Circuit Design using Classical Optimization Methods

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Outline

- Nominal circuit design optimization
- Design parameters and optimization variables
- Independent variables
- Optimizable responses
- A general formulation to circuit design optimization
- Objective functions
- Minimax optimization to circuit design
- Constrained formulation to design optimization
Nominal Design Optimization

For design optimization it is generally assumed that

- The topology of the circuit and the component types are already selected by the designer and are fixed
- There is already available a reasonable starting point

Additionally, for nominal design it is assumed that
- The design parameters are not subject to statistical fluctuations, i.e., manufacturing tolerances are neglected

Design Parameters and Optimization Variables

- Not all the available design parameters in a circuit should be selected as optimization variables
- In practice, many of the available parameters in a circuit are considered fixed or pre-assigned
- In practice, the optimization variables are restricted to a region $X$ of valid design parameter values
- $x \in X \subseteq \mathbb{R}^n$ represent the $n$ optimization variables of the electronic circuit to be optimized
- $z \in \mathbb{R}^m$ represent the $m$ pre-assigned parameters of the electronic circuit (usually fixed)
Independent Variables

- Usually there is a number of independent variables in the circuit to be optimized
- Examples: frequency, time, bias voltages, etc.
- These independent variables define the region of operation of the electronic circuit
- Vector $\psi$ contains all the independent variables

Optimizable Responses

- Circuit responses are typically obtained from an analytical model (Matlab, Excel, Mathcad, etc.) or from a CAD tool (circuit simulator, electromagnetic simulator, multi-physics simulator, etc.)
- The optimizable circuit responses are denoted by $R \in \mathbb{R}^r$ where $r$ is the number of responses to be optimized
- In general, $R$ depends on the optimization variables, the pre-assigned parameters, and the independent variables,
  \[ R = R(x, z, \psi) \]
- From the optimization perspective, the responses of interest can be treated as a multidimensional vector function, $R : X \rightarrow \mathbb{R}^r$
  \[ R = R(x) \]

- The desired response \( R^* \in \mathbb{R}^r \) is expressed in terms of design specifications or design goals.
- The problem of circuit design can be formulated as
  \[
  x^* = \arg \min_{x \in \Lambda} U(R(x))
  \]
  where \( x^* \) is the optimal design, \( \Lambda \) is the feasible region, \( U \) is a suitable objective function, and hopefully \( R(x^*) = R^* \).
- In general, the above problem corresponds to a constrained nonlinear programming problem.
- If the same circuit model \( R(x) \) is used during optimization, a simpler notation can be used,
  \[
  x^* = \arg \min_{x \in \Lambda} U(x)
  \]

The Objective Function \( U \)

- \( U \) is typically a combination of multiple objectives with conflicting criteria.
- When designing electronic circuits,
  - inequality design specifications are usually incorporated in a minimax formulation.
  - equality design specifications are either treated as equality constraints, or they are incorporated in a minimax formulation.
  - box constraints are usually either neglected or incorporated through variable transformations.
Minimax Formulation to Design Optimization

- An error function $e_k(x)$ is defined for each upper and/or lower specification for each response and independent variable sample (frequency, time, temperature, etc.)
- Each equality specification can be transformed to a couple of upper and lower specifications
- The minimax formulation to design optimization with no box constraints is

$$x^* = \arg \min_{x \in \mathcal{X}} U(x) = \arg \min_{x} \max \{e_k(x)\}$$

where a negative value in the $k$-th error function, $e_k(x)$, implies that the corresponding design specification is satisfied, otherwise it is violated.

Minimax Formulation to Design Opt. (cont)

$$x^* = \arg \min_{x} \max \{e_k(x)\}$$

where

$$e_k(x) = \begin{cases} \frac{R_k(x)}{S_k^{ub}} - 1 & \text{for all } k \in I^{ub} \\ 1 - \frac{R_k(x)}{S_k^{lb}} & \text{for all } k \in I^{lb} \\ \frac{|R_k(x) - S_k^{eq}|}{\varepsilon} - 1 & \text{for all } k \in I^{eq} \end{cases}$$

- $R_k(x)$ is the $k$-th model response at point $x$
- $S_k^{ub} > 0$ and $S_k^{lb} > 0$ are upper and lower bound specifications, and $S_k^{eq}$ are equality specifications
- $I^{ub}$, $I^{lb}$ and $I^{eq}$ are index sets (not necessarily disjoint)
- $\varepsilon$ is an arbitrary small positive number
Minimax Formulation to Design Opt. (cont)

- A minimax formulation attempts making all errors as negative as possible

  \[ U(x) = \max\{e_k(x)\} \]

- Since

  if \( U(x^*) < 0 \), the optimal solution found satisfies all the specifications

  if \( U(x^*) \geq 0 \), at least one of the design specifications is being violated at the optimal solution found

Constrained Formulation to Design Optimization

Constrained formulation with no box constraints

\[ x^* = \arg \min_{x \in \mathcal{X}} U(x) = \arg \min_{x \in \mathcal{X}} u(x) + p(x, r^h, r^g) \]

where

- \( u(x) \) includes the main unbounded design goals (if available),

- \( p(x, r^h, r^g) \) includes the equality and inequality design specifications (or constraints) through penalty functions,

\[
p(x, r^h, r^g) = r^h \|h(x)\|_2^2 + r^g \|G(x)\|_2^2
\]

\[ G_j = \max\{0, g_j(x)\} \quad j = 1, 2, \ldots, I \]

and \( r^h \) and \( r^g \) are penalty coefficients
Constrained Formulation to Design Opt. (cont)

Since $p(x, r^h, r^g)$ includes all the equality and inequality design specifications, and

$$p(x, r^h, r^g) = r^h \|h(x)\|_2^2 + r^g \|G(x)\|_2^2$$

if $p(x^*) = 0$ the optimal solution found satisfies all the design specifications (or constraints)

if $p(x^*) > 0$ at least one design specification or constraint is being violated at the optimal solution found

Mixed Formulation to Design Optimization

Constrained formulation with no box constraints

$$x^* = \arg \min_{x \in X} U(x) = \arg \min_x u(x) + r^v v(x) + r^h \|h(x)\|_2^2$$

where

$u(x)$ includes the main unbounded design goals (if available),

$v(x)$ includes the inequality design specifications through a minimax formulation,

$$v(x) = \max \{\ldots e_k(x) \ldots\}$$

$h(x)$ includes the equality design specifications through a penalty function

and $r^v$ and $r^h$ are used as scaling factors
Minimizing $U$ with Classical Opt. Methods

- “Classical” or “conventional” methods to solve
  \[ x^* = \arg\min_x U(R(x)) \]
  include Line Search and Trust Region strategies, based on methods such as Conjugate Gradient, Newton and Quasi-Newton, etc.

- Methods that use only function evaluations are more suitable for problems that are very nonlinear or have many discontinuities (Search Methods)

- Methods that use derivative information are more effective when the function is continuous in the first (and second) derivatives (Gradient Methods)