Circuit Design using Classical Optimization Methods

Dr. José Ernesto Rayas-Sánchez

1

Outline

- Nominal circuit design optimization
- Design parameters and optimization variables
- Independent variables
- Optimizable responses
- A general formulation to circuit design optimization
- Objective functions
- Minimax optimization to circuit design
- Constrained formulation to design optimization

Dr. J. E. Rayas-Sánchez 2

Nominal Design Optimization

For design optimization it is generally assumed that

- The topology of the circuit and the component types are already selected by the designer and are fixed
- There is already available a reasonable starting point

Additionally, for nominal design it is assumed that

- The design parameters are not subject to statistical fluctuations, i.e., manufacturing tolerances are neglected

Dr. J. E. Rayas-Sánchez 3

Design Parameters and Optimization Variables

- Not all the available design parameters in a circuit should be selected as optimization variables
- In practice, many of the available parameters in a circuit are considered fixed or pre-assigned
- In practice, the optimization variables are restricted to a region X of valid design parameter values
- $\boldsymbol{x} \in X \subseteq \Re^n$ represent the n optimization variables of the electronic circuit to be optimized
- $\boldsymbol{z} \in \Re^m$ represent the m pre-assigned parameters of the electronic circuit (usually fixed)

Dr. J. E. Rayas-Sánchez 4

Independent Variables

- Usually there is a number of independent variables in the circuit to be optimized
- Examples: frequency, time, bias voltages, etc.
- These independent variables define the region of operation of the electronic circuit
- Vector $\boldsymbol{\psi}$ contains all the independent variables

Optimizable Responses

- Circuit responses are typically obtained from an analytical model (Matlab, Excel, Mathcad, etc.) or from a CAD tool (circuit simulator, electromagnetic simulator, multi-physics simulator, etc.)
- The optimizable circuit responses are denoted by $\boldsymbol{R} \in \Re^r$ where r is the number of responses to be optimized
- In general, $\boldsymbol{R}$ depends on the optimization variables, the pre-assigned parameters, and the independent variables,

$$\boldsymbol{R} = \boldsymbol{R}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\psi})$$

- From the optimization perspective, the responses of interest can be treated as a multidimensional vector function, $\boldsymbol{R} : X \rightarrow \Re^r$

$$\boldsymbol{R} = \boldsymbol{R}(\boldsymbol{x})$$

A General Formulation to Nominal Design Opt.

- The desired response $\boldsymbol{R}^* \in \Re^r$ is expressed in terms of design specifications or design goals
- The problem of circuit design can be formulated as

$$\boldsymbol{x}^* = \arg \min_{\boldsymbol{x} \in X} U(\boldsymbol{R}(\boldsymbol{x}))$$

  where $\boldsymbol{x}^*$ is the optimal design, X is the feasible region, U is a suitable objective function, and hopefully $\boldsymbol{R}(\boldsymbol{x}^*) = \boldsymbol{R}^*$
- In general, the above problem corresponds to a constrained nonlinear programming problem
- If the same circuit model $\boldsymbol{R}(\boldsymbol{x})$ is used during optimization, a simpler notation can be used,

$$\boldsymbol{x}^* = \arg \min_{\boldsymbol{x} \in X} U(\boldsymbol{x})$$

The Objective Function U

- U is typically a combination of multiple objectives with conflicting criteria
- When designing electronic circuits,
  - inequality design specifications are usually incorporated in a minimax formulation
  - equality design specifications are either treated as equality constraints, or they are incorporated in a minimax formulation
  - box constraints are usually either neglected or incorporated through variable transformations

Minimax Formulation to Design Optimization

- An error function $e_k(\boldsymbol{x})$ is defined for each upper and/or lower specification for each response and independent variable sample (frequency, time, temperature, etc.)
- Each equality specification can be transformed to a couple of upper and lower specifications
- The minimax formulation to design optimization with no box constraints is

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}\in X} U(\boldsymbol{x}) = \arg\min_{\boldsymbol{x}} \max\{\ldots e_k(\boldsymbol{x})\ldots\}$$

where a negative value in the k-th error function, $e_k(\boldsymbol{x})$, implies that the corresponding design specification is satisfied, otherwise it is violated

Minimax Formulation to Design Opt. (cont)

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} \max\{\ldots e_k(\boldsymbol{x})\ldots\}$$

where

$$e_k(\boldsymbol{x}) = \begin{cases} \dfrac{R_k(\boldsymbol{x})}{S_k^{\text{ub}} + \varepsilon} - 1 & \text{for all } k \in I^{\text{ub}} \\ 1 - \dfrac{R_k(\boldsymbol{x})}{S_k^{\text{lb}} + \varepsilon} & \text{for all } k \in I^{\text{lb}} \\ \dfrac{|R_k(\boldsymbol{x}) - S_k^{\text{eq}}|}{\varepsilon} - 1 & \text{for all } k \in I^{\text{eq}} \end{cases}$$

- $R_k(\boldsymbol{x})$ is the k-th model response at point $\boldsymbol{x}$
- $S_k^{\text{ub}} > 0$ and $S_k^{\text{lb}} > 0$ are upper and lower bound specifications, and S_k^{eq} are equality specifications
- I^{ub}, I^{lb} and I^{eq} are index sets (not necessarily disjoint)
- ε is an arbitrary small positive number

Minimax Formulation to Design Opt. (cont)

- A minimax formulation attempts making all errors as negative as possible
- Since

$$U(\boldsymbol{x}) = \max\{\ldots e_k(\boldsymbol{x})\ldots\}$$

if $U(\boldsymbol{x}^*) < 0$, the optimal solution found satisfies all the specifications

if $U(\boldsymbol{x}^*) \geq 0$, at least one of the design specifications is being violated at the optimal solution found

Dr. J. E. Rayas-Sánchez 11

Constrained Formulation to Design Optimization

Constrained formulation with no box constraints

$$\boldsymbol{x}^* = \arg \min_{\boldsymbol{x}\in X} U(\boldsymbol{x}) = \arg \min_{\boldsymbol{x}} u(\boldsymbol{x}) + p(\boldsymbol{x}, r^{\mathrm{h}}, r^{\mathrm{g}})$$

where

$u(\boldsymbol{x})$ includes the main unbounded design goals (if available),

$p(\boldsymbol{x}, r^{\mathrm{h}}, r^{\mathrm{g}})$ includes the equality and inequality design specifications (or constraints) through penalty functions,

$$p(\boldsymbol{x}, r^{\mathrm{h}}, r^{\mathrm{g}}) = r^{\mathrm{h}}\|\boldsymbol{h}(\boldsymbol{x})\|_2^2 + r^{\mathrm{g}}\|\boldsymbol{G}(\boldsymbol{x})\|_2^2$$

$$G_j = \max\{0, g_j(\boldsymbol{x})\} \qquad j = 1,2,\ldots,I$$

and r^{h} and r^{g} are penalty coefficients

Dr. J. E. Rayas-Sánchez 12

Constrained Formulation to Design Opt. (cont)

Since $p(\boldsymbol{x}, r^{\mathrm{h}}, r^{\mathrm{g}})$ includes all the equality and inequality design specifications, and

$$p(\boldsymbol{x}, r^{\mathrm{h}}, r^{\mathrm{g}}) = r^{\mathrm{h}} \|\boldsymbol{h}(\boldsymbol{x})\|_2^2 + r^{\mathrm{g}} \|\boldsymbol{G}(\boldsymbol{x})\|_2^2$$

if $p(\boldsymbol{x}^*) = 0$ the optimal solution found satisfies all the design specifications (or constraints)

if $p(\boldsymbol{x}^*) > 0$ at least one design specification or constraint is being violated at the optimal solution found

Dr. J. E. Rayas-Sánchez 13

Mixed Formulation to Design Optimization

Constrained formulation with no box constraints

$$\boldsymbol{x}^* = \arg \min_{\boldsymbol{x} \in X} U(\boldsymbol{x}) = \arg \min_{\boldsymbol{x}} u(\boldsymbol{x}) + r^{\mathrm{v}} v(\boldsymbol{x}) + r^{\mathrm{h}} \|\boldsymbol{h}(\boldsymbol{x})\|_2^2$$

where

$u(\boldsymbol{x})$ includes the main unbounded design goals (if available),

$v(\boldsymbol{x})$ includes the inequality design specifications through a minimax formulation,

$$v(\boldsymbol{x}) = \max\{\ldots e_k(\boldsymbol{x}) \ldots\}$$

$\boldsymbol{h}(\boldsymbol{x})$ includes the equality design specifications through a penalty function

and r^{v} and r^{h} are used as scaling factors

Dr. J. E. Rayas-Sánchez 14

Minimizing *U* with Classical Opt. Methods

- "Classical" or "conventional" methods to solve

$$\boldsymbol{x}^* = \arg\min_{\boldsymbol{x}} U(\boldsymbol{R}(\boldsymbol{x}))$$

  include Line Search and Trust Region strategies, based on methods such as Conjugate Gradient, Newton and Quasi-Newton, etc.

- Methods that use only function evaluations are more suitable for problems that are very nonlinear or have many discontinuities (Search Methods)
- Methods that use derivative information are more effective when the function is continuous in the first (and second) derivatives (Gradient Methods)

Dr. J. E. Rayas-Sánchez 15