ is the vector of target responses
LS Formulation to Parameter Extraction

- The problem of parameter extraction can be formulated as a nonlinear least-squares problem

\[ x^* = \arg \min_x \| R(x) - R^t \|_2 \]

where \( x^* \) contains the extracted parameters

- \( x^* \) is the solution that makes \( R(x^*) \) as close as possible to \( R^t \)

- In a PE problem, we usually want to make zero the error between the circuit response and the target,

\[ e(x) = R(x) - R^t \]

General \( p \)-th Norm Formulations to PE

- The problem of circuit design can be expressed as a \( p \)-th norm minimization problem

\[ x^* = \arg \min_x \| e(x) \|_p \]

- \( l_1 \) formulation (Manhattan)

\[ x^* = \arg \min_x \sum_{i=1}^k |e_i(x)| \]

- \( l_2 \) formulation (Least Squares or Euclidean)

\[ x^* = \arg \min_x \| e(x) \|_2 = \arg \min_x e(x)^T e(x) = \arg \min_x \sum_{i=1}^k e_i^2(x) \]

- Chebyshev formulation

\[ x^* = \arg \min_x \max_i \{ \cdots \ | e_i(x) | \ | \cdots \} \]
PE Example – Problem Description

Find $R_F$, $L_F$, and $C_F$ such that $|Z_{in}|$ is as close as possible to the target impedance magnitude shown below.

$R_p = 0.5\Omega$ and $L_p = 0.1\text{nH}$ (they are fixed)

PE Example – Problem Formulation

Let $x = [R_F(\Omega) \ L_F(\text{nH}) \ C_F(\text{pF})]^{T}$

$z = [R_p(\Omega) \ L_p(\text{nH})]^{T} = [0.5 \ 0.1]^{T}$

$R^t$ : target response

$\psi = [1\text{GHz} \ \ldots \ 5\text{GHz}]$

$\psi \in \mathbb{R}^{51}$

$x^* = \arg \min_x \| R(x) - R^t \|

R(x) = |Z_{in}(x, z, \psi)|
PE Example – Starting Point

\[ \mathbf{x}_0 = [50 \ 0.2683 \ 10.4938]^T \]

**Before Parameter Extraction**

**Target** | \[ \mathbf{Z}_{in} \]

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PE Example – Solution A: Manhattan Norm

\[ \mathbf{x}^* = \arg \min_{\mathbf{x}} \| \mathbf{R}(\mathbf{x}) - \mathbf{R}^t \| \]

\[ \mathbf{x}^* = [113.5570 \ 0.3536 \ 6.1751]^T \]

- Using Nelder-Mead method:
- Iterations: 189
- Circuit evaluations needed: 335

**Optimal matching using L1-norm**

**Target** | \[ \mathbf{Z}_{in} \]
PE Example – Solution B: Euclidean Norm

\[ x^* = \arg \min_x \| R(x) - R^T \|_2 \]
\[ x^* = [105.6749 \ 0.3582 \ 6.1074]^T \]

- Using Nelder-Mead method
- Iterations: 112
- Circuit evaluations needed: 200

![Optimal matching using L2-norm](image)

PE Example – Solution C: Chebyshev Norm

\[ x^* = \arg \min_x \| R(x) - R^T \|_\infty \]
\[ x^* = [92.8499 \ 0.3735 \ 5.9067]^T \]

- Using Nelder-Mead method
- Iterations: 190
- Circuit evaluations needed: 338

![Optimal matching using Chebyshev norm](image)
Local Minima in PE

- There are usually many local minima in the PE problem
- Some of the local minima can be good solutions
- A “poor” or “bad” local minimum is a solution such that
  \[ \| R(x^*) - R^* \|_\infty > \epsilon_{PE} \]
  where \( \epsilon_{PE} \) is the maximum acceptable error in matching the responses
- There are some advanced strategies to escape from poor local minima

Statistical PE

- It consists of randomly selecting a different starting point when a poor local minimum is found
- The new starting point is calculated by perturbing the original starting point by some amount
- Several strategies can be followed to calculate this perturbation
Illustrating the Need of Statistical PE

Illustrating the Need of Statistical PE (cont)
Illustrating the Need of Statistical PE (cont)

after conventional PE

Illustrating the Need of Statistical PE (cont)

starting point (○)
PE solution (•)

after statistical PE
Illustrating the Need of Statistical PE (cont)

\[ |S_{21}| \]

frequency (GHz)

Illustrating the Need of Statistical PE (cont)

parameter extraction objective function

starting point (o)
PE solution (*)

after conventional PE
Illustrating the Need of Statistical PE (cont)

after conventional PE

Illustrating the Need of Statistical PE (cont)

starting point (○)
PE solution (*)

after statistical PE
Illustrating the Need of Statistical PE (cont)

A Basic Statistical PE Algorithm

\[ x^* = \text{SPE}(R, R', x_0) \]

\[ R: \mathbb{R}^n \rightarrow \mathbb{R} ; \ x_0, x^* \in \mathbb{R}^n ; \ R' \in \mathbb{R}' \]

begin
  \[ k = 0, \ \text{set} \ \varepsilon_{\text{PE}}, N_a, \alpha \]
  \[ x^* = \arg \min_x \left| R(x) - R' \right| \]
  while \[ \frac{|R(x^*) - R'|}{|R'|} > \varepsilon_{\text{PE}} \land k < N_a \]
    \[ x_0 = x_0 (1 + \alpha r) \]
    \[ x^* = \arg \min_x \left| R(x) - R' \right| \]
  \[ k = k + 1 \]
end

\( N_a \): Maximum number of attempts to escape from a poor local minimum

\( r \in \mathbb{R}^n \): Random vector with element values between -1 and +1

\( 0 < \alpha < 1 \): Maximum relative perturbation