Frequency-Domain Analysis of Transmission Line Circuits
(Part 3)

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Outline

- Differential transmission lines
- Common mode signaling
- Differential mode signaling
- Mode conversion
- Even and odd modes
- 2-coupled lossless transmission line theory
- Termination techniques
- Differential or Mixed-Mode S-parameters
Differential Transmission Lines

For high data rates, differential signaling is more used due to:

- Radiation is reduced (cancellation of fields)
- Receiver rejects signals that are common to both lines (high CMRR at the receiver)
- Signal voltage amplitudes can be smaller

![Diagram of differential transmission lines]

(M. Resso, 2005)

Electromagnetic Fields in a Microstrip Line

![Diagram of electromagnetic fields in a microstrip line]

(M. Resso, 2005)
Common Mode Signaling

As data rates go up, frequencies increase, lines become antennas (both send and receive) and corrupt the communication (BER, crosstalk, etc)

E and H fields addition outside

Differential Mode Signaling

Using differential excitations (differential transmission lines), most of the outside electromagnetic field cancels

E and H fields cancellation outside
Mode Conversion

- Is produced by asymmetries in the differential pairs
- Can cause a differential signal to be converted to a common mode signal (radiation, crosstalk, etc.)

![Differential Stimulus Differential Response](image)

Even Mode and Odd Mode

- Practical differential pairs operate at even and odd modes simultaneously
- Even mode – excited in phase with equal amplitudes
- Odd mode – driven 180° out of phase with equal amplitudes
Even Mode and Odd Mode (cont)

Differential Mode
- Magnetic Field
- Electric Field

Common Mode
- Magnetic Field
- Electric Field

(P. Huray and S. Pytel, 2009)

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Differential Signaling for High-Speed Links

- Differential signaling can operate at much higher data rates
- High speed links operating in excess of ~1 Gb/s use differential signaling (e.g. Infiniband, PCI-Express).
- In fact, differential signals are already used for high speed clocks

(D. Heck 2002)
Lossless Transmission Lines

\[ V(z) = V_0^+e^{-j\beta z} + V_0^-e^{+j\beta z} \]
\[ I(z) = I_0^+e^{-j\beta z} + I_0^-e^{+j\beta z} \]
\[ \beta = \omega \sqrt{LC} \]
\[ Z_0 = \sqrt{\frac{L}{C}} \quad v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \]
\[ dV(z) = -(j\omega LI) \]
\[ dI(z) = -(j\omega CV) \]
\[ -\frac{d}{dz} \begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} 0 & Z_L \\ Y_C & 0 \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix} \]
\[ Z_L = j\omega L \quad Y_C = j\omega C \]

2-Coupled Lossless Symmetrical TLs

\[ \frac{dV_1}{dz} = -j\omega L_s I_1 - j\omega L_m I_2 \]
\[ \frac{dI_1}{dz} = -j\omega (C_s + C_m)V_1 + j\omega C_m V_2 \]
\[ \frac{dV_2}{dz} = -j\omega L_m I_1 - j\omega L_s I_2 \]
\[ \frac{dI_2}{dz} = j\omega C_m V_1 - j\omega (C_s + C_m)V_2 \]
2-Coupled Lossless Symmetrical TLs (cont)

\[
\begin{align*}
\frac{dV_1}{dz} &= -j\omega L_s I_1 - j\omega L_m I_2 \\
\frac{dV_2}{dz} &= -j\omega L_m I_1 - j\omega L_s I_2 \\
\frac{dI_1}{dz} &= -j\omega (C_s + C_m)V_1 + j\omega C_m V_2 \\
\frac{dI_2}{dz} &= +j\omega C_m V_1 - j\omega (C_s + C_m)V_2
\end{align*}
\]

\[
V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad I = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}
\]

\[
Y_c = j\omega \begin{bmatrix} (C_s + C_m) & -C_m \\ -C_m & (C_s + C_m) \end{bmatrix}
\]

\[
Z_L = j\omega \begin{bmatrix} L_s & L_m \\ L_m & L_s \end{bmatrix}
\]

\[-\frac{d}{dz} \begin{bmatrix} V \\ I \end{bmatrix} = \begin{bmatrix} 0 & Z_L \\ Y_c & 0 \end{bmatrix} \begin{bmatrix} V \\ I \end{bmatrix}\]

LC Matrices of 2-Coupled TLs

\[
C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} (C_s + C_m) & -C_m \\ -C_m & (C_s + C_m) \end{bmatrix}
\]

\[
L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} = \begin{bmatrix} L_s & L_m \\ L_m & L_s \end{bmatrix}
\]

\[
Z_0 = ? \\
v_p = ?
\]
Even Mode in 2-Coupled Symmetrical TLs

\[
\frac{dV_1}{dz} = -j\omega L_s I_1 - j\omega L_m I_2 \quad \frac{dV_2}{dz} = -j\omega L_m I_1 - j\omega L_s I_2 \\
\frac{dI_1}{dz} = -j\omega (C_s + C_m)V_1 + j\omega C_m V_2 \quad \frac{dI_2}{dz} = j\omega C_m V_1 - j\omega (C_s + C_m)V_2
\]

Since \( V_1 = V_2 \) and \( I_1 = I_2 \)

\[
\frac{dV_1}{dz} = -j\omega (L_s + L_m)I_1 \quad \frac{dI_1}{dz} = -j\omega C_s V_1
\]

The effective \( L \) and \( C \) are

\[
L_{eff} = L_s + L_m \quad C_{eff} = C_s
\]

Hence

\[
Z_{0-even} = \frac{L_s + L_m}{C_s} \quad v_{p-even} = \frac{1}{\sqrt{(L_s + L_m)C_s}}
\]

**Even Mode in 2-Coupled Symmetrical TLs (cont)**

\( V_1 = V_2 \); \( I_1 = I_2 \)

\[
Z_{0-even} = \frac{L_s + L_m}{C_s} \quad v_{p-even} = \frac{1}{\sqrt{(L_s + L_m)C_s}}
\]

\( Z_{0-even} \) is the characteristic impedance of one of the conductors when the coupled line is operated in even mode

(R. Mellitz 2003)
Odd Mode in 2-Coupled Symmetrical TLs

\[
\frac{dV_1}{dz} = -j\omega L_s I_1 - j\omega L_m I_2 \quad \frac{dV_2}{dz} = -j\omega L_m I_1 - j\omega L_s I_2
\]

\[
\frac{dI_1}{dz} = -j\omega (C_s + C_m) V_1 + j\omega C_m V_2 \quad \frac{dI_2}{dz} = j\omega C_m V_1 - j\omega (C_s + C_m) V_2
\]

Since \( V_1 = -V_2 \) and \( I_1 = -I_2 \)

\[
\frac{dV_1}{dz} = -j\omega (L_s - L_m) I_1 \quad \frac{dI_1}{dz} = -j\omega (C_s + 2C_m) V_1
\]

The effective \( L \) and \( C \) are

\[
L_{\text{eff}} = L_s - L_m \quad C_{\text{eff}} = C_s + 2C_m
\]

Hence

\[
Z_{0-\text{odd}} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \quad v_{p-\text{odd}} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}
\]

Odd Mode in 2-Coupled Symmetrical TLs (cont)

\[
V_1 = -V_2 \quad I_1 = -I_2
\]

\[
Z_{0-\text{odd}} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \quad v_{p-\text{odd}} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}
\]

\( Z_{0-\text{odd}} \) is the characteristic impedance of one of the conductors when the coupled line is operated in odd mode

(R. Mellitz 2003)
Distributed Capacitances in Coupled Lines

\[ C_{\text{eff}} = C_s \]

\[ C_{\text{eff}} = C_s + 2C_m \]

(D. M. Pozar, Microwave Engineering, Wiley, 2005)


**Z₀ and \( v_p \) for Even and Odd Modes**

- If \( Z₀ \) is the characteristic impedance of each isolated conductor, and \( v_p \) is the propagation velocity or wave speed in each isolated conductor

  \[
  Z₀ = \sqrt{\frac{L_s}{C_s}} \quad v_p = \frac{1}{\sqrt{L_s C_s}}
  \]

- Since

  \[
  Z_{0\text{-even}} = \sqrt{\frac{L_s + L_m}{C_s}} \quad v_{p\text{-even}} = \frac{1}{\sqrt{(L_s + L_m)C_s}}
  \]

  \[
  Z_{0\text{-odd}} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \quad v_{p\text{-odd}} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}
  \]

- Then

  \[
  Z_{0\text{-odd}} < Z₀ < Z_{0\text{-even}} \quad v_{p\text{-odd}} < v_{p\text{-even}} < v_p
  \]

**Termination Techniques**

- A single-resistor termination for each conductor is not enough for coupled lines
- Proper terminations are needed to avoid reflections in both even and odd modes
- The most common termination networks are the T and Pi configurations
T-Termination

\[ R_c = \frac{1}{2} (Z_{0\text{-even}} - Z_{0\text{-odd}}) \]

\[ R_d = Z_{0\text{-odd}} \]

Pi-Termination

\[ R_c = Z_{0\text{-even}} \]

\[ R_d = \frac{2Z_{0\text{-even}} Z_{0\text{-odd}}}{Z_{0\text{-even}} - Z_{0\text{-odd}}} \]
Adding Buffers for Differential Signaling

Differential to Differential

\[ T \text{ or } \pi \text{ network} \]

Differential to Single-ended

\[ T \text{ or } \pi \text{ network} \]

Single-ended to Single-ended

\[ T \text{ or } \pi \text{ network} \]

S-Parameters for Two-Coupled Lines

Port 1
Port 3
Port 2
Port 4

Four-port single-ended device

Stimulus

Far End Crosstalk (FEXT)
Near End Crosstalk (NEXT)
Insertion Loss (IL)
Return Loss (RL)

Response

\( S_{11}, S_{12}, S_{13}, S_{14} \)
\( S_{21}, S_{22}, S_{23}, S_{24} \)
\( S_{31}, S_{32}, S_{33}, S_{34} \)
\( S_{41}, S_{42}, S_{43}, S_{44} \)
Single-ended to Balanced S-Parameters

Single-ended

Balanced

Stimulus

Response

Naming Convention:

$S_{\text{mode res. mode stim. port res. port stim.}}$

(M. Resso, 2005)

Balanced, Differential or Mixed-Mode S-Param.